

## 1 Introduction

This note addresses some points raised by Charles Engel and Shiu-Sheng Chen about the monte-carlo experiments in our paper. We have checked our results and done a few more Monte Carlo experiments. Our results confirms the existence of the aggregation bias that we discuss in the paper.

## 2 Experiment 1

We repeat our original experiment using the following DGP:

$$\begin{aligned} y_{i,t} &= \lambda_i y_{i,t} + v_{i,t} \\ v_{i,t} &\sim N(0, 1), \\ N &= 221, T = 100 \end{aligned} \tag{1}$$

$\lambda_i$  is sampled from cross section specific OLS estimates obtained from our original panel. That is, we approximate the distribution of  $\lambda$  by drawing iid “bootstrap” samples from the  $N \times 1$  vector of estimates of  $\lambda_i$  obtained via cross section specific OLS on the panel of relative prices. At each iteration,  $N$  samples of  $\lambda$  are drawn and  $N$  AR(1) series generated. These are then averaged and an AR(1) model is estimated. Table 1 gives the result from 1000 iterations:

	$\lambda$	True
	0.95830609	0.93067258

This indicates a considerable positive OLS bias.

## 3 Experiment 2

The above results may suggest that the experimental design used by Charles Engel may be inappropriate as it restricts the degree of heterogeneity too much. It can also be argued that given the fact that unit roots are not rejected for some relative prices, excluding all non-stationary series from the monte-carlo samples may not be valid.

Also, for near unit root processes that Charles Engel considers, there is a competing negative bias in the OLS estimates that we discuss in some detail in the paper. In this case, then the experiment will give a distorted picture if the estimates are compared to the mean of the  $\lambda$ 's used to start the experiment (because the estimated coefficients are biased downwards).

In order to deal with these issues we modify the experiment suggested by Charles Engel in the following way. We assume  $N, T=200$  and draw the AR series from the following DGP:

$$\begin{aligned}
y_{i,t} &= (\lambda + \eta_i) y_{i,t} + v_{i,t} & (2) \\
\lambda &= 0.8 \\
\eta_i &\sim Uniform(\pm 0.19) \\
v_{i,t} &\sim N(0, 1)
\end{aligned}$$

This ensures that problems associated with a small sample and near unit root processes are minimised and a larger degree of heterogeneity can be allowed for without including explosive series. Results are in table 2.

	$\lambda$	True
	0.85371825	0.80021305

Again, as above we see that the aggregation bias is a real issue as we argue in the paper.

## 4 Experiment 3

Charles Engel's experiment also appears to be sensitive to the assumption of a uniform distribution for  $\eta_i$ . We repeat the experiment above (N=100, T=150), using a normal distribution for  $\eta$  truncated around  $\pm 0.059$  and with  $\lambda = 0.94$ . The estimated persistence of the average series is 0.95428 compared to a true value of 0.94012. We also consider an experiment with  $\lambda = 0.9$  with  $\eta_i$  following a normal truncated around  $\pm 0.099$ . The estimate from the *average series is 0.934476* compared to the true average value of 0.899634.

## 5 Experiment 4

We now investigate the claims in Shiu-Sheng Chen (2003). The dgp is the following:

$$\begin{aligned}
y_{i,t} &= \alpha_i + (\lambda + \eta_i) y_{i,t} + v_{i,t} & (3) \\
\lambda &= 0.94 \\
\alpha_i, \eta_i &\sim Uniform(\pm 0.59) \\
v_{i,t} &\sim N(0, 1)
\end{aligned}$$

The generated data is organised into a panel with N, T=100 and fixed effects estimation is carried out. The average persistence estimated by FE is 0.977778 compared to a true value of 0.9400837.

We also estimate aggregate fixed effects on this panel. The 100 cross sections are averaged down to 10. Fixed effects on this panel produces average persistence of 0.976014 compared to a true value of 0.9400837.

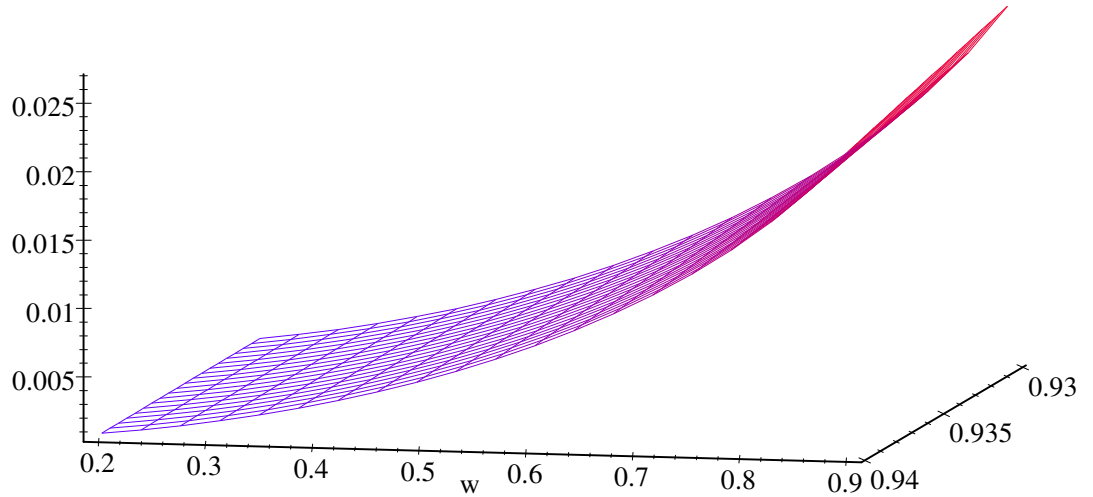
There is clear evidence of aggregation bias. Shiu-Sheng Chen uses 0.7 as a value for  $\lambda$ . The experiment in figure 2b of our paper shows that the bias is more important for more persistent panels and this is what we find here, even under restrictive assumptions.

## 6 Experiment 5

We now consider the expression for the bias derived in the paper and examine the movement of the bias for different values of  $\lambda$  and the degree of heterogeneity.

$$\hat{\lambda} - \lambda = \frac{E\left(\frac{\eta_i}{1-\lambda_i^2}\right)}{E\left(\frac{1}{1-\lambda_i^2}\right)} \quad (4)$$

Assuming that  $\eta_i \sim Uniform(\pm\varpi((1-\lambda)))$ ,  $0 \leq \varpi < 1$ ,  $0 \leq \lambda < 1$  and using (3) one can derive an expression for  $\hat{\lambda} - \lambda$  which can be plotted for different values of  $\varpi$  and  $\lambda$ . Note that explosive roots are ruled out. Figure 1 shows the resulting graph



In terms of the half-life, the bias is substantial even without explosive roots.

## 7 Conclusions

It is worth noting that quite a lot of monte-carlo work has been carried out on the aggregation bias in other papers, where results similar to ours have been

found. Examples include Robertson and Symons (1992) and Pesaran, Smith and Im (1996).

## References

- [1] Pesaran, H. R. P. Smith and K. S. Im, Dynamic Linear Models for Heterogeneous Panels. In *The Econometrics of Panel Data* (eds) L. Mátyás and P. Sevestre, 1996, chapter 8, pp.145-195, Kluwer Academic Publishers, Dordrecht, The Netherlands. ISBN 0 7923 3787 5.
- [2] D. Robertson; J. Symons. Some Strange Properties of Panel Data Estimators. *Journal of Applied Econometrics*, Vol. 7, No. 2. (Apr. - Jun., 1992), pp. 175-189.