Global Real Rates: A Secular Approach*

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Abstract

The current environment is characterized by low real rates and by policy rates close to or at their effective lower bound in all major financial areas. We analyze these unusual economic conditions from a secular perspective using data on aggregate consumption, wealth and asset returns. Our present-value approach decomposes fluctuations in the global consumption-to-wealth ratio over long periods of time and show that this ratio anticipates future movements of the global real risk-free rate. Our analysis identifies two historical episodes where the consumption-to-wealth ratio declined rapidly below its historical average: in the roaring 1920s and again in the exuberant 2000s. Each episode was followed by a severe global financial crisis and depressed real rates for an extended period of time. Our empirical estimates suggest that the world real rate of interest is likely to remain low or negative for an extended period of time.

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1 Introduction

The current macroeconomic environment remains a serious source of worry for policymakers and of puzzlement for academic economists. Global real rates, which have been trending down since the 1980s, are at historical lows across advanced economies, both at the short and long end of the term structure. Policy rates are close to or at their Effective Lower Bound in all major financial areas. Figures 1 and 2 report the nominal policy rates and long yields for the U.S., the Eurozone, the U.K. and Japan since 1980. Large amounts of wealth are invested at zero or negative yields.[1]

Despite the aggressive global monetary policy treatment administered in advanced economies, levels of economic activity have only recently normalized, suggesting a decline in the *natural interest rate*, i.e. the real interest rate at which the global economy would reach its potential output.

Understanding whether natural rates are indeed low, for how much longer, and the source of their decline has become a first-order macroeconomic question. More generally, understanding what drives movements in real rates in the long run is one of the most intriguing questions in macroeconomics.

In a celebrated speech given at the International Monetary Fund in 2013, five years after the onset of the Global Financial Crisis, Summers (2015) ventured that we may have entered an age of '*secular stagnation*', i.e. an era where output remains chronically below its potential, or equivalently real rates remain above their natural rate. Not coincidentally, the secular stagnation hypothesis was first voiced by Hansen (1939), ten years after the onset of the Great Depression. Whether we are indeed in a period of '*secular stagnation*', and why, remains to be elucidated. Several hypotheses have been put forward for a secular decline in real rates: a global savings glut (Bernanke (2005)), i.e. a rise in desired savings due to the fast growth of emerging market economies with relatively underdeveloped financial sectors; a decline in investment rates due to a lack of investment opportunities, potentially because of a technological slowdown (Gordon (2012)); a decline in the relative price of investment goods such as machine and robots, which depresses the level of investment; a decline in the rate of population growth; an increase in the demand for safe assets (Caballero, Farhi and Gourinchas (2015)); or the long

[1] According to FitchRatings (2017), the total amount of fixed-rate sovereign debt trading at negative yields was \$9.7 trillion as of December 2017, slightly below its peak of \$11.7 trillion in June 2016.

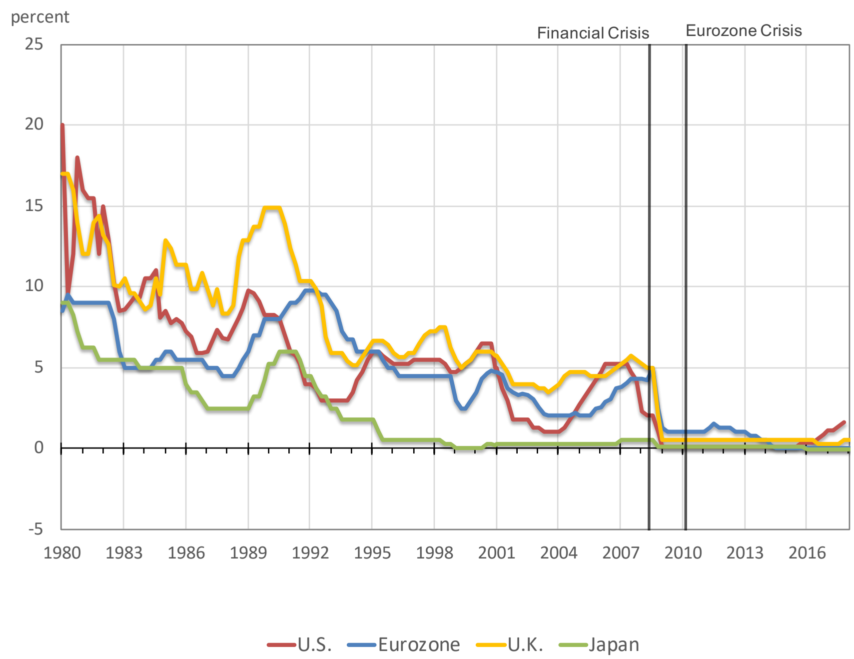

Figure 1: Policy Rates, 1980-2017. Sources: U.S.: Federal Funds Official Target Rate; Eurozone: until Dec. 1998, Germany's Lombard Rate. After 1998, ECB Marginal Rate of Refinancing Operations; U.K.: Bank of England Base Lending Rate; Japan: Bank of Japan Target Call Rate. Data from Global Financial Database.

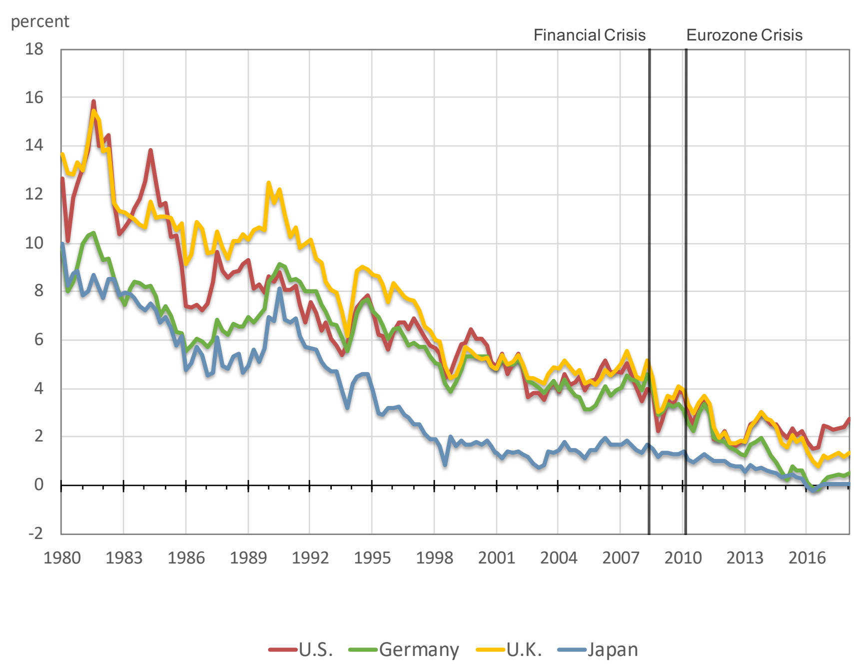

Figure 2: Yields on Long Term Debt. 1980-2017. Sources: U.S.: 10-year bond constant maturity rate; Germany: 10-year benchmark bond; U.K.: 10-year government bond yield; Japan: 10-year government bond yield. Data from Global Financial Database

shadow cast by a major financial crisis and the slow process of deleveraging associated with it (Lo and Rogoff (2015)).

This paper is an empirical contribution to this debate. We take a 'secular view,' building from recent contributions in macroeconomic history from Jordà, Schularick and Taylor (2016) or Piketty and Zucman (2014a) that have made a number long macroeconomic time-series available to researchers. Our focus is to analyze movements in real rates since 1870 in the U.S., and since 1920 for a group of four advanced economies: the U.S., the U.K., Germany and France.

A long historical perspective is important. As noted by others before us (e.g. Hamilton, Harris, Hatzius and West (2016) for a sample of 17 countries or Vlieghe (2017) for the U.K.), real rates have historically fluctuated a lot, and the current low real rates are not unprecedented when seen from an historical perspective. Figure 3 reports estimates of the annualized ex-post 3-months real interest rates for the United States, the United Kingdom, Germany and France. The figure illustrates that real short rates were high and declining from 1870 to WW1, reached low and volatile levels in the interwar period, remained low in the post WWII period, until the early 1980s when they rose sharply before gradually declining again.

To understand the evolution of global real rates over such long periods of time, we propose an approach based on standard present-value decompositions often used in the modern finance literature (Campbell and Shiller (1988), Lettau and Ludvigson (2001) and more recently Binsbergen, Jules and Koijen (2010)). We apply this long run decomposition to more than a century of historical data. Under very modest assumptions, this decomposition establishes that the global consumption-to-wealth ratio encodes information about future risk-free rates, future risk-premia and/or future consumption growth. The intuition is quite straightforward: times when consumption is high relative to wealth must be followed either by lower consumption growth (the numerator), or higher returns on wealth (the denominator). Higher returns on wealth can result either from higher real risk-free rates, or from higher risk-premia. Because consumption growth and risk-premia are difficult to predict, we expect the consumption-to-wealth ratio to contain information mostly about current and future real rates. This intuition turns out to be correct: empirically, the consumption-to-wealth ratio, reported on Figure 4,

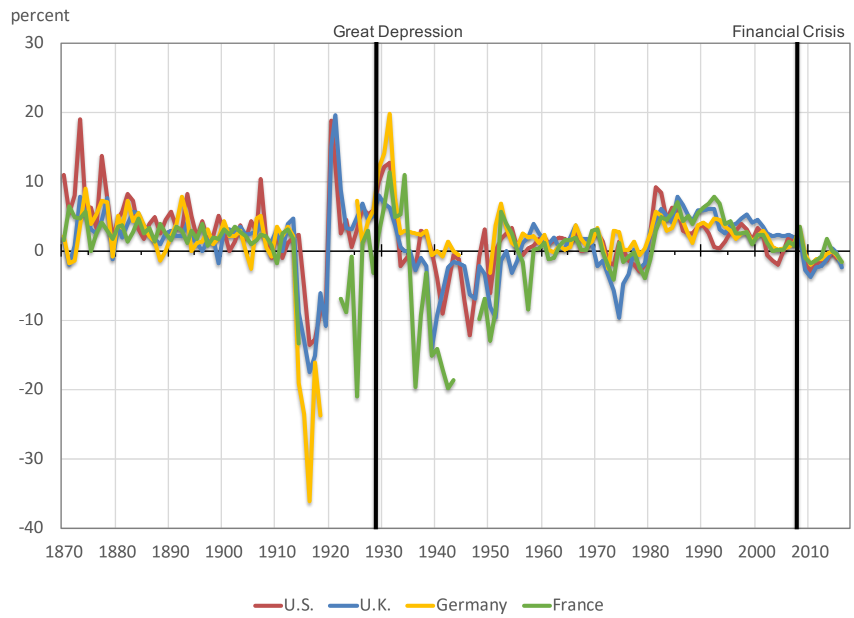

Figure 3: 3-Months Ex-Post Real Rates (p.a.), 1870-2015. Sources: Jordà et al. (2016). Ex-post real rates are constructed as the nominal interest rate on 3-months Treasuries minus realized CPI inflation.

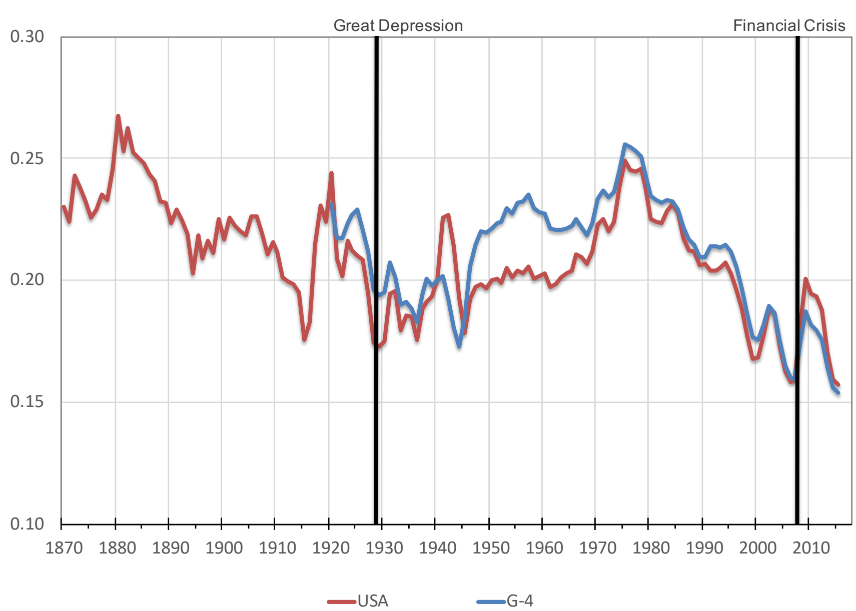

Figure 4: Consumption-to-Private Wealth Ratio, 1870-2015, United States and G-4 (U.S., U.K., Germany and France). Sources: Jordà et al. (2016), Piketty et al. (2017) and WDI.

is an excellent predictor of the low-frequency movements in global real rates. In particular, our estimation suggests that real rates will remain low for an extended period of time: our baseline empirical estimates predict an average real short-term rate of -2.35% between 2015 and 2025 for the U.S., and of -3.1% for the U.S., U.K., Germany and France combined.

Establishing the importance of the global consumption-to-wealth ratio for predictive purposes is an important result. But this brings an immediate question: why does the consumption-to-wealth ratio fluctuate over time? Returns, consumption and wealth are all endogenous. This is an identification question and as such it is much harder to answer. Yet, our decomposition does provide some useful hints. While consumption growth and risk premia are difficult to forecast, their present value still contributes to movements in the consumption-to-wealth ratio, alongside the present value of risk-free rates. However, different fundamental shocks will imply different patterns of co-movements between the different components. Consider for instance, the impact of productivity shocks. A decline in productivity growth is often claimed as a reason behind the recent decline in real rates. Standard Euler equation reasoning suggests that lower productivity growth should be associated with lower real rates, with the strength of that effect controlled by the intertemporal elasticity of substitution. Since lower productivity growth also means lower consumption growth, it follows that productivity shocks will two opposite effects on the consumption-to-wealth ratio: lower real rates will tend to decrease the ratio; lower future consumption growth to increase it. By looking at the empirical pattern of co-movements between the different components of the ratio, we can hope to recover some information about the key drivers. We consider four such shocks: productivity growth, demographics (specifically population growth), deleveraging shocks and risk appetite. The can think of the first two as *'macro' shocks*. The latter two are *'finance' shocks* that have been the object of much recent focus in the literature.

Our results indicate that both macro and financial forces play a role. For the former, we do find evidence that demographic and productivity shocks play a small but significant role, especially at lower frequencies. On the financial side, we find that two historical episodes stand out, during which the consumption-to-wealth ratio was abnormally low: in the 1930s and since 2000. In both cases the decline in the consumption-to-wealth ratio was largely driven by a rapid increase in wealth during the financial boom that preceded a major financial crisis: the Great Depression in 1929 and the Great Financial Crisis in 2008. Our decomposition suggests that low real rates in the aftermath of these crisis was driven in part by a pro-tracted and still on-going deleveraging process associated with the financial cycle.

The next section presents our empirical decomposition, based on the present-value relationship similar to Campbell and Shiller (1991) and Lettau and Ludvigson (2001). Section 3 proposes some elements of theory. Section 4 estimates the present value model while Section 5 presents predictive regressions based on our framework. Section 6 then presents an estimation of the model using simulated method of moments.Section 7 concludes.

Review of the Literature. This is a placeholder for the literature review. It will include:

- A discussion of papers that estimate the natural rate: Holston, Laubach and Williams (2017); Laubach and Williams (2003, 2016); Barro and Sala-i Martin (1990); Farooqui (2016); Hamilton et al. (2016); Del Negro, Giannone, Giannoni and Tambalotti (2017); Pescatori and Turunen (2015)

- A discussion of paper papers on the decline in global real rates due to heterogenous risk aversions: Barro, Fernandez-Villaverde, Levintal and Mollerus (2014); Gourinchas, Rey and Govillot (2010); Hall (2016).

- A discussion of papers on 'secular stagnation': Caballero et al. (2015); Eggertsson and Mehrotra (2014); Eggertsson, Mehrotra, Singh and Summers (2015); Hansen (1939); Sajedi and Thwaites (2016); Summers (2015)

- A discussion of papers on financial crises and deleveraging: Lo and Rogoff (2015); Jordà et al. (2016); Schularick and Taylor (2012)

- A discussion of papers on the present value approach: Binsbergen et al. (2010); Campbell and Shiller (1991); Gourinchas and Rey (2007); Lettau and Ludvigson (2001); Lustig, Van Nieuwerburgh and Verdelhan (2013)

- A discussion of papers on integrated macroeconomic accounts and historical data, Piketty et al. (2017), Piketty and Zucman (2014a), Jordà et al. (2016).

2 The Dynamics of the Consumption-to-Wealth Ratio and Natural Rates

We are interested in understanding the drivers behind the low-frequency movements in the global natural rate of interest. Our key methodological contribution consists in connecting expected current and future global risk-free rates to fluctuations in the consumption-to-wealth ratio, using a simple Present Value model (PV). This Present Value model can be derived under a minimal set of assumptions, which we make explicit, and builds from the generic implications of the global resource constraint.

2.1 The Global Resource Constraint: A Present Value Relation

Since we are interested in understanding global returns, the relevant unit of analysis is the global (i.e. world) resource constraint. Let $\bar{W}_t$ denote the beginning-of-period global *total* private wealth, composed of the sum of global private wealth W_t and global human wealth H_t. Private wealth W_t consists of financial assets, including private holdings of government assets, and non-financial assets such as land and real estate. Human wealth H_t consists of the present value of current and future non-financial income.[2] Total private wealth evolves over time according to:

$$\bar{W}_{t+1} = \bar{R}_{t+1}(\bar{W}_t - C_t). \tag{1}$$

In equation (1), C_t denotes global private consumption expenditures and $\bar{R}_{t+1}$ the gross return on total private wealth between periods t and $t + 1$. All variables are expressed in real terms. Equation (1) is simply an accounting identity that holds period-by-period.

We add some structure to this identity by observing that, in almost any sensible income-fluctuation and portfolio-choice model, optimizing households aim to smooth consumption. This tends to stabilize the consumption-to-wealth ratio, i.e. the average propensity to consume. For instance, if consumption decisions are taken by an infinitely lived representative-household maximizing welfare defined as the

[2] See appendix A for a detailed data description. We focus on *private* wealth and consumption, and not *national* wealth or consumption, which includes government's net wealth and consumption.

expected present value of a logarithmic period utility $u(C) = \ln C$, then the consumption-to-wealth ratio is constant and equal to the discount rate of the representative agent.

Assumption 1. *The (log) consumption-to-wealth ratio is stationary and we denote $\overline{\ln(C/W)} < 0$ its unconditional mean.*

If the (log) average propensity to consume out of wealth is stationary, equation (1) can be log-linearized around its steady state value. Denote $0 < \rho_w \equiv 1 - \exp(\overline{\ln(C/W)}) < 1$, Δ the difference operator so that $\Delta x_{t+1} \equiv x_{t+1} - x_t$, and $\bar{r}_{t+1} \equiv \ln \bar{R}_{t+1}$, the continuously compounded real return on wealth.[3] Following the same steps as Campbell and Mankiw (1989) or Lettau and Ludvigson (2001), we obtain the following log-linearized expression (ignoring an unimportant constant term) :[4]

$$\ln C_t - \ln \bar{W}_t \backsimeq \rho_w \left(\ln C_{t+1} - \ln \bar{W}_{t+1} + \bar{r}_{t+1} - \Delta \ln C_{t+1}\right). \tag{2}$$

Equation (2) indicates that if today's consumption-to-wealth ratio is high, then either (a) tomorrow's consumption-to-wealth ratio will be high, or (b) the return on wealth between today and tomorrow $\bar{r}_{t+1}$ will be high, or (c) aggregate consumption growth $\Delta \ln C_{t+1}$ will be low.

Since $\rho_w < 1$, Equation (2) can be iterated forward under the usual transversality condition, $\lim_{j\to\infty} \rho_w^j (\ln C_{t+j} - \ln W_{t+j} = 0$. Denoting $\mathbb{E}_t[.]$ the conditional expectations at time t, we obtain the following ex-ante Present Value (PV) relation:

$$\ln C_t - \ln \bar{W}_t \backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(\bar{r}_{t+s} - \Delta \ln C_{t+s}\right). \tag{3}$$

To understand equation (3), suppose that the (log) consumption-to-wealth ratio is currently higher than its unconditional mean, $\overline{\ln(C/W)}$. Since $\ln(C/\bar{W})$ is stationary, this ratio must be expected to decline in the future. Equation (3) states that this decline can occur in one of two ways. First, expected future return on total private wealth $\bar{r}_{t+s}$ could be high. This would increase future wealth, i.e. the

[3]In steady state, it follows from equation (??) that $\Gamma/\bar{R} = 1 - \exp \overline{\ln(C/W)} \equiv \rho_w$, where Γ denotes the steady state growth rate of total private wealth and $\bar{R}$ the steady state gross return on total private wealth. The requirement that $\rho_w < 1$ is equivalent to $\bar{R} > \Gamma$, i.e. that the real interest rate exceeds the gross rate of the economy.

[4]See appendix B for a full derivation.

denominator of $C/\bar{W}$. Alternatively expected future aggregate consumption growth could be low. This would reduce the numerator of $C/\bar{W}$.

At this stage, it is important to emphasize that the assumptions needed to derive equation (3) are *minimal*: we start from a global budget constraint, equation (1), which is an accounting identity. We then perform a log-linearization under very mild stationarity conditions, and impose a transversality condition that rules out paths where wealth grows without bounds in relation to consumption. Equation (3)'s main economic message is that today's average propensity to consume out of wealth *encodes* relevant information about future consumption growth and/or future returns to wealth.

2.2 From the Present Value Relation to Empirics

Before we can exploit this expression empirically, we need to make two important adjustments. First, as mentioned above, *total* private wealth is the sum of *private* wealth W_t and *human* wealth H_t. The former is -partly- observable, from existing wealth surveys and historical integrated macroeconomic accounts such as Piketty and Zucman (2014a) or Jordà et al. (2016). The latter is not, and often needs to be estimated with the help of auxiliary assumptions on the stochastic process of the discount factor and/or future labor income. For instance, Lettau and Ludvigson (2001) approximate human wealth with current aggregate labor income and construct a proxy for the left hand side of equation (??) by estimating a co-integration relation between consumption, financial wealth and labor income. Lustig et al. (2013) follow a different approach. Using data on bond yields and stock returns, they estimate an affine Stochastic Discount Factor (SDF) consistent with no-arbitrage. They then solve for total wealth $\bar{W}$ as the market value of a claim to current and future aggregate consumption expenditures, evaluated at the estimated SDF. An advantage of their method is that it does not require any wealth data. A disadvantage is that one needs to put a lot of faith on the particular SDF that is estimated.

We follow a different route. Specifically, denote $\omega_t = W_t/\bar{W}_t$ the aggregate share of private wealth in total private wealth. If ω_t is stationary around a mean ω, we can approximate (log) total wealth as $\ln \bar{W}_t = \omega \ln W_t + (1-\omega) \ln H_t$, and the log return on total wealth as $\bar{r}_t = \omega r_t^w + (1-\omega) r_t^h$

where r_t^w (resp. r_t^h) denotes the log return on private wealth (resp. human wealth).[5] Substituting these expressions into equation (3) and re-arranging we obtain:

$$
\begin{aligned}
0 \backsimeq & \ \omega\left(\ln C_t - \ln W_t - \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^w - \Delta \ln C_{t+s}\right)\right) \\
& +(1-\omega)\left(\ln C_t - \ln H_t - \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^h - \Delta \ln C_{t+s}\right)\right).
\end{aligned} \tag{4}
$$

This equation makes clear that if the Present Value relation holds for private wealth (the first term of the equation), then it holds for human wealth (the second term of the equation), and vice versa. More generally, we can re-arrange this expression into:

$$
\ln C_t - \ln W_t \backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^w - \Delta \ln C_{t+s}\right) + \varepsilon_t. \tag{5}
$$

where ε_t represents an error term induced by ignoring human wealth that can be expressed as:

$$
\varepsilon_t = (1-\omega)\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^h - r_{t+s}^w\right) + (1-\omega)\left(\ln H_t - \ln W_t\right)
$$

This error term is small when expected returns on human and private wealth are similar, and when the ratio of private to human wealth is stationary (since we are ignoring constants). Equation equation (??) states that the consumption to private wealth ratio may be high if either (a) future returns on private wealth are high; (b) future consumption growth is low; (c) the error term is high, which can occur either if the returns on human wealth r^h are high relative to the returns on private wealth r^w or when human wealth is high relative to private financial wealth. Because human wealth and the return on human wealth are difficult to observe, we will simply assume that the error term is negligible and ignore it

Assumption 2. *The Present Value Relation equation* (??) *holds for total private wealth. Equivalently:* $\varepsilon_t \approx 0$.

[5] The gross return on human wealth may be defined as $R_{t+1}^h = \exp\left(r_{t+1}^h\right) = H_{t+1}/(H_t - WL_t)$ where WL_t denotes aggregate non-financial income in period t. See Campbell (1996).

The recent evidence on the decline in the labor share (see e.g. Karabarbounis and Neiman (2014)) and on the increase in income ineqality (see e.g. Piketty and Saez (2003)) could invalidate these assumptions: in recent years, the return on private wealth r^w may have exceeded the return on human wealth r^h. Similarly, it is possible growing wealth inequality imply that human wealth H declined relative to private wealth W. This could translate into downward trends in the consumption-to-private wealth ratio C/W, even if consumption-to-total wealth $C/\bar{W}$ remained stationary. However, our focus on long run data should mitigate these concerns. For instance, as documented by Piketty and Saez (2003), the dynamics of income inequality over the last century is characterized by large and persistent fluctuations, but no historical trend: income and wealth inequality in the U.S. are today close to what they were at the beginning of the XXth century.

The second adjustment is to realize that the return on private wealth r^w_{t+1} can always be decomposed into the sum of a real risk-free rate r^f_t (known at time t) and an excess return er^w_{t+1} according to: $r^w_{t+1} = r^f_t + er^w_{t+1}$. While we can construct reasonably accurate estimates of the real risk free rate r^f_t, it is harder to measure the excess return on private wealth er^w_{t+1}, or equivalently, the return to private wealth r^w_{t+1}. This is so since private wealth includes a variety of traded financial assets such as portfolio holdings, whose return could reasonably be approximated, but also non-financial or non-traded assets such as real estate, agricultural land and equipments whose returns are more difficult to measure. Our approach consists in proxying the excess return on private wealth with a vector of N excess returns on existing assets $\tilde{\mathbf{er}}_{t+1}$, such as equity or bond returns, as follows:

$$r^w_{t+1} = r^f_t + \nu' \tilde{\mathbf{er}}_{\mathbf{t+1}}, \tag{6}$$

where ν is an $N \times 1$ vector that will be estimated.

Substituting (6) into the present value relation (5), we obtain our fundamental representation:

$$\begin{aligned} \ln C_t - \ln W_t \backsimeq \; & \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s r^f_{t+s-1} && + \nu' \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \mathbf{rp}_{\mathbf{t+s-1}} && - \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \Delta \ln C_{t+s} && + \varepsilon_t. \\ \equiv \; & cw^f_t && + cw^{rp}_t && + cw^c_t && + \varepsilon_t, \end{aligned} \tag{7}$$

where $\mathbf{rp_t} = \mathbb{E}_t[\tilde{\mathbf{er}}_{\mathbf{t+1}}]$ is the $N \times 1$ vector of one period-ahead risk premia. This equation states that the consumption-to-private wealth ratio C/W should contain information either about (a) future safe rates r_t^f, (b) future risk premia, $\mathbf{rp_t}$, or (c) future aggregate consumption growth, $\Delta \ln C_t$. The terms cw_t^f, cw_t^{rp} and cw_t^c summarize the relative contributions of the risk free rate, the risk premia and consumption growth, respectively.

Inspecting (7), we make two final observations. First, under a particular data generating process, it is relatively straightforward to estimate the present value terms cw_t^f and cw_t^c. However, since the vector of loadings of private wealth excess returns on market returns ν is unknown, we cannot infer the contribution of risk-premia cw_t^{rp} without additional assumptions. We will estimate ν so as to minimize the residuals in equation (7). This way of proceeding opens up the possibility that our estimate of the risk-premium component may be contaminated by the human capital component error term ε_t. For instance, if we proxy excess returns on private wealth with equity excess returns only, $N = 1$ and the OLS estimate of ν satisfies $\hat{\nu} = \nu + cov\,(\varepsilon, cw^{rp})/\text{var}(cw^{rp})$ where $cw^{rp} = \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \tilde{er}_{t+s}$ is the estimated present value of future excess equity returns. The possible bias on $\hat{\nu}$ attributes to the risk-premium component the part of the variation in $\ln C - \ln W$ coming from fluctuations in human wealth that co-moves with the equity risk premium.

Second, and importantly for us, it is well-known that aggregate consumption expenditures is close to a random walk, while the risk premium is volatile and difficult to predict. Therefore, we expect equation (7) to connect the aggregate consumption-to-wealth ratio to the expected path of future real risk-free rates r_{t+s}^f via cw_t^f. The last step of the argument is to realize that, under the generally admitted assumption that monetary policy aims to target the risk-free rate to the natural rate denoted r_t^*, $\mathbb{E}_t r_{t+s}^f = \mathbb{E}_t r_{t+s}^*$, and the risk free component can be expressed as:

$$cw_t^f = \mathbb{E}_t \sum_{s=1}^{\infty} \rho^w r_{t+s-1}^* = cw_t^*$$

In other words, we expect to recover from the behavior of the global consumption-to-wealth ratio information about the discounted path of future natural rates.[6]

[6]The assumption that $\mathbb{E}_t r_{t+s}^f = \mathbb{E}_t r_{t+s}^*$ could be violated if the economy is stuck at the Effective Lower Bound (See

3 Consumption-to-Wealth Ratio: Some Elements of Theory

Before we lay out our empirical strategy in more details, we discuss how different fundamental shocks can affect returns, consumption and the consumption/wealth ratio. We then show how, under more restrictive assumptions, a full characterization of the consumption-to-wealth ratio can be obtained.

3.1 Present Value Relation and Structural Shocks

Our fundamental representation (7) does not provide a *causal* decomposition: the risk-free, risk-premium and consumption growth components cw^i are endogenous and interdependent. Different fundamental shocks will imply different patterns of co-movements between risk-free rates, risk premia and consumption growth. We begin by fleshing out the implications for our fundamental representation equation (**??**) by considering productivity shocks, demographic shocks, deleveraging shocks and changes in risk appetite.

3.1.1 Productivity shocks.

To focus on the purest implications of productivity shocks, consider a closed endowment economy with no government, so consumption C is equal to the endowment Y. Equation (7) takes the form:

$$\ln C_t - \ln W_t \backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^w - \Delta \ln Y_{t+s} \right). \tag{8}$$

Suppose that total output growth is expected to decline in the future, $\Delta \ln Y_{t+s} < 0$, holding output growth unchanged at other periods. For a given path of expected future returns, this should exert *upward* pressure on the current consumption-to-wealth ratio. However, and this is the key insight, expected future returns will not remain constant. Faced with a future slowdown in output growth, households may want to save more today. This will depress expected returns, up to the point where consumption remains equal to output. The decline in expected returns will exert *downward* pressure

discussion below). In that case, $\mathbb{E}_t r_{t+s}^f \geqslant \mathbb{E}_t r_{t+s}^*$ and $cw_t^f \geqslant cw_t^*$ would provide an upper bound on the discounted path of future natural rates.

on the consumption-to-wealth ratio. Which of these two effects will dominate? The answer depends on whether the Intertemporal Elasticity of Substitution (IES) is above or below 1.

To see this mechanism explicitly, assume that the representative household has additively separable preferences over consumption, with a constant intertemporal elasticity of substitution (IES) $1/\gamma$ and discount rate ρ: $U_t = \mathbb{E}_t \sum_{s=0}^{\infty} e^{-\rho s} c_{t+s}^{1-\gamma}/(1-\gamma)$. The usual log-linearized Euler equation takes the following form (up to the second order):

$$\gamma \mathbb{E}_t \Delta \ln C_{t+1} = \mathbb{E}_t r_{t+1}^w - \rho + \frac{1}{2}\sigma_{z,t}^2,$$

where $\sigma_{z,t}^2$ denotes the conditional variance of $z_{t+1} = r_{t+1}^w - \gamma \Delta \ln Y_{t+1}$ at time t.

Denote $g_t = \Delta \ln Y_t$ the (exogenous) aggregate endowment growth, which coincides here with productivity, and $\sigma_{g,t}^2$ its conditional variance. The Euler equation can be solved for the expected return on wealth:

$$\mathbb{E}_t r_{t+1}^w = \rho + \gamma \mathbb{E}_t g_{t+1} - \frac{1}{2}\sigma_{z,t}^2. \tag{9}$$

This expression encodes precisely the extent to which the expected return on wealth needs to respond to changes in expected output growth so as to clear the goods market: if output growth is expected to increase by 1%, the expected return on private wealth must increase by $\gamma\%$. Substituting the Euler equation (9) into equation (8) and ignoring constants, one obtains:

$$\ln C_t - \ln W_t \backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left((\gamma - 1) g_{t+s} - \frac{1}{2}\sigma_{z,t+s-1}^2 \right). \tag{10}$$

It is immediate from equation (10) that whether the consumption-to-wealth ratio increases or decreases with output growth depends on the sign of $\gamma - 1$, i.e. on the relative strength of the substitution and income effects. If $\gamma > 1$, the IES is low and expected returns need to decline a lot in order to stimulate consumption growth. The impact of productivity changes on returns dominates and C/W co-moves *positively* with expected future productivity growth. If instead $\gamma < 1$, the IES is high and a modest decline in expected returns is sufficient to push consumption growth down. The direct impact of productivity growth dominates and C/W co-moves *negatively* with expected future productivity growth.[7]

Following similar steps, one can compute the various components cw_t^i as (up to some unimportant constants):

$$\begin{aligned}
cw_t^f &= \gamma \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(g_{t+s} - \frac{\gamma}{2} \sigma_{g,t+s-1}^2 \right) \\
cw_t^{rp} &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(\gamma cov_t(r_{t+s}^w, g_{t+s}) - \frac{1}{2} \sigma_{r,t+s-1}^2 \right) \\
cw_t^c &= -\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s g_{t+s},
\end{aligned}$$

where $\sigma_{r,t}^2$ is the conditional variance of the return on private wealth.

These expressions make clear that expected changes in future productivity have direct opposite effects on the risk free and consumption components, scaled by the inverse of the IES, while the risk premium component only depends on the present value of co-movements between the return on wealth and output growth. In the limit where there is no time-variation in second moments, the risk premium component is constant while the risk free and consumption components are perfectly negatively correlated, and $var(cw^f)/var(cw^c) = \gamma$.[8]

3.1.2 Demography.

Consider now the effect of demographic forces on the consumption-to-wealth ratio. To do so, decompose total consumption growth $\Delta \ln C_{t+1}$ into per capita consumption growth $\Delta \ln c_{t+1}$, and population growth n_{t+1}: $\Delta \ln C_{t+1} = \Delta \ln c_{t+1} + n_{t+1}$. Substituting into (7) we obtain:

$$\begin{aligned}
\ln c_t - \ln w_t &\backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^w - \Delta \ln c_{t+s} - n_{t+s} \right), \\
&\backsimeq cw_t^f + cw_t^{cp} + cw_t^n
\end{aligned}$$

[7] In the special case where $\gamma = 1$, the consumption-to-wealth ratio is constant and independent from $\mathbb{E}_t g_{t+s}$.

[8] Of course, risk premia may not be constant. For most models of interest, however, the correlation between excess returns on wealth and consumption growth is relatively small, indicating a small role for the macroeconomic risk premium that we measure here.

where w_t denotes real private wealth per capita. cw_t^{cp} and cw_t^n represent respectively the contribution of future growth in consumption per capita and future population growth. It is obvious from this expression that an expected decline in population growth ($\mathbb{E}_t n_{t+s} < 0$) has a direct and positive effect on $c - w$, given a path of returns and consumption per capita. The effect of a decline in population growth on equilibrium returns, and therefore the indirect effect on the consumption-to-wealth ratio, is more complex. As population growth slows down, capital per worker increases, pushing down the marginal product of capital and r^w. At the same time, a decline in population growth increases the dependency ratio, i.e. the ratio of retirees to working-age population. Since retirees save less than workers, aggregate savings may decline, pushing interest rates up. Finally, increases in life expectancy, which have been a major driver of demographic developments in the last century, lead to increased saving and therefore a decline in interest rates.

The empirical evidence as well as calibrated overlapping generation models such as see Carvalho, Ferrero and Nechio (2016) generally indicate that slowdowns in population growth are associated with increased savings.[9] This should push down expected returns and the consumption-to-wealth ratio, with the strength of that effect, again, controlled by the IES $1/\gamma$. In this case, as in the case of productivity shocks, the impact of demographic shocks will have opposite effects on the risk-free and population growth components: $corr(cw_t^f, cw_t^n) < 0$. We can measure the direct effect of demographic shocks on the consumption-to-wealth ratio by constructing an empirical counterpart to $cw_t^n = -\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s n_{t+s}$.

3.1.3 Deleveraging shock.

Consider next what happens if there is an expected shift in individuals' desire to save. At an abstract level, one can model this shift as a decrease in ρ, the discount rate of households. Such deleveraging shocks have been studied by Eggertsson and Krugman (2012), as well as Guerrieri and Lorenzoni (2011). To understand how these shocks may affect the consumption-to-wealth ratio, we need to consider two cases, depending on whether the economy is above or at the Effective Lower Bound on nominal interest rates (ELB). In the presence of nominal rigidities, the ELB may constrain the equilibrium real interest

[9] With open economies, the same phenomenon manifests itself in the form of current account surpluses for countries, such as Japan, Germany and China, with more rapid slowdown in population growth and aging.

rate in the economy at a level that is excessively high, pushing the economy into a recession.

Consider first the case where the economy is above the ELB. For simplicity, assume that (potential) output is constant. With the economy outside the ELB, it is possible for the real interest rate to adjust so that consumption equals output. The Euler equation takes the form:

$$\mathbb{E}_t r^w_{t+1} = \rho_t - \frac{1}{2}\sigma^2_{r,t}, \tag{11}$$

where ρ_t is the now time-varying discount rate of the representative household between periods t and $t+1$, known at time t. A decline in ρ_t pushes down the equilibrium expected return on wealth. Under the assumption that the economy remains permanently above the ELB, the present-value equation (8) becomes:

$$\ln C_t - \ln W_t = \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (\rho_{t+s-1} - \frac{1}{2}\sigma^2_{r,t+s-1}).$$

We can express the different components as:

$$\begin{aligned} cw_t^f &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \rho_{t+s-1} \\ cw_t^{rp} &= -\frac{1}{2}\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \sigma^2_{r,t+s-1} \\ cw_t^c &= 0. \end{aligned}$$

An expected deleveraging shock, i.e. a decline in $\mathbb{E}_t \rho_{t+s}$, has a direct negative effect on the consumption-to-wealth ratio because it lowers the real risk-free rate one for one, but it has no effect on the consumption or risk-premia components.

Consider now what happens at the ELB. If prices are nominally rigid and real interest rates cannot decrease further to satisfy (11), the economy will experience a recession, as in Eggertsson and Krugman (2012) or Caballero and Farhi (2015). For simplicity, suppose that the effective lower bound is zero and that prices are permanently fixed so that $r^f = 0$ while the economy remains at the ELB (i.e. while the natural rate ρ_t remains negative). The Euler equation for the risk-free rate requires that:

$$\gamma \mathbb{E}_t \Delta \ln C_{t+1} = -\rho_t + \frac{\gamma^2}{2} var_t \left(\Delta \ln C_{t+1} \right).$$

Consumption is expected to increase at a rate that reflects the (positive) gap between the real interest rate (0) and the natural real interest rate ($\rho_t < 0$). Since potential output is constant this expression makes clear that the economy must experience a recession today (i.e. consumption and output need to be below potential). The expected return on wealth (equal to the expected excess return) now satisfies:

$$\mathbb{E}_t r^w_{t+1} = \gamma cov_t(r^w_{t+1}, \Delta \ln C_{t+1}) - \frac{1}{2}\sigma^2_{r,t},$$

and may increase as the economy hits the ELB, as emphasized by Caballero, Farhi and Gourinchas (2016). If the economy is expected to remain permanently at the ELB, the different components of the consumption-to-wealth ratio can be expressed as:

$$\begin{aligned}
cw^f_t &= 0 \\
cw^{rp}_t &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho^s_w \left(\gamma cov_{t+s-1}(r^w_{t+s}, \ln C_{t+s}) - \frac{1}{2}\sigma^2_{r,t+s} \right) \\
cw^c_t &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho^s_w \left(\frac{1}{\gamma}\rho_{t+s-1} - \frac{\gamma}{2} var_{t+s-1} \left(\Delta \ln C_{t+s} \right) \right).
\end{aligned}$$

This expression makes clear that at the ELB, the adjustment in the consumption-to-wealth ratio occurs through the consumption component. Expected future consumption growth requires that the consumption-to-wealth ratio be low today. In the general case where the economy does not remain stuck at the ELB permanently, the adjustment will occur both via a decline in future real risk free rates -when the economy is expected to leave the ELB and via an increase in consumption growth while the economy is at the ELB. Both terms depress the consumption-to-wealth ratio, so cw^f and cw^c will be *positively* correlated.

3.1.4 Risk Appetite.

A deleveraging shock increases the demand for savings and therefore depresses the returns on all assets, leaving risk-premia largely unchanged outside the ELB. Let's now consider a shock to risk appetite, i.e. a shift in the demand for *safe* versus *risky* assets. The safe asset scarcity, arising from instance from an increase in desired holdings of safe assets, has been one of the leading explanations for the secular decline in real risk-free rates (Hall (2016), Caballero et al. (2015)).

An easy way to capture such a shift would be via an increase in risk aversion. However, with CES preferences, it is well known that the coefficient of risk aversion is also the inverse of the intertemporal elasticity of substitution $1/\gamma$. In order to isolate the effect of a shift in risk appetite from that of a change in the IES, assume that the representative household has Epstein-Zin recursive preferences:

$$U_t = \left\{(1-e^{-\rho})C_t^{1-\sigma} + e^{-\rho}\left(\mathbb{E}_t U_{t+1}^{1-\gamma_t}\right)^{\frac{1-\sigma}{1-\gamma_t}}\right\}^{\frac{1}{1-\sigma}},$$

where γ_t is the now time-varying coefficient of relative risk aversion. The IES is assumed constant and equal to $1/\sigma$. Given these preferences, we can solve the Euler equation for the risk-free rate:

$$r_t^f = \rho + \sigma \mathbb{E}_t \Delta \ln C_{t+1} + \frac{\theta_t - 1}{2}\sigma_{r,t}^2 - \frac{\theta_t \sigma^2}{2}\sigma_{g,t}^2$$

where $\theta_t \equiv (1-\gamma_t)/(1-\sigma)$. When $\theta_t = 1$, this formula collapses to the Euler equation equation (**??**) for the risk-free return. By contrast, when $\theta_t \neq 1$, the risk free rate depends on the variance of the market return $\sigma_{r,t}^2$. Standard derivations provide the following expression for the expected risk premium:

$$\mathbb{E}_t r_{t+1}^w - r_t^f = \theta_t \sigma\, cov_t(r_{t+1}^w, g_{t+1}) + (1-\theta_t)\sigma_{r,t}^2.$$

To highlight the role of fluctuations in risk appetite, consider an environment where output is constant, so $\sigma_{g,t}^2 = 0$. It follows that the consumption-to-wealth ratio can be expressed as (up to some constant):

$$\ln C_t - \ln W_t = \frac{1}{2}\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(1 - \theta_{t+s-1}\right)\sigma_{r,t+s-1}^2.$$

An increase in risk aversion γ_t raises $1-\theta_t = (\gamma_t - \sigma)/(1-\sigma)$ and leads to an increase in the consumption-to-wealth ratio. This is intuitive: while current consumption is unchanged (by assumption), the decline in risk appetite lowers the present value of future income, hence the current value of wealth. The decomposition (7) yields :

$$
\begin{aligned}
cw_t^f &= -\frac{1}{2}\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(1-\theta_{t+s-1}\right) \sigma_{r,t+s-1}^2 \\
cw_t^{rp} &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (1-\theta_{t+s-1}) \sigma_{r,t+s-1}^2 \\
cw_t^c &= 0.
\end{aligned}
$$

An expected increase in future risk aversion increases the risk premium component cw^{rp} and decreases the risk free rate component cw^f. The consumption component remains unchanged. This is also intuitive: the decline in risk appetite requires an increase in risk premia. This increase in risk-premia is achieved via an increase in the expected return on risky assets and a decline in the risk-free rate. It follows that $corr(cw_t^f, cw_t^{rp}) = -1$. Overall, the increase in risk premia dominates, driving up the consumption-to-wealth ratio so $corr(cw_t, cw_t^f) = -1$.

Summary. The preceding discussion highlights that, while the decomposition (7) does not provide a causal interpretation of the different components, the co-movements of the different components offers a natural signature about the various economic forces at play: If the consumption and risk free rate components are negatively correlated, we would conclude that productivity and/or demographic shocks play an important role. If instead the consumption and risk free rate components are either poorly correlated or positively correlated and the consumption-to-wealth ratio is positively correlated to the risk-free component, then we would conclude that deleveraging shocks are likely to be more relevant. Finally, if we find that the risk free component is both negatively correlated with the risk premium components and the consumption-to-wealth ratio, we would infer that shocks to risk appetite are an important part of the story. Table 1 summarizes the different co-movements implied by the theory.

Shock	$\ln C/W$	cw^f	cw^{rp}	cw^c
Productivity	sign of $\gamma - 1$	$+$	$\sim$	$-$
Population Growth	sign of $\gamma - 1$	$+$	$\sim$	$-$
Deleveraging (outside ELB)	$-$	$-$	$\sim$	0
Deleveraging (at the ELB)	$-$	0	$\sim$	$-$
Risk Appetite	$-$	$+$	$-$	0

Table 1: Summary of Sign Restrictions. The table reports the sign of $\ln C/W$ and its components cw^i in response to various expected future structural shocks. For instance, in response to an expected future positive productivity shock, cw^c decreases, cw^f increases, cw^{rp} is mostly unchanged and the sign of $\ln C/W$ depends on $\gamma - 1$.

3.2 Orders of Magnitude

Our empirical approach is flexible: it does not require imposing a particular stochastic discount factor, and allows for a flexible parametrization of the data generating process. Under additional restrictions, it is possible to express the consumption-to-wealth ratio in closed form as a function of the underlying fundamental parameters. For instance, following Martin (2013) and Vlieghe (2017), assume that there is a representative agent with separable constant elasticity preferences, so that the real Stochastic Discount Factor takes the form: $\mathcal{M}_{t,t+1} = e^{-\rho-\gamma\Delta \ln C_{t+1}}$. Assume further that consumption growth is i.i.d. with Cumulant Generating Function $\mathcal{C}(\theta) = \ln \mathbb{E}[\exp(\theta\Delta \ln C_{t+1})]$. Then, the consumption-to-total wealth ratio is constant and satisfies:[10]

$$\ln C_t - \ln \bar{W}_t = \ln \left(1 - e^{-\rho+\mathcal{C}(1-\gamma)}\right).$$

The risk-free return is also constant,

$$r_t^f = \ln R_t^f = -\ln \mathbb{E}_t \left[\mathcal{M}_{t,t+1}\right] = \rho - \mathcal{C}(-\gamma)$$

[10]To obtain this result, observe that we can substitute the return $\bar{R}_{t+1}$ into the fundamental asset pricing equation $\mathbb{E}_t[\mathcal{M}_{t,t+1}R_{t+1}] = 1$ and iterate forward to obtain

$$\bar{W}_t = C_t \sum_{s=0}^{\infty} \mathbb{E}_t \left[e^{-\rho s}\left(\frac{C_{t+s}}{C_t}\right)^{1-\gamma}\right] = C_t \sum_{s=0}^{\infty} e^{-\rho s}\left(\mathbb{E}\left[\exp \Delta \ln C_{t+1}\right]\right)^s = \frac{C_t}{1 - e^{-\rho+\mathcal{C}(1-\gamma)}}.$$

.

while the return on total wealth follows

$$\bar{r}_{t+1} = \ln \bar{R}_{t+1} = \ln\left(\frac{\bar{W}_{t+1}}{\bar{W}_t - C_t}\right) = \ln\left(\frac{1}{1 - C/\bar{W}}\frac{C_{t+1}}{C_t}\right) = \rho - \mathcal{C}(1-\gamma) + \Delta \ln C_{t+1},$$

and the expected risk premium satisfies:

$$ERP \equiv \ln \mathbb{E}_t \bar{R}_{t+1} - \ln R^f = \mathcal{C}(1) + \mathcal{C}(-\gamma) - \mathcal{C}(1-\gamma)$$

This representation is obviously too restrictive, since it implies a constant consumption-to-wealth ratio and a constant risk-free rate, but it nevertheless allows us to consider some relevant orders of magnitudes. For instance, if we follow Martin (2013) and postulate that log consumption growth follows a jump-diffusion process $\Delta \ln C_{t+1} = g + \sigma_g^2 \varepsilon_{t+1} + v_{t+1}$ where ε_{t+1} is a standard normal and v_{t+1} is a Poisson 'disaster' process with arrival rate p per unit of time and, where the disaster size is distributed $\mathcal{N}(-b, s^2)$, then the cumulant generating function satisfies:

$$\mathcal{C}(\theta) = g\theta + \frac{1}{2}\sigma_g^2\theta^2 + p\left(e^{-\theta b + \frac{1}{2}\theta^2 s^2} - 1\right).$$

Substituting the parameters from Barro (2006), $\rho = 0.03$, $\gamma = 4$, $g = 0.025$, $\sigma_g = 0.02$, $p = 0.017$, $b = 0.39$ and $s = 0.25$, we obtain $C/\bar{W} = 0.0465$, with a real risk-free rate $r^f = 1.04\%$ and an expected risk premium $ERP = 5.73\%$.

As we will see in the empirical section, the observed consumption-to-private wealth ratio for the U.S. between 1870 and 2015 has a mean of 0.209, which implies that the ratio of private wealth to total wealth is equal to $0.0465/0.209 = 22.25\%$. According to this crude calculation, human wealth represents the bulk of total wealth (77.75%), a figure that is roughly in line with -albeit smaller than- the calculations of Lustig et al. (2013) who estimate that human wealth represents 92% of total wealth. Similar calculations for the U.S., the U.K., Germany and France between 1920 and 2015 yield a consumption-to-private wealth ratio of 0.210, which implies a very similar estimate of the ratio of private wealth to total wealth (22.14%).

4 Estimating the Present Value Relation

We implement our empirical strategy in three steps. First, we construct estimates of the consumption-to-wealth ratio over long periods of time. Next, we evaluate the empirical validity of equation (7) by constructing the empirical counterparts of the right hand side of that equation, and testing whether they capture movements in the consumption-to-wealth ratio. Lastly, we investigate the role of various drivers of the consumption-to-wealth ratio.

4.1 Data description and Long-run Covariability

We use historical data on private wealth, population and private consumption for the period 1870-2015 for the United States, the United Kingdom, Germany and France from Piketty and Zucman (2014a), Piketty et al. (2017), the World Inequality Database, as well as Jordà et al. (2016) to construct measures of real per capita consumption and (beginning of period) private wealth, expressed in constant 2010 US dollars. Private wealth is defined as the sum of non-financial assets, including housing and other tangible assets such as software, equipment and agricultural land, and net financial assets, including equity, pensions, life insurance and bonds. Private wealth does not include government assets, but includes privates holdings of government issued liabilities as an asset.

Figure 5 reports real per capita private wealth and consumption for the United States between 1870 and 2015. As expected, historical time series on consumption and private wealth show a long term positive trend. U.S. real per capita consumption increased from \$2,829 in 2010 dollars in 1870 to \$35,771 in 2015, while real per capita private wealth increased over the same period from \$12,304 to \$227,283. The resulting consumption-to-wealth ratio, already reported on Figure 4 appears relatively stable over this long period of time, with a mean of 20.94 percent, decreasing from roughly 23 percent in 1870 to about 16 percent in the latter part of the sample. As noted above, we observe two periods during which the consumption-to-wealth ratio was significantly depressed: the first one spans the 1930s, starting

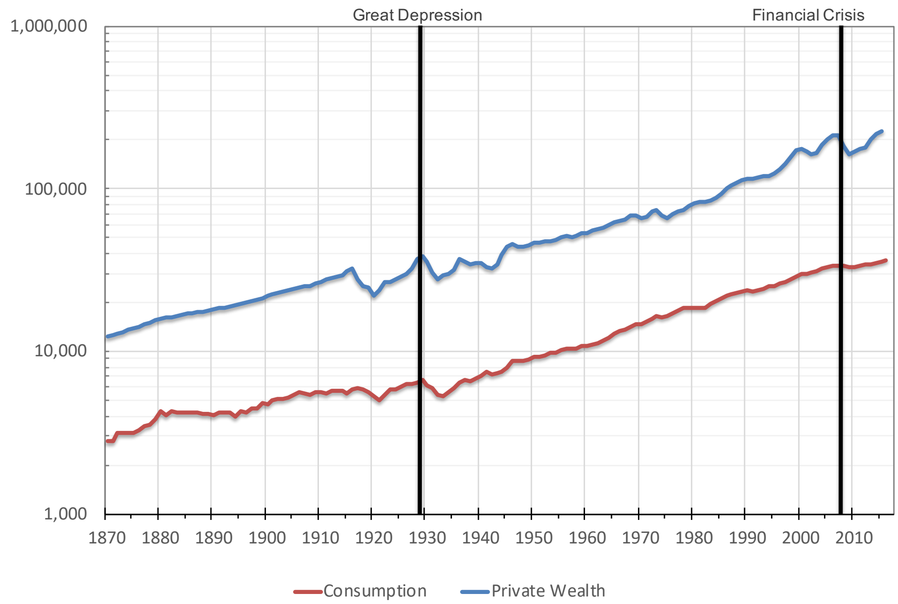

Figure 5: Real Consumption and Private Wealth per capita, 2010 USD, United States, 1870-2015.

shortly before the Great Depression and ending at the beginning of the 1940s. Interestingly, in 1939 Professor Alvin Hansen writes his celebrated piece about 'secular stagnation' (Hansen (1939)). The second episode of low consumption-to-wealth ratio starts around 1995 with a pronounced downward peak in 2008. The consumption-to-wealth ratio temporarily rebounds after 2008 largely as a result of the decline in private wealth. Perhaps not coincidentally, in the Fall 2013 at a conference at the International Monetary Fund, Larry Summers resuscitates the idea of secular stagnation, an idea which is still haunting us in 2018 (Summers (2015)).

Figure 6 reports real consumption and wealth per capita for an aggregate of the U.S., the U.K., Germany and France since 1920. We label this aggregate the 'G-4'. Over the period considered, these four countries represent a sizable share of the world's financial wealth and consumption. London, New-York, and to a lesser extent Frankfurt and Paris, represent major financial centers. As for the U.S., real consumption and wealth per capita for the G-4 show a long term positive trend with a few major declines during the two World Wars and the Great Depression.[11] Real per capita consumption

[11]The effect of the wars on the financial wealth and consumption of Germany and France is most dramatic during WWI. The U.S. consumption-to-wealth ratio is somewhat insulated and does not show swings of the amplitude of the U.K., French and German consumption wealth ratios at the time during that period. This, and concerns about data quality prior to 1920

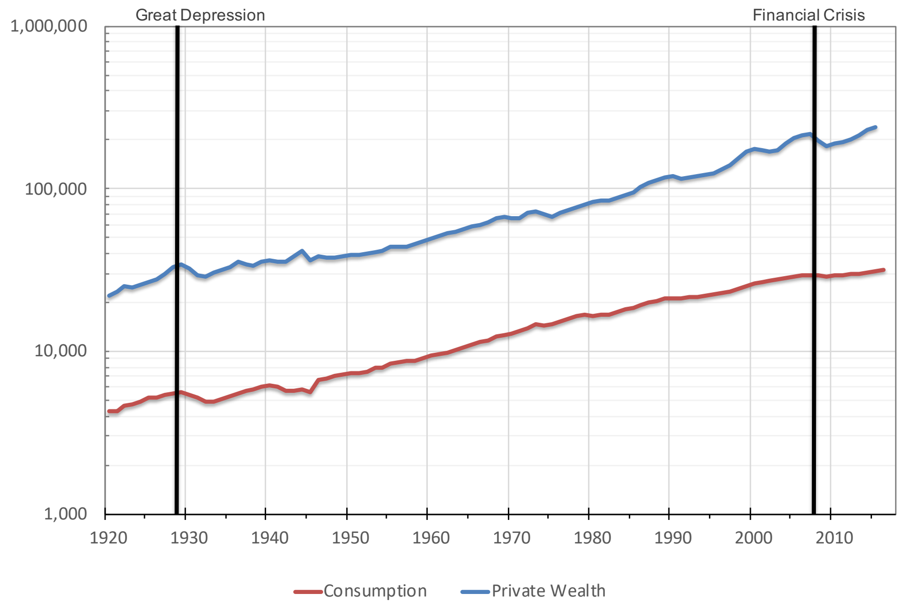

Figure 6: Real Consumption and Private Wealth per capita, 2010 USD, United States, United Kingdom, Germany and France, 1920-2015.

increased from \$4,282 in 1920 to \$31,198 in 2015 in 2010 constant dollars while real per capita private wealth increased from \$21,818 to \$238,535 over the same period. The consumption-to-wealth ratio exhibits the same pattern as that of the U.S., with a mean of 20.97 percent. While both consumption and wealth per capita look quite smooth over long periods of time, the ratio C/W exhibits substantial fluctuations, as seen in Figure 4.

Looking at Figures 4 and 5-6, it is clear that the decline in the consumption-to-wealth ratio observed in the 1930s and in the 2000s was associated in both cases with faster growth in private wealth, rather than slower growth in consumption. The growth rate of U.S. real private wealth per capita reached 4.88% p.a. between 1920 and 1930 and 4.35% between 1997 and 2007. Over the same periods, the growth rate of real consumption per capita was 1.56% and 2.4% respectively.[12]

Figure 7 uses the Piketty and Zucman (2014a) data to decompose U.S. real private wealth per capita

are the two reasons we only begin the 'G4' aggregate after 1920. In particular, as discussed in the appendix, wealth data is not available annually before 1954 for France, 1950 for Germany, 1920 for the U.K. and 1916 for the U.S. and is imputed based on savings data and estimates of the rate of capital gains on wealth for each country.

[12] Over the 1870-2015 period, the average growth rate of U.S. real private wealth per capita was 2.01%, that of real consumption per capita was 1.75%.

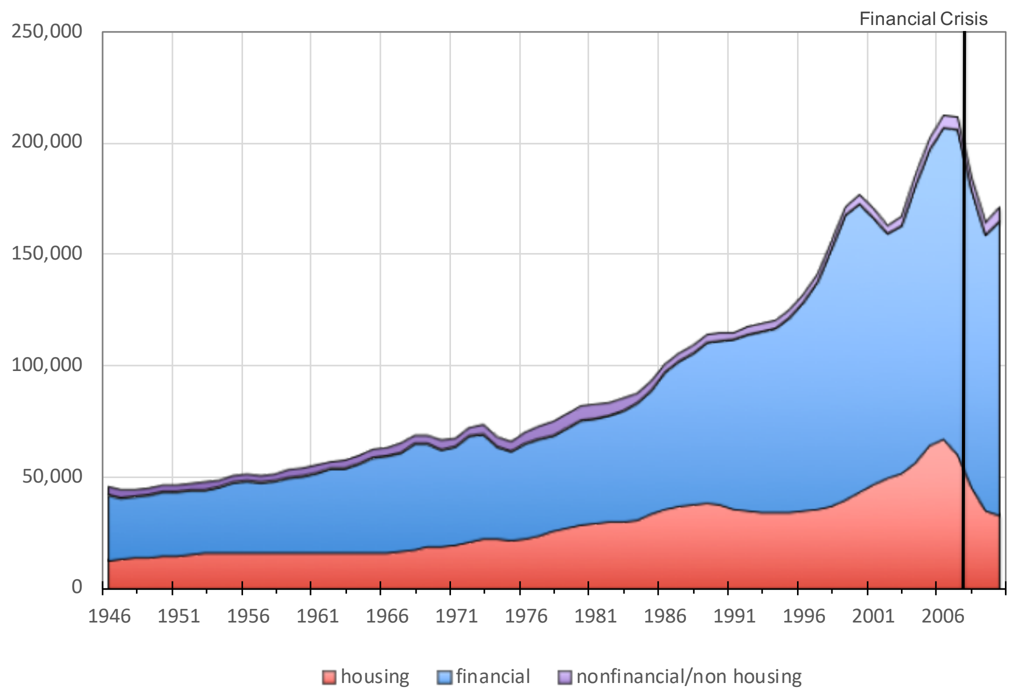

Figure 7: Housing, Financial and Private Wealth per capita, 2010 USD, United States, 1946-2010. Source: Piketty and Zucman (2014a).

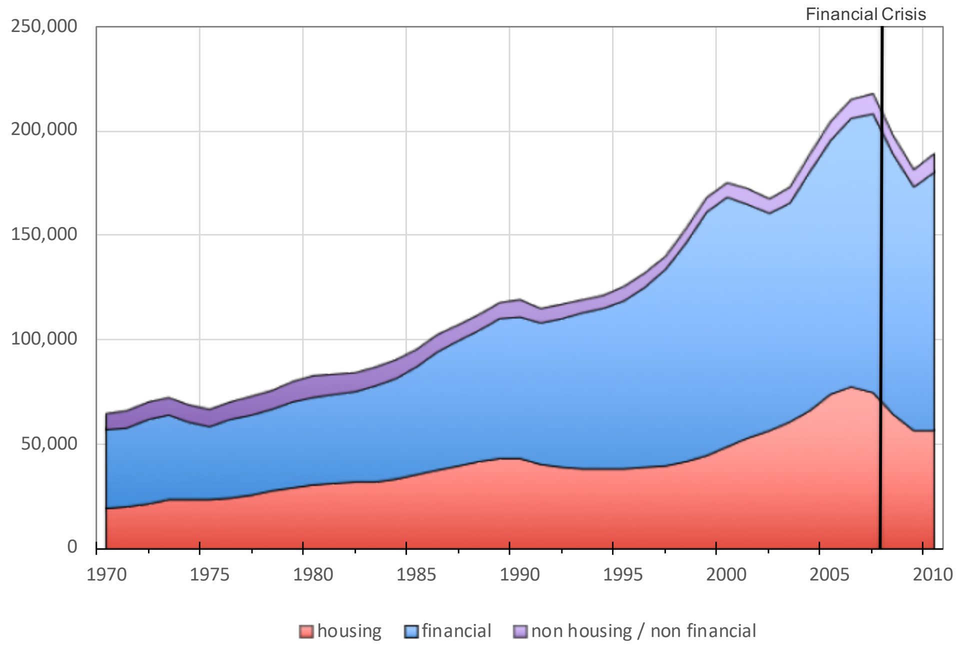

Figure 8: Housing, Financial and Private Wealth per capita, 2010 USD, United States, United Kingdom, Germany and France, 1970-2010. Source: Piketty and Zucman (2014a).

into housing, financial and a non-housing/non-financial residual components between 1946 and 2010.[13] The figure illustrates that housing wealth declined as a fraction of private wealth during that period, from 28-30% in 1946 to 20% by 2010. The figure also illustrates that the first decline in C/W in 2000 was associated with an increase in financial wealth (the growth rate of real financial wealth per capita between 1990 and 2000 was 5.66%, at the time of the dotcom boom), while the second decline in 2007 was associated with rapid growth in housing wealth (5.2% p.a. between 1997 and 2007 during the U.S. housing boom). Figure 8 reports a similar decomposition for our G-4 group, but on the shorter period 1970-2010 and shows a similar pattern, with rapid growth in housing wealth, but also financial wealth in the 2000s, when the ratio C/W was rapidly decreasing.[14]

For each country or group of country, we measure the real ex-post interest rate as the 3-month nominal yield minus realized CPI inflation.[15] Lastly, we use excess returns on equities r^e, long term

[13]For the U.S., the non housing/non financial component includes software, equipment and agricultural land. This represent a very small share of private wealth.

[14]Our consumption measure includes input rent for homeowners. Hence, increases in housing prices could be reflected in higher inputed rents.

[15]For the G4 aggregate, we use the average of the U.S. and U.K. real interest rates, weighted by relative wealth. Appendix

bonds r^l and the rate of growth of house prices r^h to instrument for the risk premium on private wealth.[16]

Table 2 reports some summary statistics for the different variables.

	C/W	$\Delta \ln c$	$\Delta \ln w$	n	r^f	$r^e - r^f$	$r^l - r^f$	$r^h - r^f$
Panel A: United States. Sample: 1870-2015								
Mean	20.941	1.750	2.011	1.432	1.984	4.530	0.400	4.320
Median	20.868	1.643	2.197	1.344	2.060	6.369	0.037	3.717
Standard deviation	2.295	3.422	4.897	0.523	4.932	17.741	7.441	7.998
Maximum	26.772	10.798	14.537	2.515	19.029	41.137	22.485	36.755
Minimum	15.739	-10.001	-15.181	0.489	-13.482	-53.303	-16.672	-31.065
Panel B: G-4. Sample: 1920-2015								
Mean	20.965	2.022	2.356	0.774	2.144	4.829	1.121	4.616
Median	21.471	2.127	3.129	0.755	2.267	8.109	0.829	4.854
Standard deviation	2.410	2.394	4.061	0.372	4.470	16.934	8.258	5.384
Maximum	25.599	10.954	9.273	1.412	18.710	38.011	21.429	22.003
Minimum	15.391	-5.784	-11.024	-1.491	-11.704	-50.285	-13.836	-10.033

Table 2: Summary Statistics. The table shows summary statistics for the consumption-to-wealth ratio C/W, the growth rate of real consumption per capita $\Delta \ln c$, the growth rate of real private wealth per capita $\Delta \ln w$, population growth n, the risk-free real rate r^f, the equity excess return $r^e - r^f$, the term premium $r^l - r^f$, housing capital gain minus the risk free rate $r^h - r^f$. r^h is available starting in 1891 for the U.S.

The table exhibits a number of interesting findings. First, the consumption-wealth ratio declines slightly over time since the rate of growth of consumption is small than that of private wealth by about 0.3% p.a. Second, wealth growth is more volatile than consumption growth. Hence, as discussed above, fluctuations in the consumption-to-wealth ratio will likely be driven by endogenous changes in. Third, the realized excess return on equities is sizable, around 4.5% for the U.S. and 5.3% for the G-4, numbers that are consistent with historical estimates of the equity premium. Third, the capital gain on housing is slightly lower than the risk free rate, and this excess return is highly volatile. As discussed above, r^h does not represent the full return on housing since it does not include rental income. Nevertheless, this suggests that the long run return to housing is largely driven by rental income and not capital gains.

C provides the details of the aggregation procedure. We do not include the real rate for Germany and France, since episodes of monetary instability in the 1920s and during WWII in both countries generate very volatile measures of the ex-post real interest rate.

[16] As for the risk free rate, we use a wealth-weighted average of the equity excess returns, term premium and housing returns for the global excess return. Note that the rate of growth of house prices differs from the true return on housing by the rent/price ratio which was not available to us. Since the rent/price ratio is positive, the rate of growth of housing price underestimates the actual return on housing.

Variable \Variable	C/W	$\Delta \ln c$	$\Delta \ln w$	n	r^f	$r^e - r^f$	$r^l - r^f$
$\Delta \ln c$	0.05						
	[-0.20,0.60]						
	[-0.40,0.65]						
$\Delta \ln w$	-0.12	0.64					
	[-0.44,0.13]	[0.40,0.78]					
	[-0.44,0.25]	[0.38,0.78]					
n	0.46	-0.12	-0.13				
	[-0.10,0.85]	[-0.35,0.50]	[-0.25,0.20]				
	[-0.11,0.85]	[-0.40,0.60]	[-0.25,0.20]				
r^f	0.18	-0.00	0.10	0.30			
	[-0.30,0.80]	[-0.35,0.36]	[-0.15,0.43]	[-0.10,0.75]			
	[-0.48,0.85]	[-0.44,0.46]	[-0.40,0.46]	[-0.27,0.80]			
$r^e - r^f$	-0.13	0.27	0.38	-0.05	-0.15		
	[-0.70,0.08]	[-0.18,0.58]	[0.02,0.64]	[-0.45,0.32]	[-0.65,0.16]		
	[-0.70,0.20]	[-0.35,0.63]	[-0.01,0.68]	[-0.50,0.46]	[-0.75,0.23]		
$r^l - r^f$	-0.41	0.05	0.21	-0.32	-0.05	0.28	
	[-0.80,0.03]	[-0.40,0.42]	[-0.10,0.55]	[-0.75,0.10]	[-0.65,0.40]	[-0.15,0.65]	
	[-0.85,0.03]	[-0.45,0.52]	[-0.10,0.60]	[-0.80,0.20]	[-0.70,0.55]	[-0.30,0.70]	
$r^h - r^f$	0.01	0.30	0.27	0.05	-0.65	0.08	0.00
	[-0.55,0.33]	[-0.05,0.59]	[-0.08,0.55]	[-0.26,0.60]	[-0.82,-0.33]	[-0.21,0.45]	[-0.34,0.45]
	[-0.60,0.50]	[-0.15,0.64]	[-0.16,0.58]	[-0.45,0.65]	[-0.82,-0.23]	[-0.28,0.60]	[-0.55,0.60]

Table 3: Long-Run Co-Variability — US, 1870-1915. The table reports the long-run correlation between any two variables, estimated as in Müller and Watson (2018). 67% and 90% CI reported in brackets. Variables: C/W: consumption-to-wealth ratio; $\Delta \ln c$: growth rate of real consumption per capita; $\Delta \ln w$: growth rate of real private wealth per capita; n: population growth; r^f: real ex-post risk free rate; $r^e - r^f$: realized excess equity return; $r^l - r^f$: term premium (10-year minus 3-months); $r^h - r^f$: excess of housing capital gains over risk-free rate.

Table 3 reports estimates of long-run covariability between pairs of variables for the US while Table 4 presents the same results for the G-4. Long run covariability estimates developed by Müller and Watson (2018) are designed to allow long run inference on the co-movements between two variables that is robust to the degree of long-run persistence in the data. For a pair of variables x_t and y_t, Müller and Watson (2018) estimates the long-run correlation as the correlation between low frequency transformations of the variables, using low-pass filters. The table also present 67% and 90% confidence intervals, estimated using Müller and Watson ABcde model. Not surprisingly, the results indicate that consumption and wealth co-move positively in the long run, with a long run correlation of the growth rates of 0.64. Beyond this finding, the table illustrates the absence of strong long-run co-movements between real risk-free rates and the usual suspects: real interest rates do not systematically co-move with real consumption growth or population growth. Real risk-free rates do appear to covary negatively

Variable \Variable	C/W	$\Delta \ln c$	$\Delta \ln w$	n	r^f	$r^e - r^f$	$r^l - r^f$
$\Delta \ln c$	0.32						
	[-0.03,0.80]						
	[-0.05,0.80]						
$\Delta \ln w$	0.01	0.46					
	[-0.32,0.45]	[0.01,0.72]					
	[-0.55,0.60]	[-0.20,0.80]					
n	0.27	0.60	0.33				
	[-0.40,0.70]	[0.26,0.85]	[-0.10,0.61]				
	[-0.45,0.75]	[0.13,0.90]	[-0.40,0.70]				
r^f	0.13	-0.07	0.12	-0.03			
	[-0.45,0.75]	[-0.65,0.55]	[-0.21,0.65]	[-0.65,0.65]			
	[-0.50,0.80]	[-0.70,0.65]	[-0.35,0.80]	[-0.70,0.70]			
$r^e - r^f$	-0.02	0.16	0.34	0.42	-0.10		
	[-0.50,0.44]	[-0.22,0.56]	[-0.13,0.65]	[-0.00,0.71]	[-0.70,0.31]		
	[-0.60,0.46]	[-0.50,0.65]	[-0.40,0.68]	[-0.30,0.80]	[-0.75,0.55]		
$r^l - r^f$	-0.53	-0.34	0.03	-0.28	0.00	0.12	
	[-0.92,-0.13]	[-0.75,0.05]	[-0.34,0.42]	[-0.75,0.16]	[-0.65,0.55]	[-0.25,0.65]	
	[-0.93,-0.01]	[-0.80,0.11]	[-0.55,0.60]	[-0.80,0.27]	[-0.75,0.65]	[-0.35,0.75]	
$r^h - r^f$	-0.01	0.35	-0.01	0.04	-0.72	-0.00	-0.12
	[-0.55,0.50]	[-0.05,0.75]	[-0.41,0.38]	[-0.34,0.60]	[-0.92,-0.38]	[-0.42,0.55]	[-0.60,0.32]
	[-0.70,0.65]	[-0.13,0.80]	[-0.60,0.55]	[-0.42,0.70]	[-0.94,-0.29]	[-0.50,0.70]	[-0.75,0.41]

Table 4: Long-Run Co-Variability — G4, 1920-1915. The table reports the long-run correlation between any two variables, estimated as in Müller and Watson (2018). 67% and 90% CI reported in brackets. Variables: C/W: consumption-to-wealth ratio; $\Delta \ln c$: growth rate of real consumption per capita; $\Delta \ln w$: growth rate of real private wealth per capita; n: population growth; r^f: real ex-post risk free rate; $r^e - r^f$: realized excess equity return; $r^l - r^f$: term premium (10-year minus 3-months); $r^h - r^f$: excess of housing capital gains over risk-free rate.

with the term premium, i.e. the difference between the yield on 10-year government bonds and the 3-months rate. This is consistent with the expectation hypothesis, with long term rates encoding future short term real rates and the later mean reverting slowly over time. Similarly, while the consumption-to-wealth ratio does not seem to covary strongly with the level of the risk free rate, it is strongly and statistically negatively correlated with the term premium (-0.52).[17]

4.2 Vector-Auto-Regression Results

We construct an empirical estimate of the right hand side of equation (7) using a Vector Auto Regression (VAR). We form the vector $\mathbf{z}_t = \left(\ln C_t - \ln W_t, r_t^f, \mathbf{er_t}', \Delta \ln C_t\right)'$ and estimate a Vector Auto

[17] Results are similar for the G-4. We omit that table in the interest of space.

Regression (VAR) of order p. Using this VAR, we then construct the forecasts $\mathbb{E}_t \mathbf{z}_{t+k}$ to construct:[18]

$$\hat{cw}_t^f \backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s r_{t+s-1}^f$$
$$\hat{cw}_t^{rp} \backsimeq \hat{\nu} \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \mathbf{er_{t+s}}$$
$$\hat{cw}_t^c \backsimeq -\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \Delta \ln C_{t+s}.$$

Each of these components has a natural interpretation as the contribution of the risk free rate, the risk premium and the consumption growth components to the consumption-to-wealth ratio. We assume an annual discount rate $\rho_w = 1-0.0465$. Recall that according to our derivations $\rho_w = 1-C/\bar{W}$ and that we calibrated $C/\bar{W} = 0.0465$ in section 3.2. Importantly, observe that we do not need to identify structural shocks to form the forecasts $\hat{cw}_t^i$.

Our approach requires an estimate of ν. As indicated earlier, we estimate this parameter by regressing $\ln C_t - \ln W_t - \hat{cw}_t^f - \hat{cw}_t^c$ on $\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \mathbf{er_{t+s}}$. Recall that we do not observe the return on private wealth, so this method gives the highest chance to the model to match the observed consumption-to-wealth ratio. This calls for two observations. First, as noted above, this method leaves cw^f and cw^c unchanged so the correlation between the consumption growth component and the risk free rate component is unaffected by $\hat{\nu}$. Second, as we noted, while this method is appropriate if there is measurement error in the return to private wealth, it may induce some spurious movements if the residual in Eq. (7) due to fluctuations in human wealth relative to private wealth, is correlated with the excess return on equities and bonds. In that case, $\hat{cw}_t^{rp}$ is best interpreted as capturing both the risk premium as well as the component of the excess return on human wealth that is correlated with it. We start by using the equity excess return $r^e - r^f$ to forecast the risk premium component and discuss later how our results change as we include the term premium and a proxy for housing returns.

Figure 9 shows the consumption wealth ratio as well as the components of the right hand side of equation (7) for the US. The results are striking. First, we note that the fit of the VAR is excellent.[19]

[18]See the details of the empirical VAR methodology in Appendix D.
[19]The lags of the VAR are selected by standard criteria.

The grey line reports the predicted consumption-to-wealth ratio, i.e. the sum of the three components $cw_t^f + cw_t^{rp} + cw_t^c$.[20] Our empirical model is able to reproduce quite accurately the annual fluctuations in the consumption-to-wealth ratio over more than a century of data. This is all the more striking since the right hand side of equation (7) is constructed entirely from the reduced form forecasts implied by the VAR estimation.

Second, most of the movements in the consumption-to-wealth ratio reflect expected movements in the future risk-free rate, i.e. the cw_t^f component. The estimated risk-premium component cw^{rp} (in black) is never very significant economically. We do observe, however, a negative co-movement between the consumption cw^c and both $\ln C/W$ and the risk-free component cw^f. This is consistent with productivity and/or demographic shocks driving part of the movements in $\ln C/W$ as discussed in section 6. It follows that the consumption-to-wealth ratio contains significant information on current and future real short term rates, as encoded in equation (7). As discussed above, the two historical episodes of low consumption-to-wealth ratios occurred during periods of rapid asset price and wealth increases followed each time by a severe financial crisis. Our empirical results indicate that in the aftermath of these crises real short term rates remain low (or negative) for an extended period of time.

Table 5 decomposes the variance of $\ln C - \ln W$ into components reflecting news about future real risk-free rates, future risk premia, and future consumption growth. The decomposition accounts for 96 percent of the variance in the average propensity to consume, with the risk free rate representing 102 percent of the variation and the consumption growth component -45 percent.

Figure 10 reports a similar decomposition for the 'G-4' aggregate between 1920 and 2015. The results are very similar. First, the overall fit of the VAR remains excellent.[21] As before, we find that the risk-free component explains most of the fluctuations in the consumption-to-wealth ratio. The adjusted risk premium and consumption growth components remain smaller and the risk free component remains strongly negatively correlated with the consumption growth component Finally, the variance decomposition, presented in Table 5 confirms again the importance of the risk free compo-

[20]The overall fit is excellent, with an $R^2 = 0.92$. However, this result is obtained with a some attenuation of the equity excess return since we estimate $\hat{\nu} = 0.64$.

[21]The attenuation of the equity risk premium is stronger, however since we estimate $\hat{\nu} = 0.33$.

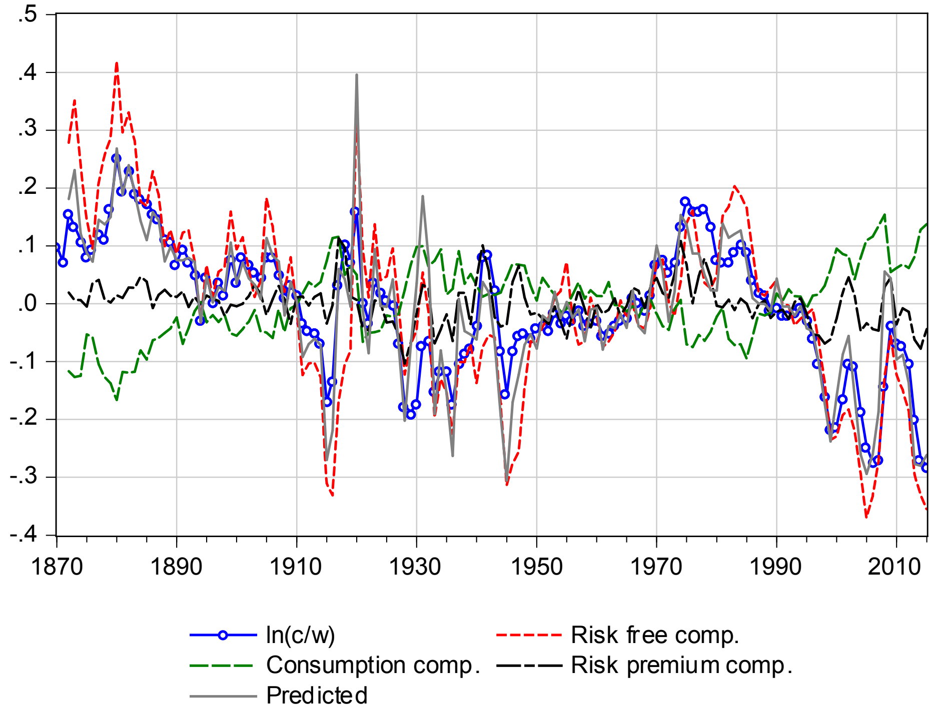

Figure 9: Consumption Wealth, Risk-free, Equity Premium and Consumption Growth Components. United States, 1870-2015. Note: The graph reports the (log, demeaned) private consumption-wealth ratio together with the risk-free, risk premium and consumption growth components. Estimates a VAR(2) with $\hat{\nu} = 0.64$. Source: Private wealth from WID. Consumption and short term interest rates from Jordà et al. (2016). Equity return from Global Financial Database.

nent. Overall, these results are consistent with the main drivers of being deleveraging shocks as well as productivity/demographic shocks.

To explore further the distinction between productivity and demongraphic shocks, Figure 11 reports an alternate decomposition where we separate total consumption growth into growth in consumption per capita and population growth: $\Delta \ln C = \Delta \ln c + n$. The results are largely unchanged. Table 5 provides the unconditional variance decomposition. This suggests that productivity shocks and demographic shocks play similar role in the dynamics of C/W. Both are negatively correlated with the risk free component

The fact that equity risk premia account for almost none of the movements in C/W is perhaps surprising in light of Lettau and Ludvigson (2001)'s findings that a cointegration relation between aggregate consumption, wealth and labor income predicts reasonably well U.S. equity risk premia. A

#	percent	USA		G4	
1	β_{rf}	1.023	1.242	1.681	1.685
2	β_{rp}	0.383	0.325	0.242	0.375
3	β_c	-0.450	-0.427	-0.714	-0.853
	of which:				
4	β_{cp}		-0.096		-0.758
5	β_n		-0.331		-0.095
6	Total	0.955	1.140	1.209	1.207
7	$\hat{\nu}$	0.760	0.734	0.296	0.471

Table 5: Unconditional Variance Decomposition of $\ln C - \ln W$

Note: β_{rf} (resp.β_{rp}, and β_c) represents the share of the unconditional variance of $\ln C - \ln W$ explained by future risk free returns (resp. future risk premia and future total consumption growth); β_{cp} (β_n) represents the share of the unconditional variance of $\ln C - \ln W$ explained by per capita consumption growth (population growth). The sum of coefficients $\beta_{cp} + \beta_n$ is not exactly equal to β_c due to numerical rounding in the VAR estimation. Sample: U.S: 1870-2015; G4: 1920:2015

number of factors may account for this result. First and foremost, we assume that $\ln C/W$ is stationary over the long run, and thus do not estimate a cointegrating vector with labor income. Second, we consider a longer sample period, going back to 1870 for the U.S and 1920 for the other countries. Thirdly, as argued above, our sample is dominated by two large financial crises and their aftermath. Lastly, we view our analysis as picking up low frequency determinants of real risk-free rates while Lettau and Ludvigson (2001) seem to capture business cycle frequencies.

5 Predictive regressions

The third step consists in directly evaluating the forecasting performance of the consumption-wealth variable for future risk-free interest rates, risk premia and aggregate consumption growth.

Our decomposition exercise indicates that the consumption-wealth ratio contains information on future risk-free rates. We can evaluate directly the predictive power of $\ln C_t/W_t$ by running regressions of the form:

$$y_{t+k} = \alpha + \beta \ln (C_t/W_t) + \epsilon_{t+k} \tag{12}$$

where y_{t+k} denotes the variable we are trying to forecast at horizon k . We consider the following candidates for y: the average real risk free rate between t and $t+k$; the average one-year excess return

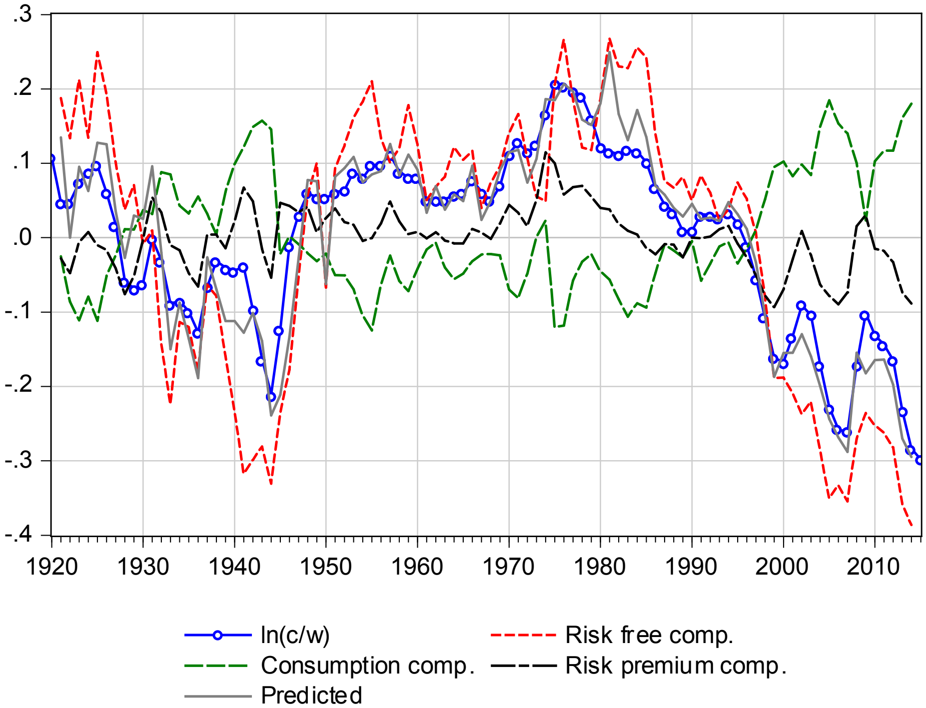

Figure 10: Consumption Wealth: Risk-free, Equity Premium and Consumption Growth Components. United States, United Kingdom, Germany and France, 1920-2015. Note: The graph reports the (log, demeaned) private consumption-wealth ratio for the U.S. U.K., Germany and France, together with the risk-free, risk premium and consumption growth components. Estimates a VAR(2) with $\hat{\nu} = 0.329$. Source: Private wealth from Piketty and Zucman (2014a). Consumption and short term interest rates from Jordà et al. (2016). Equity return from Global Financial Database.

between t and $t+k$; the average annual real per capita consumption growth between t and $t+k$; the average annual population growth between t and $t+k$; the average term premium between $t$and $t+k$; the average growth of real credit to the non-financial sector per capita between t and $t+k$.

Tables 6 presents the results for the US and the G4 aggregate. We find that the consumption-to-wealth ratio always contains substantial information about future short term risk free rates (panel A). The coefficients are increasing with the horizon and become strongly significant. They also have the correct sign, according to our decomposition: a low $\ln C/W$ strongly predicts a period of below average real risk-free rates up to 10 years out. By contrast, the consumption-to-wealth ratio has almost no predictive power for the equity excess returns, and more limited predictive power for per-capita consumption growth. The regressions indicate some predictive power for population growth for the

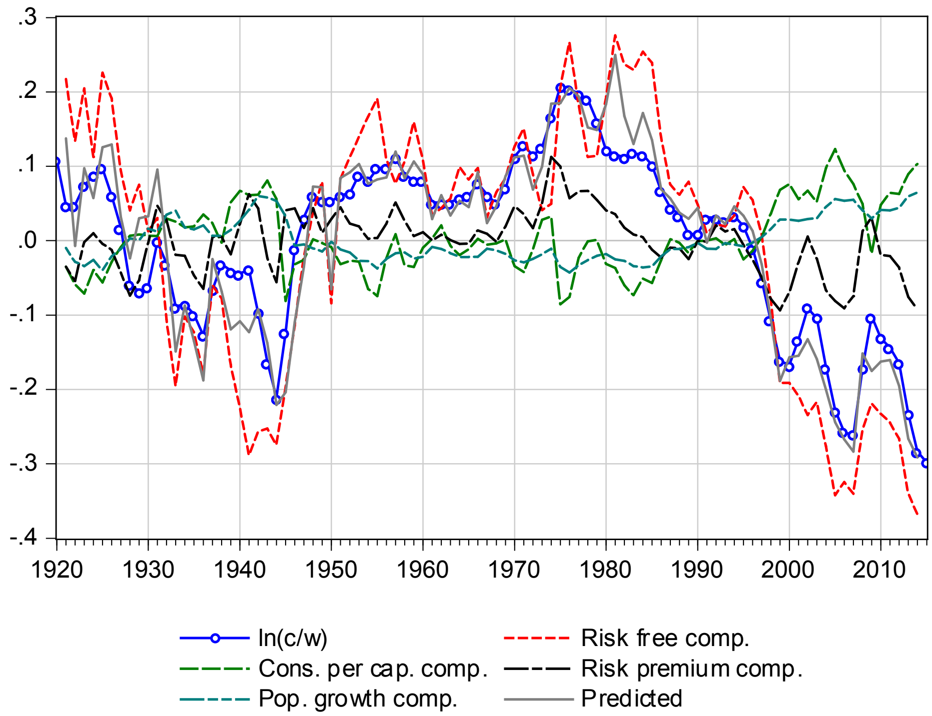

Figure 11: Consumption Wealth: Risk-free, Equity Premium, Consumption per capita and Population Growth Components. United States, United Kingdom, Germany and France, 1920-2015. Note: The graph reports the (log, demeaned) private consumption-wealth ratio together with the riskfree, risk premium, consumption per capita and population growth components. Estimates a VAR(2) with $\hat{\nu} = 0.19$. Source: Private wealth from Piketty and Zucman (2014a). Consumption, population and short rates from Jordà et al. (2016). Equity return from Global Financial Database.

U.S.: a low $\ln C/W$ predicts a low future population growth which suggests that the indirect effect (via changes in real risk-free rates) dominates the direct effect. Finally, there is significant predictive power for the term premium, i.e. the difference between the yield on 10-year Treasuries and short term rates. According to the estimates, a decrease in C/W is associated with a significant increase in term premia. This result is consistent with our long-run co-variability estimates.

Figures 12-18 report our forecast of the risk free rate, equity premium, population growth, cumulated consumption growth per capita, term premium and the growth rate of credit to the nonfinancial sector, using the G-4 consumption-to-wealth ratio at 1, 2, 5 and 10 year horizon. For each year t, the graph reports $y^f_{t,k} = \frac{1}{k}\sum_{s=0}^{k-1} y^f_{t+s}$, the average of the variable z to forecast one-year real risk-free rate between t and $t + k$, where k is the forecasting horizon. The graph also reports the predicted value

	United States				U.S., U.K., France and Germany			
	Forecast Horizon (Years)				Forecast Horizon (Years)			
	1	2	5	10	1	2	5	10
	A. Short term interest rate				A. Short term interest rate			
$\ln C_t/W_t$	0.13	0.14	0.14	0.15	0.07	0.08	0.12	0.18
	(0.04)	(0.05)	(0.04)	(0.03)	(0.04)	(0.05)	(0.05)	(0.06)
R^2	[0.08]	[0.11]	[0.19]	[0.29]	[0.02]	[0.04]	[0.14]	[0.32]
	B. Consumption growth (per-capita)				B. Consumption growth (per-capita)			
$\ln C_t/W_t$	-0.03	0.00	0.01	-0.01	0.04	0.04	0.04	0.03
	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
R^2	[0.00]	[-0.01]	[0.00]	[0.00]	[0.02]	[0.06]	[0.10]	[0.11]
	C. Population Growth				C. Population Growth			
$\ln C_t/W_t$	0.03	0.03	0.03	0.02	0.01	0.01	0.01	0.00
	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)	(0.01)
R^2	[0.30]	[0.30]	[0.33]	[0.29]	[0.05]	[0.06]	[0.06]	[0.03]
	D. Equity Risk Premium				D. Equity Risk Premium			
$\ln C_t/W_t$	0.13	0.12	0.02	-0.04	0.15	0.16	0.09	-0.02
	(0.16)	(0.14)	(0.08)	(0.08)	(0.19)	(0.15)	(0.09)	(0.10)
R^2	[0.00]	[0.00]	[-0.01]	[0.00]	[0.00]	[0.01]	[0.01]	[-0.01]
	E. Long Bond Risk Premium				E. Long Bond Risk Premium			
$\ln C_t/W_t$	-0.11	-0.11	-0.12	-0.09	-0.18	-0.17	-0.18	-0.12
	(0.06)	(0.05)	(0.04)	(0.04)	(0.09)	(0.06)	(0.05)	(0.05)
R^2	[0.02]	[0.05]	[0.15]	[0.13]	[0.05]	[0.11]	[0.28]	[0.17]
	F. Housing Risk Premium				F. Housing Risk Premium			
$\ln C_t/W_t$	0.01	0.04	0.03	-0.06	-0.05	-0.06	-0.14	-0.19
	(0.07)	(0.08)	(0.07)	(0.04)	(0.06)	(0.08)	(0.09)	(0.06)
R^2	[-0.01]	[0.00]	[0.00]	[0.03]	[0.00]	[0.01]	[0.10]	[0.26]
	G. Credit Growth				G. Credit Growth			
$\ln C_t/W_t$	0.06	0.09	0.13	0.09	0.15	0.15	0.14	0.06
	(0.05)	(0.05)	(0.07)	(0.08)	(0.04)	(0.05)	(0.07)	(0.06)
R^2	[0.00]	[0.02]	[0.08]	[0.06]	[0.09]	[0.12]	[0.14]	[0.02]

Table 6: Long Horizon Regressions. Note: The table reports the point estimates, Newey-West corrected standard errors and the R^2 of the forecasting regression.

$\hat{y}^f_{t,k}$ based on predictive regression (12) together with a 2-standard error confidence band, computed using Newey-West robust standard errors. For two variables, the average future global short rate and the average future global term premium, the fit of the regression improves markedly with the horizon. The last forecasting point is 2015, indicating a forecast of -3.1 percent for the global short real interest rate until 2025 (bottom right graph). The corresponding figure using U.S. data is -2.35 percent.

Finally, figure **??** reports, for the U.S., the forecast of the average risk-free rate at 10 years, together with a Kalman-Filter estimate constructed using the Present Value representation, as in Ventura (2001). The Kalman-Filter estimate tracks the realized 10-year average riskfree rate extremely well. The estimated risk free rate for 2015-2025 is slightly higher, at -1.37 percent, but still remarkably low compared to historical averages.

6 Estimating the Shocks: Macro vs Finance

In this section, we present and estimate a structural version of the model from Section 3. Our goal here is twofold. First, we use the model to interpret our empirical results, and second, we use it to spell out several broad channels that could be behind the secular trends described previously, and quantify their relative importance. We keep the model deliberately simple so as to be as close as possible to our empirical exercise. Notably, we stick to a representative-agent framework, still widely used in practice, to assess where the secular trends we have described would show up. An interesting next step would be to map where channels related to the heterogeneity between agents would be captured in our framework.

6.1 Set-up and general results

Consider a global (therefore closed) economy with no government so that consumption equals endowment: $C_t = Y_t$. Further, decompose total output into output per capita y_t and population N_t : $Y_t = y_t N_t$. In what follows, we treat y_t and N_t as exogenous processes and denote $g_t = \Delta \ln y_t$ and $n_t = \Delta \ln N_t$, the growth rates of output per capita and population, respectively.

Assume that preferences take the following Epstein-Zin form:

$$U_t = \left\{(1-\beta_t)C_t^{1-\sigma} + \beta_t \left(\mathbb{E}_t U_{t+1}^{1-\gamma_t}\right)^{\frac{1-\sigma}{1-\gamma_t}}\right\}^{\frac{1}{1-\sigma}} \tag{13}$$

where γ_t is a time-varying coefficient of relative risk aversion that captures *shocks to risk appetite*, while $\beta_t = \exp(-\rho_t)$ is a time-varying discount factor that captures, among other things, *deleveraging shocks*.

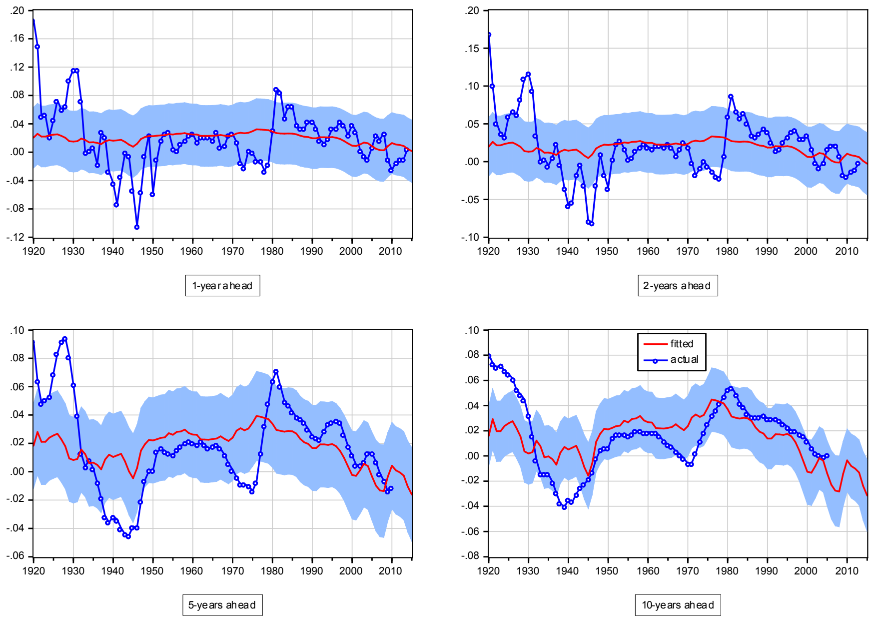

Figure 12: Predictive Regressions: Risk Free Rate, 1920-2015. Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global real risk free rate from a regression on past $\ln(C/W)$.

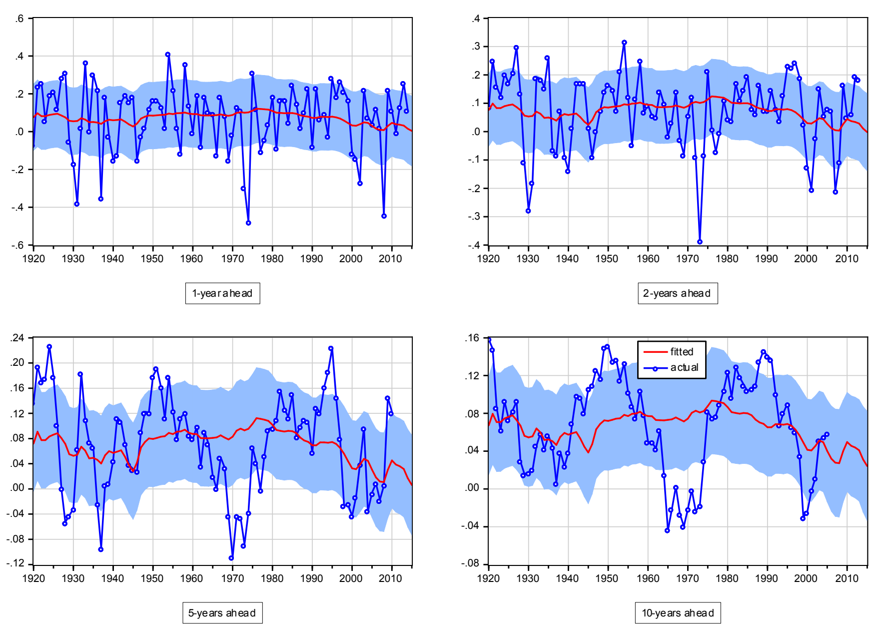

Figure 13: Predictive Regressions: Equity Premium, 1920-2015. Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized equity premium from a regression on past $\ln(C/W)$.

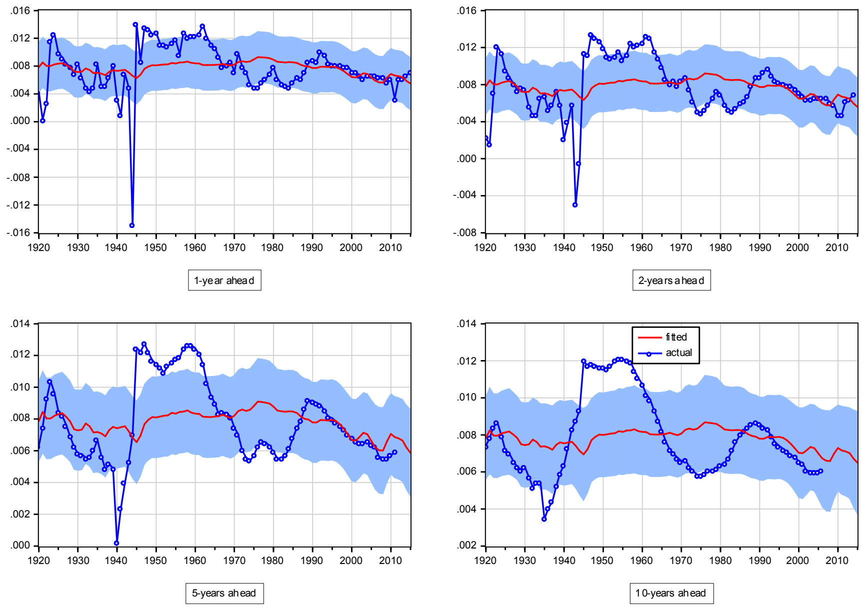

Figure 14: Predictive Regressions: Population Growth, 1920-2015. Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global population growth rate from a regression on past $\ln(C/W)$.

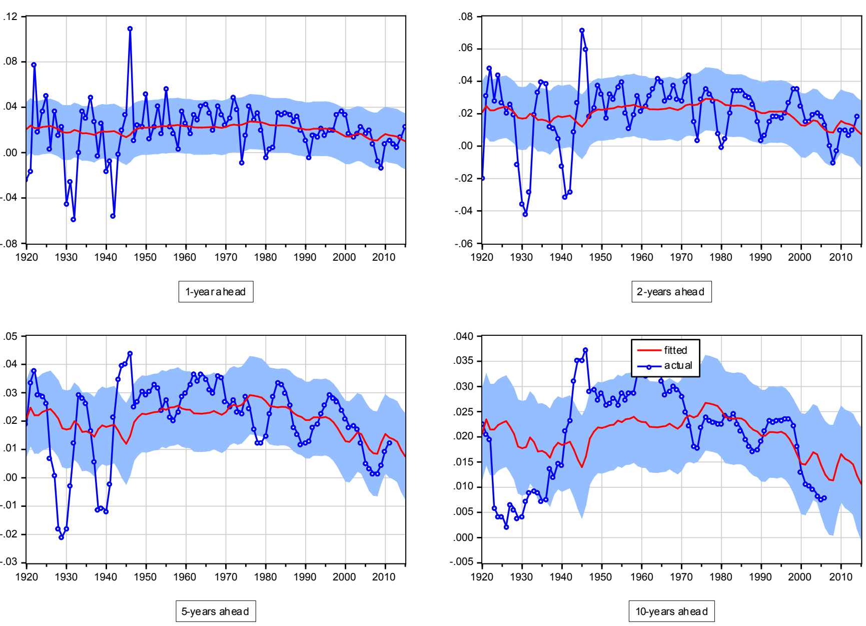

Figure 15: Predictive Regressions: Consumption growth per capita, 1920-2015. Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global per capita real consumption growth from a regression on past $\ln(C/W)$.

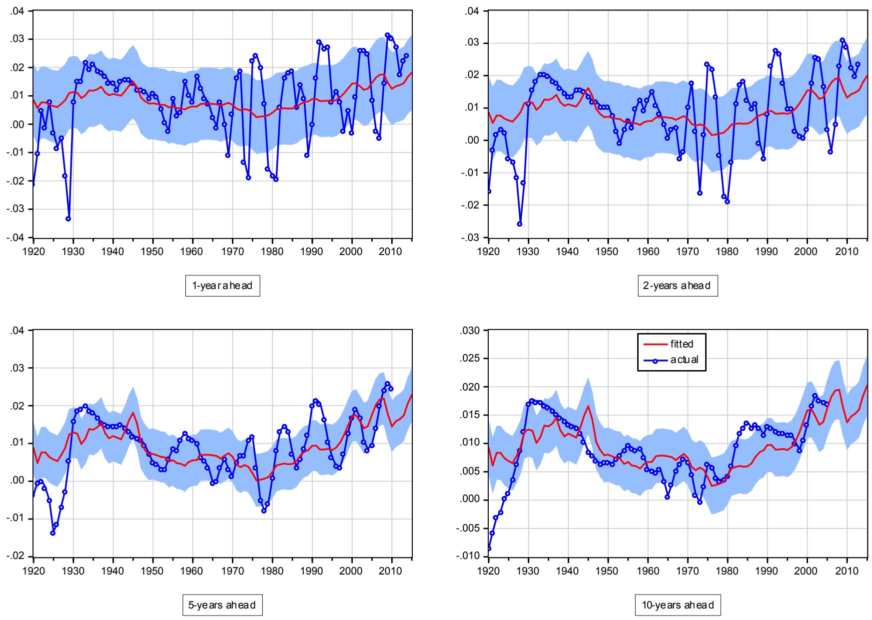

Figure 16: Predictive Regressions: Term premium, 1920-2015. Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global term premium from a regression on past $\ln(C/W)$.

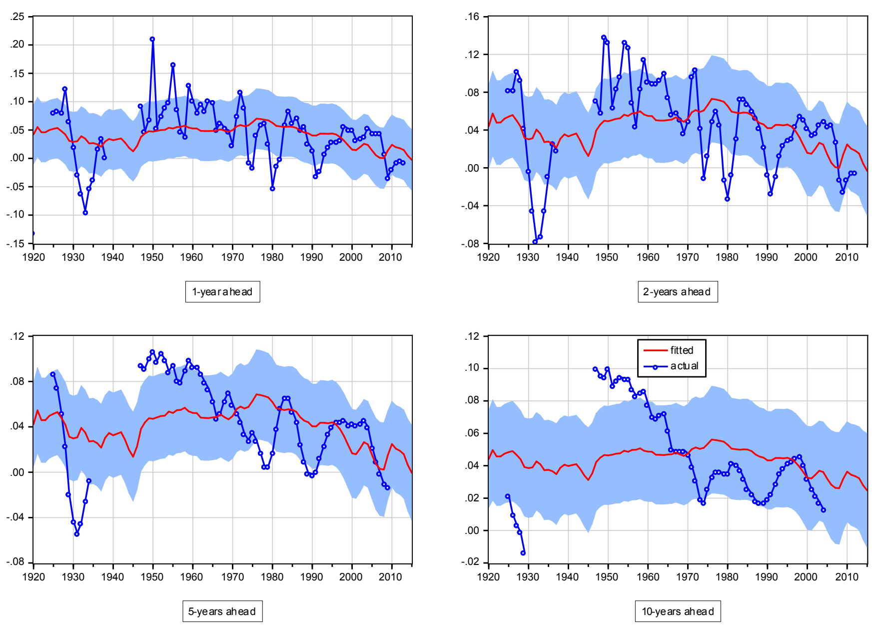

Figure 17: Predictive Regressions: Growth rate of credit to non-financial sector, 1920-2015. Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global term premium from a regression on past $\ln(C/W)$.

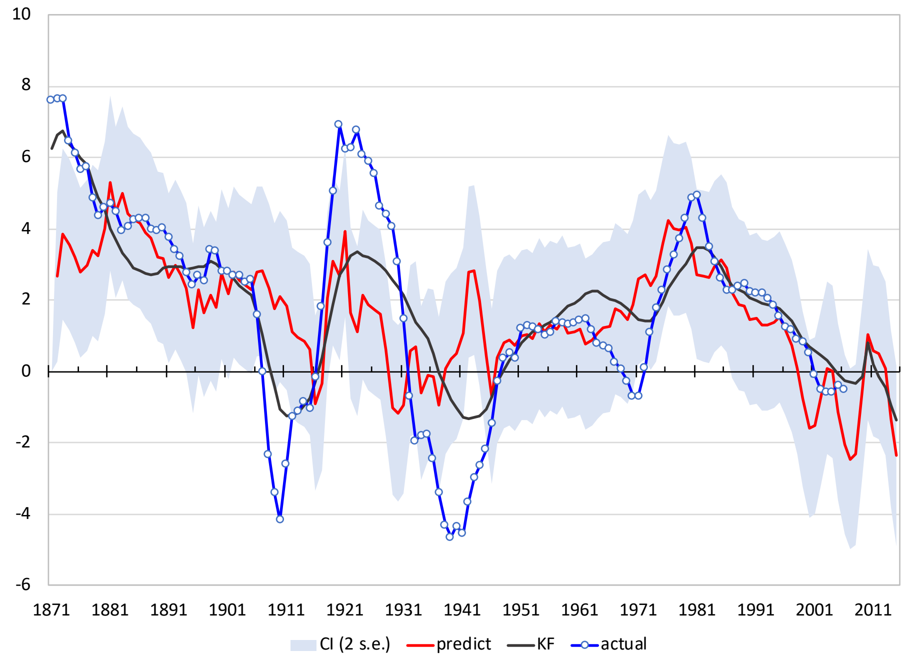

Figure 18: Predictive Regressions: Real interest rate, 1870-2015. Note: The graph reports forecasts at 10 years of the annualized risk-free rate: a simple forecast using $\ln(C/W)$ and a Kalman Filter estimate using Ventura (2001).

σ is the inverse of the elasticity of intertemporal substitution (EIS), and is assumed to be constant.[22]

The budget constraint is:

$$\bar{W}_{t+1} = \bar{R}_{t+1}\left(\bar{W}_t - C_t\right) \tag{14}$$

where $\bar{W}_t = W_t + H_t$ is total wealth, composed of financial wealth W_t, and human wealth H_t. Human wealth in turn follows:

$$H_{t+1} = R^h_{t+1}(H_t - YL_t) \tag{15}$$

where YL_t denotes aggregate non-financial income in period t and R^h_{t+1} is the gross return to human wealth. Substituting, and manipulating, we obtain:

$$W_{t+1} = R^w_{t+1}\left(W_t - C_t\omega_t\right) + R^h_{t+1}\left(YL_t - C_t(1-\omega_t)\right) \tag{16}$$

[22] When $\sigma = 1$, preferences take the form $\ln U_t = (1-\beta_t)\ln C_t + \left(\frac{\beta_t}{1-\gamma_t}\right)\ln \mathbb{E}_t U_{t+1}^{1-\gamma_t}$.

where $\omega_t = W_t/\bar{W}_t$ is the fraction of financial wealth in total wealth. Now, if we assume that $R^w_{t+1} = R^h_{t+1}$ and $\omega_t = \omega$ is constant, we obtain[23]:

$$W_{t+1} = R^w_{t+1}\left(W_t - \omega C_t\right) \tag{17}$$

In words, financial wealth is a claim to a constant fraction ω of aggregate consumption expenditures. In this framework, the stochastic discount factor can be expressed as:

$$\mathcal{M}_{t,t+1} = \beta_t \left(\frac{1-\beta_{t+1}}{1-\beta_t}\right)\left(\frac{U_{t+1}}{\left(\mathbb{E}_t U_{t+1}^{1-\gamma_t}\right)^{1/(1-\gamma_t)}}\right)^{\sigma-\gamma_t}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \tag{18}$$

The stochastic discount factor can also be written as a function of R^w_{t+1}, the return on wealth, as follows:

$$\mathcal{M}_{t,t+1} = \beta_t^{\theta_t}\left(\frac{1-\beta_{t+1}}{1-\beta_t}\right)^{\theta_t}\left(R^w_{t+1}\right)^{\theta_t-1}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma\theta_t} \tag{19}$$

where $\theta_t \equiv (1-\gamma_t)/(1-\sigma)$.

As a result, the return R^i_{t+1} on any asset i satisfies a standard Euler equation:

$$\mathbb{E}_t\left[\mathcal{M}_{t,t+1}R^i_{t+1}\right] = 1 \tag{20}$$

Note also that in this endowment economy, the expression for total consumption growth in equilibrium is directly obtained from market clearing and can be plugged in the stochastic discount factor:

$$\ln(C_{t+1}/C_t) = g_{t+1} + n_{t+1} \tag{21}$$

Finally, the model is closed by assuming that consumption per capita growth, population growth, discount rate shocks, and risk appetite shocks, follow autoregressive processes of order 1 with normally-distributed iid errors.

[23] To see this, observe that we can write $H_{t+1}/W_{t+1} = 1-\omega = (1-\omega)(1-YL_t/H_t)/(1-C_t/\bar{W}_t)$ from which we infer that $YL_t/C_t = 1-\omega$, so the second term on the right hand side is identically equal to zero.

Equilibrium and summary of equations The definition of the equilibrium is standard: prices $\{R_t^w, R_t^f\}$ and quantities $\{C_t\}$ are such that (i) the representative agent maximizes her utility (13) subject to her budget constraint (17), and (ii) all markets clear.

Expressed in logs, we have 9 variables $g_t, n_t, \rho_t, \theta_t, cw_t, r_t^w, r_t^f, m_{t+1}, gc_{t+1}$, where $cw_t \equiv \ln(C_t/W_t)$, $r_t^w \equiv \ln R_t^w$, $r_t^f \equiv \ln R_t^f$, $m_{t+1} \equiv \ln \mathcal{M}_{t,t+1}$, and $gc_{t+1} \equiv \ln(C_{t+1}/C_t)$. The system is summarized by the following equations:

$$gc_t = g_t + n_t \tag{22a}$$

$$r_t^w = gc_t - cw_t + cw_{t-1} - \ln(1 - \omega e^{cw_{t-1}}) \tag{22b}$$

$$1 = \mathbb{E}_t \left[e^{m_{t+1} + r_{t+1}^w} \right] \tag{22c}$$

$$1 = e^{r_t^f} \mathbb{E}_t \left[e^{m_{t+1}} \right] \tag{22d}$$

$$m_{t+1} = -\rho_t \theta_t + \theta_t \ln \left(\frac{1 - e^{-\rho_{t+1}}}{1 - e^{-\rho_t}} \right) + (\theta_t - 1) r_{t+1}^w - \sigma \theta_t gc_{t+1} \tag{22e}$$

$$\rho_{t+1} = (1 - \rho_\rho)\mu_\rho + \rho_\rho \rho_t + \sqrt{(1 - \rho_\rho^2)\mathbb{V}_\rho}\varepsilon_{\rho,t+1} \tag{22f}$$

$$\theta_{t+1} = (1 - \rho_\theta)\mu_\theta + \rho_\theta \theta_t + \sqrt{(1 - \rho_\theta^2)\mathbb{V}_\theta}\varepsilon_{\theta,t+1} \tag{22g}$$

$$g_{t+1} = (1 - \rho_g)\mu_g + \rho_g g_t + \sqrt{(1 - \rho_g^2)\mathbb{V}_g}\varepsilon_{g,t+1} \tag{22h}$$

$$n_{t+1} = (1 - \rho_n)\mu_n + \rho_n n_t + \sqrt{(1 - \rho_n^2)\mathbb{V}_n}\varepsilon_{n,t+1} \tag{22i}$$

Resolution method We currently solve the model using perturbation methods at order 3. The third order is important in order for conditional second moments, and therefore risk premia, to be time-varying. However, increasing the order further has no significant impact on the results. In ongoing investigations, we are also in the process of solving the model using global methods, specifically using projection methods based on orthogonal Chebyshev polynomials, or using a time-iteration algorithm à la Kubler & Schmedders (2003) as for instance used in Stephanchuk & Tsyrennikov (2015) and Coeurdacier, Rey & Winant (2019).

6.2 Estimation

We now turn back to the full non-linear model and describe the estimation procedure.

Estimated parameters The parameters of the model are separated into three blocks.

The first block consists in parameters that have well-established values in the literature, or that are difficult to pin down empirically, and are therefore calibrated. This block includes σ, the inverse of the elasticity of substitution, that we set to $1/2$ so that the EIS is 2, and μ_ρ the average value of the discount rate, which is set to 0.03 so that the average return in steady-state ($\beta^{-1} - 1$) is around 3%.

The second block consists in parameters that are estimated ex-ante. This includes $\mu_g, \rho_g, \mathbb{V}_g$ and $\mu_n, \rho_n, \mathbb{V}_n$, the parameters for the consumption per capita growth and population growth processes, as well as ω, the fraction of financial wealth in total wealth. The first six parameters come from the standard maximum likelihood estimation of the AR(1) processes for g and n, which are observed. ω is computed as:

$$\omega = \frac{1 - \exp\{-\mu_\rho - (\sigma - 1)(\mu_g + \mu_n)\}}{\overline{CW}} \tag{23}$$

where $\overline{CW}$ is the average consumption to wealth ratio obtained from the empirical part (0.20941 for the United States, on which we mostly focus in what follows, and 0.20965 for the G4).

The third block consists in the remaining set of parameters, denoted Θ, which are estimated by the Simulated Method of Moments[24,25] described below. This includes the parameters characterizing the discount rate shock (ρ_t) and risk appetite shock (θ_t):

$$\Theta \equiv \{\mu_\rho, \mu_\theta, \rho_\rho, \rho_\theta, \mathbb{V}_\rho, \mathbb{V}_\theta\} \tag{24}$$

[24]Formally, our method is closest to so-called Indirect Inference, as proposed in Smith (1993) and further developed by Gouriéroux, Monfort and Renault (1993). However, we take SMM to refer to the broad family of methods.

[25]In upcoming investigations, we plan, for comparison, to estimate those parameters from the approximate likelihood function obtained from a particle filter.

Table 7 in Section 6.3 summarizes all parameter values, either calibrated or resulting from the estimation.

Simulated Method of Moments Denote $\Theta \in \Omega \subseteq \mathbb{R}^P$ the vector of remaining parameters to be estimated, and $m \in \mathbb{R}^Q$ the moments used in the estimation. The estimator for Θ is:

$$\hat{\Theta} = \arg\min_{\Theta \in \Omega} d\left(\tilde{m}(\Theta), \hat{m}\right)' W d\left(\tilde{m}(\Theta), \hat{m}\right) \tag{25}$$

where $\tilde{m}(\Theta)$ are moments computed on simulated data, $\hat{m}$ are target moments computed on actual data, W is a weighting matrix taken to be the identity, and $d(\cdot,\cdot)$ is a measure of distance between simulated and actual moments. In practice, we take the distance to be either $d\left(\tilde{m}(\Theta), \hat{m}\right) = \tilde{m}(\Theta) - \hat{m}$, the simple difference between simulated and target moments, or $d\left(\tilde{m}(\Theta), \hat{m}\right) = \frac{\tilde{m}(\Theta) - \hat{m}}{\hat{m}}$, the percentage deviation. In what follows, we focus mostly on the former, which appears to be more stable and slightly faster, but the estimation gives broadly similar results if we use the latter, except for parameter $\mathbb{V}_\theta$ that is difficult to pin down regardless of the method.

Moments To maintain consistency with the empirical analysis of Section **??**, a large part of the moments we select relate to the cw_t^f, cw_t^c and cw_t^{rp} components. Specifically, we run the VAR described in that section on both actual and simulated data, and back out the corresponding cw_t^f, cw_t^c and cw_t^{rp} components for each. Six moments are taken to be the upper triangular terms of the unconditional variance-covariance matrix of $cw_t = (cw_t^f, cw_t^c, cw_t^{rp})'$. Two additional moments are the unconditional covariance of gc_t, total consumption growth, with cw_t^c and cw_t^{rf}, which could be helpful in pinning down the parameters for discount rate shocks (ρ_t) and risk appetite shocks (θ_t) (although they also relate to the EIS, that we maintain fixed). Finally, we include the unconditional variance of the risk-free rate, as well as the unconditional variance of risky returns, which corresponds to returns on wealth in the model and are proxied by equity returns in the data, as well as the unconditional risk premium on those risky returns. We expect the last three moments to be particularly helpful in pinning down the parameters for risk appetite shocks (θ_t). In summary, m is a vector of $Q = 11$ moments.

Algorithm details In the current version, the model is solved using perturbation methods, for which we use the Dynare and Dynare++ toolboxes. The resolution is wrapped in the SMM estimation to obtain the parameters that minimize distance $d(\cdot, \cdot)$. In addition, because the starting values at which we initialize the estimation procedure could matter in this non-linear context, we start the estimation at several points throughout the parameter space Ω. For this purpose, we use the dedicated *MultiStart* algorithm from Matlab's Global Optimization Toolbox. For most results, we settle on using 25 different starting points, but have also performed tests with more. Overall, the hope of such an approach is that, even though the estimation started at any particular point might converge only to a local minimum for the distance, testing different starting points could get us closer to a true global minimum.

6.3 Results

Note that, because our sample is significantly longer for the United States, we focus mostly on this country for the estimation procedure and results presented below. In as of yet unreported tests, we are also experimenting with estimating the model on the shorter G4 sample. Although some of those preliminary results are in accordance with those presented below, in particular in terms of summary statistics and variance decompositions, the sample is perhaps a bit limited for the estimation procedure at hand.

6.3.1 Estimated parameters

The values of all parameters are summarized in Table 7.

As a reminder, the value for the calibrated parameters are $\sigma = 1/2$ and $\mu_\rho = 0.03$. For the second block, estimated ex-ante, i.e. before the SMM procedure, we obtain for consumption per capital growth g a constant of $\alpha_g = 0.016$ ($std = 0.003$), an autoregression parameter $\rho_g = 0.081$ (0.062), and a variance $\sigma_g^2 = 1.16 \times 10^{-3}$ (1.15×10^{-4}), and for population growth n a constant of $\alpha_n = 8.08 \times 10^{-4}$ (5.19×10^{-4}), an autoregression parameter $\rho_n = 0.938$ (0.034), and a variance $\sigma_n^2 = 3.39 \times 10^{-6}$ (4.70×10^{-7}). The corresponding parameters in the notation of Section **??** are $\mu_g = \alpha_g/(1-\rho_g)$, $\mu_n =$

Table 7: Summary of parameter values

Parameter	Description	Value
First block: calibrated parameters		
σ	Inverse of EIS	$1/2$
μ_ρ	Average discount factor	0.03
Second block: parameters estimated ex-ante		
μ_g	Mean consumption per capita growth	0.018
ρ_g	AR(1) parameter of consumption per capital growth	0.081
$\mathbb{V}_g$	Variance of consumption per capital growth	1.16×10^{-3}
μ_n	Mean population growth	0.013
ρ_n	AR(1) parameter of population growth	0.938
$\mathbb{V}_n$	Variance of population growth	2.83×10^{-5}
ω	Share of financial wealth in total wealth	6.98%
Third block: parameters estimated by SMM		
ρ_ρ	AR(1) parameter of discount rate shock	0.663
$\mathbb{V}_\rho$	Variance of discount rate shock	2.96×10^{-5}
μ_θ	Mean risk appetite shock	-20.31
ρ_θ	AR(1) parameter of risk appetite shock	0.943
$\mathbb{V}_\theta$	Variance of risk appetite shock	19.36

$\alpha_n/(1-\rho_n), \mathbb{V}_g = \sigma_g^2/(1-\rho_g^2)$ and $\mathbb{V}_n = \sigma_n^2/(1-\rho_n^2)$.

This results in a value for the fraction of financial wealth in total wealth of:

$$\omega = \frac{1 - \exp\{-\mu_\rho - (\sigma - 1)(\mu_g + \mu_n)\}}{\overline{CW}} = 6.98\% \tag{26}$$

for the United States.

Finally, the parameters estimated via SMM are as follows: $\rho_\rho = 0.663, \mathbb{V}_\rho = 2.96 \times 10^{-5}, \mu_\theta = -20.31, \rho_\theta = 0.943$, and $\mathbb{V}_\theta = 19.36$. The implied average risk aversion is therefore $\mu_\gamma = 1 - \mu_\theta(1 - \sigma) = 11.15$. Two remarks are in order. First, note that we are in the process of computing appropriate standard errors. Second, the estimated values for ρ_ρ, $\mathbb{V}_\rho$, μ_θ, ρ_θ and the implied μ_γ appear broadly stable

Table 8: Actual vs. simulated moments at baseline estimation

Moments	Model	Data
$\text{var}(cw_t^f)$	0.0162	0.0159
$\text{cov}(cw_t^f, cw_t^{rp})$	0.0065	0.0035
$\text{cov}(cw_t^f, cw_t^c)$	-0.0009	-0.0062
$\text{var}(cw_t^{rp})$	0.0061	0.0032
$\text{cov}(cw_t^{rp}, cw_t^c)$	0.0014	-0.0018
$\text{var}(cw_t^c)$	0.0009	0.0036
$\text{cov}(cw_t^c, gc_t)$	-0.0002	-0.0005
$\text{cov}(cw_t^f, gc_t)$	0.0002	0.0002
$\text{var}(r_t^f)$	0.0053	0.0024
$\text{var}(r_{t+1}^w)$	0.0267	0.0291
$\mathbb{E}(r_{t+1}^w - r_t^f)$	0.0455	0.0455

across estimations as we vary the number of starting points and the measure of distance $d(\cdot,\cdot)$. On the other hand, we have quite a bit of difficulty pinning down $\mathbb{V}_\theta$, the variance of risk appetite shocks, with the final estimated values often quite dependent on the starting point. This suggests that there is still scope for the estimation methodology to be refined. However, the exact value of the final parameters do not appear to significantly impact the summary statistics and variance decompositions presented after. Appendix **??** provides slightly more discussion on the stability of the estimation by showing the resulting estimated parameters for several of the starting points, and for varying random seeds.

To get a sense of how well moments are matched, Table 8 shows actual moments estimated on the data, and matched moments in simulated data for our baseline estimation. Overall, the average difference between model and actual moments is 24.91%.

6.3.2 Summary statistics

To get a sense of the implications of the model, Table 9 presents its summary statistics. Observe that the model produces a reasonable unconditional average risk premium of 4.45%. This is satisfying given that the unconditional risk premium is one of the moments targeted in the SMM estimation. Observe also that the risk premium is time-varying, which comes both from time-varying risk appetite θ_t and time-varying second moments. Finally, notice that although the mean log consumption to wealth ratio

Table 9: Summary statistics

Variable	Mean (uncond.)	Std. (uncond.)
Exogenous variables		
g_t	0.0169	0.0337
n_t	0.0125	0.0054
gc_t	0.0294	0.0340
ρ_t	0.0299	0.0053
θ_t	-20.6937	4.3455
z_t	0.0000	0.1512
Endogenous variables		
m_{t+1}	-0.2474	0.5660
$\ln(C_t/W_t)$	-1.2211	0.1913
r^w_{t+1}	0.0506	0.1610
r^f_t	0.0048	0.0715
$r^w_{t+1} - r^f_t$	0.0457	0.1996
Conditional moments		
$\mathbb{E}_t[r^w_{t+1}] - r^f_t$	0.0445	0.0083
$\text{var}_t(r^w_{t+1})$	0.0210	0.0045
$\text{cov}_t(r^w_{t+1}, z_{t+1})$	-0.0187	0.0044
$\text{cov}_t(r^w_{t+1}, gc_{t+1})$	0.0012	0.0000

implies an average consumption to wealth ratio that is slightly higher than the calibrated value, its steady-state value is $\overline{CW} = 0.20941$ as expected. The differences between summary statistics and steady-state values stem from the fact that summary statistics are computed on simulated data. As the sample grows, those moments will converge to steady-state values but this might require a somewhat large sample given that we solve the model at the third order and that non-linearity could be important.

6.3.3 Variance decomposition

We now turn to assessing the contribution of each exogenous variable to the variations in risky returns, the risk-free rate, the risk premium, and the consumption-wealth ratio. To that end, we compute a variance decomposition of the endogenous variables with respect to each of the exogenous shocks: $\varepsilon_{g,t}, \varepsilon_{n,t}, \varepsilon_{\rho,t}$ and $\varepsilon_{\theta,t}$. Note that with a model solved at the third order, i.e. non-linearly, computing a variance decomposition is not necessarily obvious. The reason is that, contrary to a linear model

Table 10: (Simulated) Variance decomposition

Variable	Shock			
	$\varepsilon_{g,t}$	$\varepsilon_{n,t}$	$\varepsilon_{\rho,t}$	$\varepsilon_{\theta,t}$
m_{t+1}	55%	19%	17%	9%
$\ln(C_t/W_t)$	0%	2%	97%	1%
r^w_{t+1}	5%	1%	95%	0%
r^f_t	0%	0%	99%	1%
$r^w_{t+1} - r^f_t$	3%	0%	97%	0%
$\mathbb{E}_t[r^w_{t+1}] - r^f_t$	0%	0%	26%	73%
$\text{var}_t(r^w_{t+1})$	0%	0%	100%	0%

Note: We normalize the contributions by the total variance for each endogenous variable so that they sum to 100%. They initial do not due to non-zero correlation between shocks in small samples, and non-linearity.

in which the contributions of each shock add up exactly to the total variance, this is no longer true at higher orders. This comes from the fact that shutting certain shocks down might shut down interaction terms that represent a significant part of the total variance. In other words, the contribution of a given shock might depend on the values of the other exogenous variables, and the variance decomposition is state-dependent. For those reasons, it is no longer necessarily accurate to perform a simple variance decomposition as is usually done in the VAR literature. Keeping those caveats in mind, we proceed by computing a simulated variance decomposition, that is, we compute the contribution of a shock by simulating the model for each shock at a time. This ignores some of the interaction effects.

Table 10 shows the results for our current baseline estimation. The most striking feature is that, in this estimation, discount factor shocks (ρ_t) explain the vast majority ($> 90\%$) of variations in the consumption to wealth ration, the risky returns, and the risk-free rate. This is so despite the fact that the stochastic discount factor is impacted by all four shocks significantly. Risk appetite shocks (θ_t) appear to matter only for the risk premium, $\mathbb{E}_t[r^w_{t+1}] - r^f_t$.

Importantly, note that even though the exact values of the parameters estimated by SMM vary, the variance decomposition results seem broadly unaffected, and if anything reinforced.

7 Conclusion.

Our results suggest that macro and financial shocks are both important determinants of global real rates. On the macro side, there is some evidence that productivity growth and demographic shocks affect global rates. On the financial side, the two significant declines in C/W occurred in the years preceding -and in the aftermath- of global financial crises. These boom-bust financial cycles are a strong determinant of real short term interest rates. During the boom, private wealth increases rapidly, faster then consumption, bringing down the ratio of consumption to private wealth. This increase in wealth can occur over the course of a few years, fueled but increased leverage, financial exuberance, and increased risk appetite. Two such historical episodes for the global economy are the roaring 1920s and the 2000s. In the subsequent bust, asset prices collapse, collateral constraints bind, and households, firms and governments attempt to simultaneously de-leverage, as risk appetite wanes. The combined effect is an increase in desired saving that depresses persistently safe real interest rates. An additional force may come from a weakened banking sector and financial re-regulation or repression that combine to further constrain lending activity to the real sector. Our estimates indicate that short term real risk free rates are expected to remain low or even negative for an extended period of time.

The central object of our analysis are risk free rates. In recent years, an abundant empirical literature has attempted to estimate the natural rate of interest, r^*, defined as the real interest rate that would obtain in an equivalent economy without nominal frictions. Many estimates indicate that this natural rate may well have become significantly negative. Our analysis speaks to this debate. Outside of the effective lower bound, monetary policy geared at stabilizing prices and economic activity will set the policy rate so that the real short term rate is as close as possible to the natural rate. Therefore, to the extent that the economy is outside the ELB, our estimate of future global real rates should coincide with estimates of r^*. At the ELB, this is not necessarily the case since global real rates must, by definition of the ELB, be higher than the natural rate. Therefore, our estimates provide an upper bound on future expected natural rates. Given that our estimates are quite low (-2.35 percent on average between 2015 and 2025), we conclude that the likelihood of the ELB binding remains quite elevated.

Our empirical results suggest that over long horizons, global real rates are driven both by standard structural forces, such as productivity or demographic forces, as well as financial forces, especially the leveraging cycle that accompanied the boom and bust in the 1930s and in the 2000s.

We view these empirical results very much in line with interpretations of recent events that emphasize the global financial cycle (Miranda-Agrippino and Rey (2015), Reinhart and Rogoff (2009)).

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Appendix

A Data description

The data used in Section 4 were obtained from the following sources:

1. **Consumption:**
   Real per-capita consumption going back to 1870 and covering the two world wars was taken from Jordà et al. (2016) who in turn obtained the data from Barro and Ursúa (2010). As this consumption series is an index rather than a level, we convert it to a level using the consumption data from Piketty and Zucman (2014a). To convert to a level we could use any year we have level data for but chose to use the year 2006 (the year that the index of consumption was 100). In addition, the consumption data was adjusted so that instead of being based on a 2006 consumption basket, it was based on a 2010 consumption basket to match the wealth data.

2. **Wealth:**
   Real per capita wealth data was taken from Piketty and Zucman (2014b). The wealth concept used here is private wealth. As such it does not include government assets but includes private holdings of government issued liabilities as an asset. Where possible, wealth data is measured at market value. Human wealth is not included. Private wealth is computed from the following components: "Non-financial assets" (includes housing and other tangible assets such as software, equipment and agricultural land), and net financial assets (includes equity, pensions, value of life insurance and bonds). Prior to 1954 for France, 1950 for Germany, 1920 for the UK and 1916 for the USA, wealth data is not available every year (see Piketty-Zucman's appendix for details on when data is available for each country or refer to Table 6f in the data spreadsheets for each country). When it is available is is based on the market value of land, housing, other domestic capital assets and net foreign assets less net government assets. For the remaining years the wealth data is imputed based on savings rate data and assumptions of the rate of capital gains of wealth (see the Piketty-Zucman appendix for details of the precise assumptions on capital gains for each country. The computations can be found in Table 5a in each of the data spreadsheets for each country).

3. **Short term interest rates:**
   These were taken from Jordà et al. (2016) and are the interest rate on 3-month treasuries.

4. **Long term interest rates:**
   These were taken from Jordà et al. (2016) and are the interest rate on 10 year treasuries.

5. **Return on Equity:**
   This data is the total return on equity series taken from the Global Financial Database.

6. **CPI:**

   CPI data is used to convert all returns into real rates and is taken from Jordà et al. (2016).

7. **Population:**

   These were taken from Jordà et al. (2016).

   Figure 6 reports consumption per capita, wealth per capita, the consumption/wealth ratio as well as the short term real risk free rate for our G4 aggregate between 1920 and 2011.

B Loglinearization of the budget constraints and aggregation

For a country i the budget constraint takes the form:

$$\bar{W}^i_{t+1} = \bar{R}^i_{t+1}(\bar{W}^i_t - C^i_t) \tag{27}$$

where $\bar{W}^i_t$ denotes total wealth at the beginning of period t, C^i_t is private consumption during period t and $\bar{R}^i_{t+1}$ is the gross return on total wealth between periods $t$and $t+1$. All variables are measured in real terms. Lettau and Ludvigson (2001) propose a log-linear expansion around the steady state consumption-to-wealth ratio and steady state return. Define $cw^i_t = \ln C^i_t - \ln \bar{W}^i_t$. cw^i_t is stationary with mean $\overline{cw}^i$. Dividing both side of (27) by $\bar{W}^i_t$ and taking logs, we obtain:

$$\begin{aligned}
\ln \bar{W}^i_{t+1}/\bar{W}^i_t &= \bar{r}^i_{t+1} + \ln(1 - C^i_t/W^i_t) \\
&= \bar{r}^i_{t+1} + \ln(1 - e^{\overline{cw}^i}\exp(cw^i_t - \overline{cw}^i)) \\
&\approx \bar{r}^i_{t+1} + \ln(1 - e^{\overline{cw}^i} - e^{\overline{cw}^i}(cw^i_t - \overline{cw}^i)) \\
&\approx \bar{r}^i_{t+1} + \ln\left((1 - e^{\overline{cw}^i})\left(1 - \frac{e^{\overline{cw}^i}}{1 - e^{\overline{cw}^i}}(cw^i_t - \overline{cw}^i)\right)\right) \\
&\approx \bar{r}^i_{t+1} + \ln(1 - e^{\overline{cw}^i}) - \frac{e^{\overline{cw}^i}}{1 - e^{\overline{cw}^i}}(cw^i_t - \overline{cw}^i) \\
&\approx \bar{r}^i_{t+1} + k + \left(1 - \frac{1}{\rho_w}\right) cw^i_t
\end{aligned}$$

where $\rho_w = 1 - e^{\overline{cw}^i}$ and k is an unimportant constant. The next step is to rewrite the left hand side as

$$\ln \bar{W}^i_{t+1}/\bar{W}^i_t = \ln(\bar{W}^i_{t+1}/C^i_{t+1}) - \ln(\bar{W}^i_t/C^i_t) + \Delta \ln C^i_{t+1} = -cw^i_{t+1} + cw^i_t + \Delta \ln C^i_{t+1}$$

to obtain (again, ignoring the constant):

$$cw^i_t = \rho_w \left(cw^i_{t+1} - \Delta \ln C^i_{t+1} + \bar{r}^i_{t+1}\right) \tag{28}$$

which can be iterated forward to obtain (under the usual transversality condition):

$$c_t^i - \bar{w}_t^i = \sum_{s=1}^{\infty} \rho_w^s \left(\bar{r}_{t+s}^i - \Delta \ln C_{t+s}^i \right)$$

C Aggregation

From Eq. (1) we can aggregate across countries:

$$\sum_i \frac{\bar{W}_{t+1}^i}{\bar{R}_{t+1}^i} = \sum_i \bar{W}_t^i - C_t^i = \bar{W}_t - C_t$$

where $\bar{W}_t = \sum_i \bar{W}_t^i$ and $C_t = \sum_t C_t^i$. From this expression we can derive

$$\bar{W}_{t+1} = \bar{R}_{t+1}(\bar{W}_t - C_t)$$

where

$$\frac{1}{\bar{R}_{t+1}} = \sum_i \frac{\bar{W}_{t+1}^i}{\bar{W}_{t+1}} \frac{1}{\bar{R}_{t+1}^i}$$

The global period return on private wealth is an harmonic weighted mean of the individual country returns.

D VAR methodology

Consider the present value relation in Eq. 4. We form $\mathbf{z}_t = (\ln C_t - \ln W_t, r_t, \mathbf{ep_t}', \Delta \ln C_t,)'$ and estimate Vector AutoRegression of order p, VAR(p), which can be expressed in companion form as:

$$\bar{\mathbf{z}}_t = \bar{\mathbf{A}} \bar{\mathbf{z}}_{t-1} + \bar{\varepsilon}_t$$

where $\bar{\mathbf{z}}_t' = \left(\mathbf{z}_t', \mathbf{z}_{t-1}', ..., \mathbf{z}_{t-p}' \right)$. Using the estimated VAR matrix $\bar{\mathbf{A}}$, conditional forecasts of $\bar{\mathbf{z}}_t$ can be directly obtained as:

$$\mathbb{E}_t \bar{\mathbf{z}}_{t+k} = \bar{\mathbf{A}}^k \bar{\mathbf{z}}_t$$

from which we recover:

$$\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \bar{\mathbf{z}}_{t+s} = \sum_{s=1}^{\infty} \rho_w^s \bar{\mathbf{A}}^s \bar{\mathbf{z}}_t = \rho_w \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}} \right)^{-1} \bar{\mathbf{z}}_t.$$

Denote $\mathbf{e}_x$ the vector that 'extracts' variable x from $\bar{\mathbf{z}}$, in the sense that $\mathbf{e}_x' \bar{\mathbf{z}} = x$. It follows that

$$\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \mathrm{x}_{t+s} = \rho_w \mathbf{e}_x' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}} \right)^{-1} \bar{\mathbf{z}}_t$$

From this we can construct the various components as:

$$
\begin{aligned}
cw_t^f &= \rho_w \mathbf{e}_r' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t \\
cw_t^c &= -\rho_w \mathbf{e}_{\Delta \ln C}' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t \\
cw_t^{rp} &= \nu \rho_w \mathbf{e}_{erp}' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t \\
cw_t^{\Delta \ln c} &= -\rho_w \mathbf{e}_{\Delta \ln c}' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t \\
cw_t^n &= -\rho_w \mathbf{e}_n' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t
\end{aligned}
$$