# 1 Introduction

In this note we analyse the dataset used in the critique of our paper by Chen and Engel. We find the following:

1. *Heterogeneity does matter for the persistence of relative prices and, taking heterogeneity into account, we confirm our results of the main paper:* We find half-lives close to 2 years. We have now estimated low half-lives, when correcting for heterogeneity, in a large number of alternative datasets. In other words: (a) The results in the main paper are not due to measurement errors or sampling, and (b) heterogeneity matters.

2. In the sample selected by Chen and Engel, the RCM (random coefficients model) estimator performs worse than in any other case that we have examined. This, we show, can be attributed to the relatively small cross section used by Chen and Engel. The point of our paper was certainly not to argue in favor of one type of estimator or another, *provided* the estimator used allowed for heterogeinity in the dynamics of the subcomponents. We are aware of potential problems with the RCM estimator if the cross section used is small, and always checked our results with the MG (mean group) estimator as well. Both estimators allow for heterogeinity in the dynamics and are *equivalent asymptotically*[1]. In the sample selected by Chen and Engel, in fact, tests reject the RCM in favour of the MG model, unlike in our original paper.[2] When the proper estimator is used on the Chen Engel dataset, all our results are confirmed.

# 2 Data

Chen and Engel's dataset has 127 cross sections for the period 1981:01-1996:12. There are 9 countries and a maximum of 16 goods. Note that the coverage is considerably less than the data used in our paper. In particular, Greece, Finland and Ireland are missing, as are goods such as rents (1310) and Tobacco (1140). In fact, the number of cross sections is almost half of what was used in our paper[3]. With a smaller cross-sectional dimension and a shorter sample, it is to be expected the RCM estimator performs less well, but the MG estimator still has good small sample properties. We now discuss this in detail.

[1] See appendix A for a proof of the asymptotic equivalence of these two estimators in a simple case.

[2] Letting the data choose between RCM or MG is very much akin to a Hausman test. RCM assumes sector-specific heterogeneity to be random, whereas MG assumes it to be deterministic. The analogy carries through in some of the following discussion: one would not want to push results based on a Random Effect model when a Fixed Effect model is a better representation of the data.

[3] Our dataset had 221 cross sections.

# 3 Results

Table 1 confirms that we closely reproduce Chen and Engel's RCM and FE estimates when using their data. However, the Mean group estimator produces a much shorter half-life[4]. Figure 1 shows that this remains true for all possible lag lengths. The implication is clear. *Allowing for heterogeneity makes a significant difference in this sample as well. The Hausman test strongly rejects parameter homogeneity.*

Chen and Engel reject that heterogeneity matters on the basis of the RCM estimator but in this much smaller sample the RCM estimator does not perform well. Why do RCM and MG not perform equally well? Both estimators allow for slope heterogeneity, but only the former imposes distributional assumptions on heterogeneity. Using Chen and Engel's data a Hausman test for heterogeneity strongly rejects homogeneous slopes. However, the alternative hypothesis could be either heterogeneous and fixed or heterogeneous and random. In other words, the alternative hypothesis is consistent with both MG and RCM. In order to distinguish between the two we use a test devised by Pudney (1978).[5] This is a test for the assumptions underlying random coefficients. A rejection of the null hypothesis implies rejection of the random coefficient assumption. In an AR(5) model we obtain a test statistic of 126.72003 (0.0000). This implies that the mean group model is more appropriate in these data.[6]

## 3.1 Expanding the dataset

The fact that Chen and Engel's panel is narrower than ours could also explain the problems with the RCM model, especially at high lags. This is particularly important as the heterogeneous estimators we propose are essentially averages and their consistency and efficiency depends on the cross-section of the panel. We conducted a simple experiment. Data was generated from a heterogeneous data generating process, with 220 cross sections. Then we estimated RCM models only on the first 100 cross sections. Figure 2 plots the distribution of the resulting estimates and contrasts it with estimates from the whole panel. It is clear that the estimator that uses fewer cross sections has a much more dispersed distribution, i.e. the estimates are less precise. This problem is likely to be more severe as the number of parameters increases.

Direct evidence on the importance of this point can be seen in table 2. Table 2 lists estimates obtained when Chen and Engel's dataset is expanded. We add the following: 1) Data for Greece, Finland and Ireland. 2) Data for Tobacco and Rents. In each case, the data for all countries was checked and any outliers were removed, in a way similar to the selection method described on Charles

[4] All half-lives in this note are defined as the number of periods it takes for the impulse response to cross 0.5 *permanently*,.thus addressing some of the definitional concerns in Chen and Engel.

[5] The Estimation and Testing of some Error Component Models. Pudney, S.E. London School of Economics 1978.

[6] Note that in our original data the statistic was 10.29169 (0.9).

Engel's website. This gives us a panel with 191 cross sections, a number closer to our original data. The fixed effects half-life is close to Chen and Engel's estimate. The heterogeneous estimators, however, now produce shorter half-lives. In particular the MG model gives a half-life of 20 months. Figure 3 plots the half-lives obtained from these estimators against the lag-length. The MG half-lives are consistently less than 2 years. The RCM model produces half-lives close to 2 years, whereas the Fixed Effects estimator is biased upwards. Note that RCM and MG converge at higher lag lengths, as they should. Note also that we do not observe the kind of impulse responses documented by Chen and Engel.[7]

## 3.2 A discussion of outliers

Chen and Engel correct the data by removing outliers and parts of the series which appear inconsistent. We do a similar exercise when we expand their data. Clearly, the original Eurostat dataset contains typos that should be corrected, and we fully agree that some other observations may look odd and suspicious. Indeed we had thoroughly looked for typos when we performed our original estimates.

However, removing "odd" data may also be problematic since it introduces a degree of subjectivity. In other words, there is a chance that it removes informative shocks. In fact, it is possible that such a procedure may produce persistence. We can infer the impact of this from the following experiment: 10,000 AR(1) processes with a autoregressive coefficient of 0.95 were generated. Estimation was carried out on i) the generated series without any changes ii) on series where "outliers" where replaced by an average over t+1 and t-1. The mean estimated half-life in case (i) is 13.52 which is very close to the true half-life of 13.51. In case (ii) this goes up to 22.5. Figure 4 plots the distribution of the estimates. It is easy to see that in case (ii) the distribution is much more dispersed around a higher mean.

This is perhaps not very surprising because in this case outliers are erroneously removed. In reality, many of the "corrected" data may of course be true measurement errors. However, correcting for measurement errors on the basis of removing/replacing outliers, is problematic since it would for instance not remove "small" measurement errors. For that reason more objective approaches to the measurement error problem might be desirable.

In our paper, we also reported RCM estimates based on GMM estimators with instruments chosen to account for errors in variables. We showed this did not have impact the results. We now examine how this estimator performs. We generate data for AR(1) models using a coeffcient of 0.95. Then we add

[7] We tried various other combinations of the data. For example, adding data for Greece and Ireland only, produces very similar results. In addition we considered removing every series that has repeated observations (as noted by Charles Engel on his web page). Again the MG estimator gives a half-life of 23 for an AR(5) model. In any case, we should not expect this particular aspect to be a problem, because any bias that is created is positive. This was demonstrated through simulation.

a random error $\nu$ to this data where $\upsilon\tilde{}N(0, 0.3)$. A typical sample is shown in Figure 5. It can be seen that the series with the error has many possible outliers. Next we estimate models using OLS which is expected to be biased and GMM which is consistent. The distribution of the resulting estimates is shown in figure 6. There is a downward bias in OLS, however, GMM performs much better and its mean is close to the true estimate. This does indicate that if error in variables were a substantial problem we should have observed a large difference between RCM estimates based on OLS (Table 3 in the paper) and GMM (Table 4). In our paper we found very similar results from using either estimator indicating that measurement errors do not account for our results.

## 4 Conclusions

We find that the qualitative features of our results hold, even when we examine the much smaller sample proposed by Chen and Engel. We are grateful to Chen and Engel for stimulating discussion about our results and for giving us the opportunity to respond to their criticism and to show that our conclusions are very similar even when we restrict attention to their suggested alternative sample.

# 5 Appendix A: Asymptotic equivalence of RCM and MG estimators

A very simple way to show the equivalence between the RCM (Random Coefficient Model) and the MG (Mean Group) estimators is as follows.

Consider a panel with two cross sections and $t_i$ observations. Let the OLS coefficients be $\hat{b}_1$ and $\hat{b}_2$ and covariance matrices $V_1$ and $V_2$ where:

$$V_i = E\left[(\hat{b}_i - b_i)(\hat{b}_i - b_i)'\right]$$

where $b_i$ is the true value of the coefficient. Consider the MG estimator in this panel:

$$\beta_{mg} = \frac{1}{2}(b_1 + b_2)$$

The RCM estimator is:

$$\begin{aligned}\beta_{rcm} &= \left(\frac{1}{2}\frac{b_1^2 + b_2^2 - 2b_1b_2 + 2V_2}{b_1^2 + b_2^2 - 2b_1b_2 + V_2 + V_1}\right) b_1 + \left(\frac{1}{2}\frac{b_1^2 + b_2^2 - 2b_1b_2 + 2V_1}{b_1^2 + b_2^2 - 2b_1b_2 + V_2 + V_1}\right) b_2 = \\ &\Psi_1 b_1 + \Psi_2 b_2\end{aligned}$$

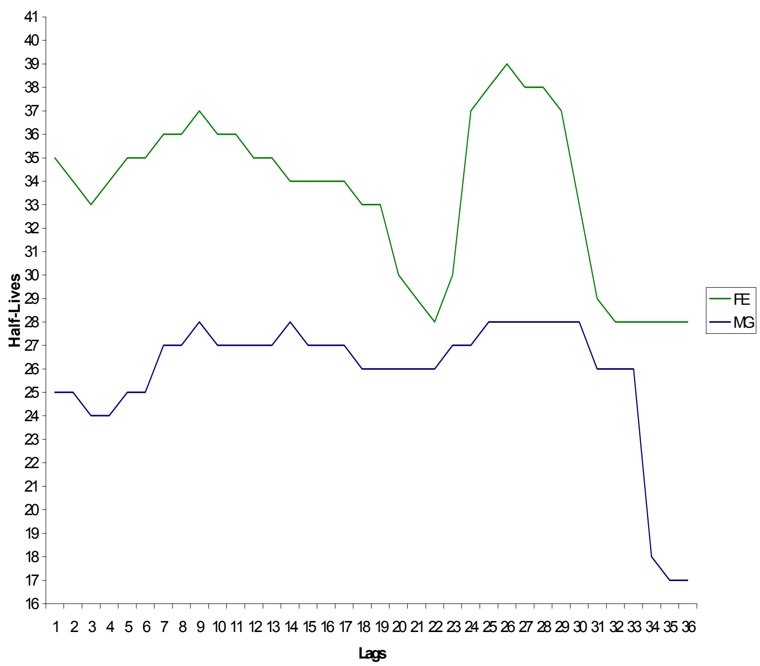
As $t_i \to \infty$, $\left(\hat{b}_i - b_i\right) \longrightarrow^p 0$ and $\Psi_i \longrightarrow \frac{1}{2}$ as $V_i$ gets smaller. In other words for large $T$, the variance of the estimators gets very small and the RCM weights approach the MG weights.

# 6 Tables and Figures

| Table 1: | | | |
|---|---|---|---|
| Method | p | $\sum \rho$ | Half-Life |
| *FE* | *12* | *0.97767* | *35* |
| *RCM* | *5* | *0.97951* | *35* |
| **MG** | **5** | **0.97063** | **25** |
| Hausman Test | 80.804(0.000) | | |

| Table 2 | | | |
|---|---|---|---|
| Method | p | $\sum \rho$ | Half-Life |
| *FE* | *12* | *0.97757* | *33* |
| *RCM* | *5* | 0.97247 | *26* |
| **MG** | **5** | 37.068 | **21** |
| Hausman Test | 37.068(0.000) | | |

<u>Figure 1</u>



41
40
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17
16
Half-Lives
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36
Lags
FE
MG

Figure 2

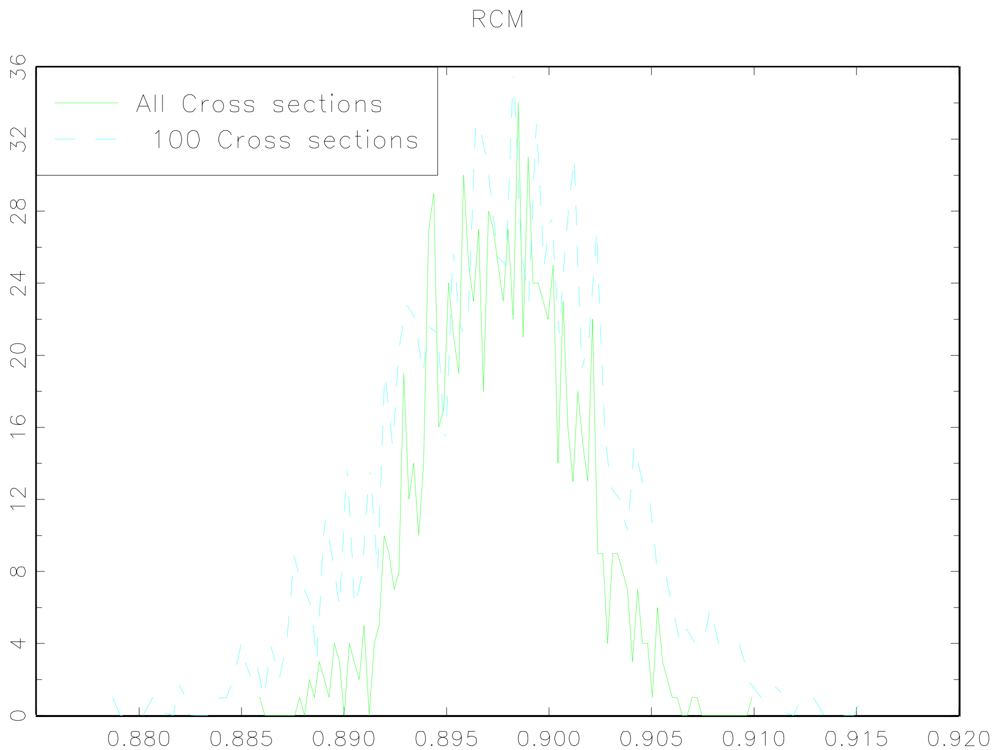
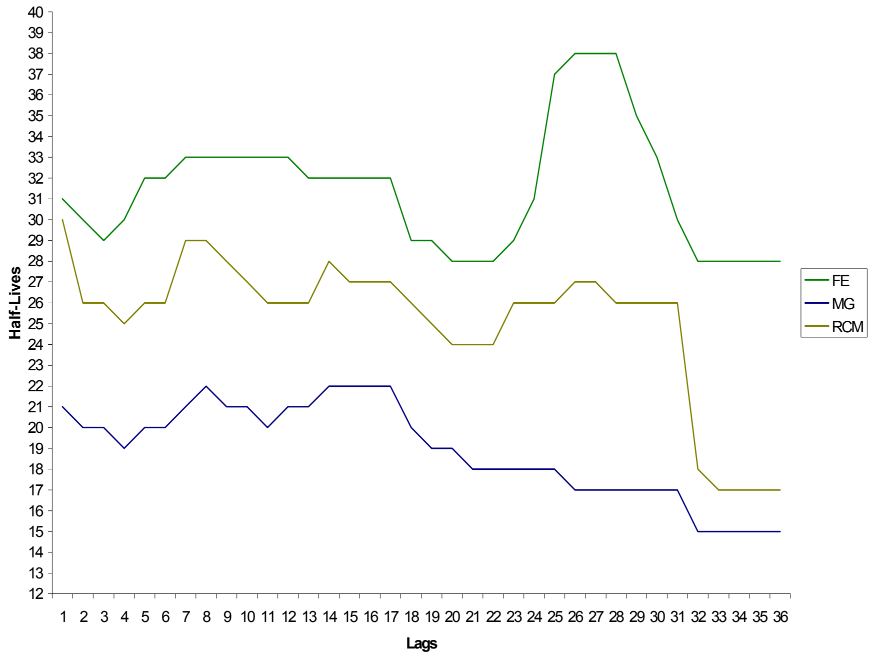

RCM
All Cross sections
100 Cross sections
0
4
8
12
16
20
24
28
32
36
0.880
0.885
0.890
0.895
0.900
0.905
0.910
0.915
0.920

Figure 3


Half-Lives
40
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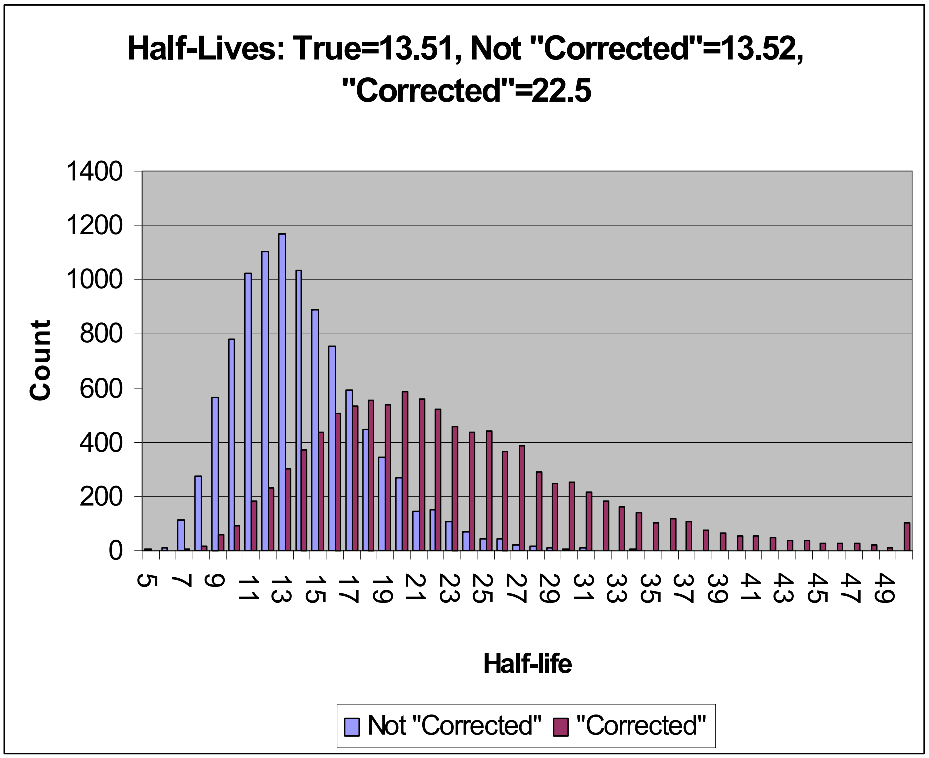
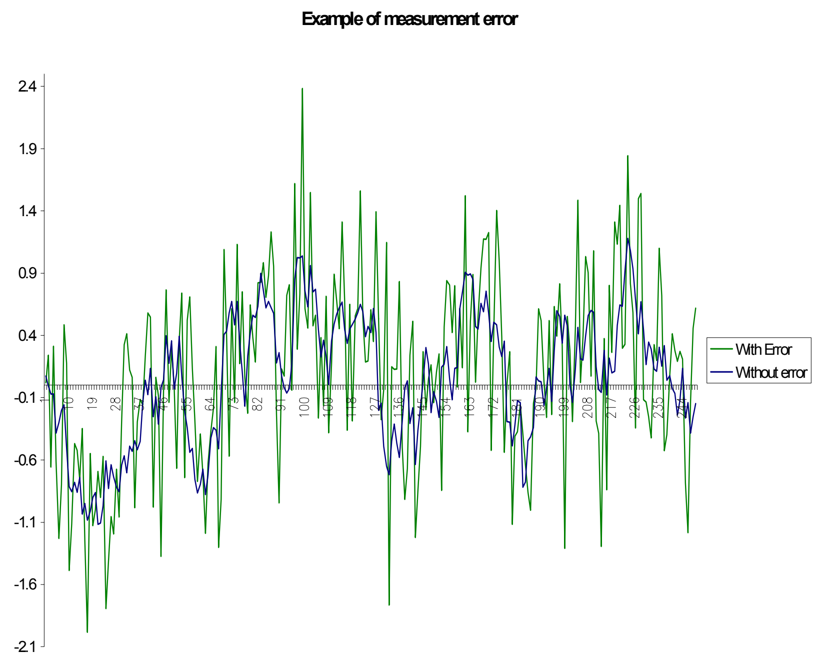
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36
Lags
FE
MG
RCM

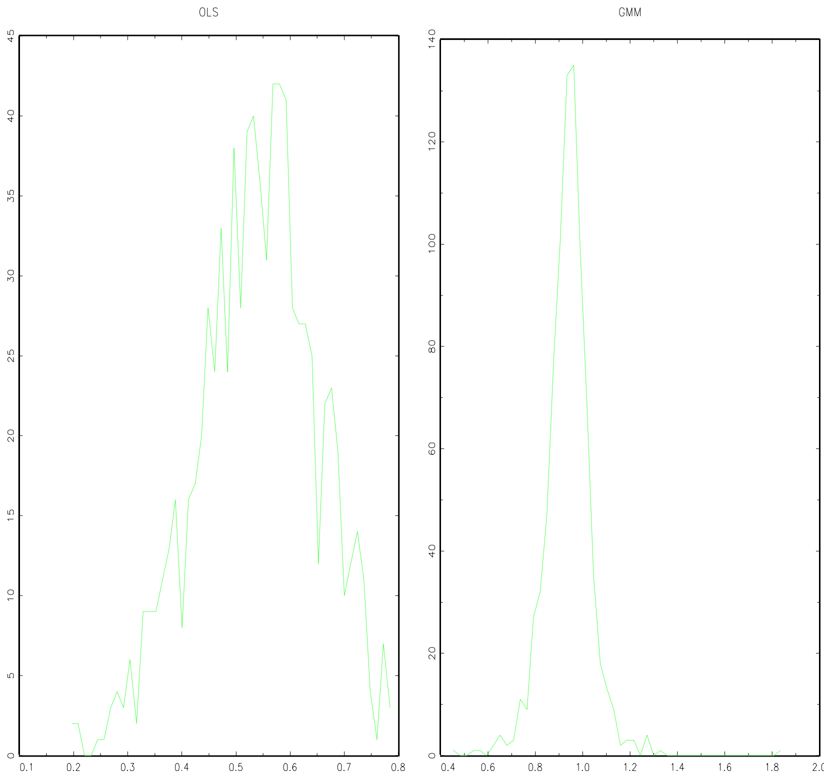
Figure 4


Half-Lives: True=13.51, Not "Corrected"=13.52, "Corrected"=22.5
1400
1200
1000
800
600
400
200
0
Count
5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41 43 45 47 49
Half-life
Not "Corrected"
"Corrected"

<u>Figure 5</u>


Example of measurement error
2.4
1.9
1.4
0.9
0.4
-0.1
-0.6
-1.1
-1.6
-2.1
With Error
Without error

<u>Figure 6</u>


OLS
GMM
0
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10
15
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25
30
35
40
45
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
20
40
60
80
100
120
140
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0