Online Appendix for "International Financial Adjustment"

Pierre-Olivier Gourinchas
UC Berkeley, NBER and CEPR

Hélène Rey
Princeton, NBER and CEPR

June 2007

1 Appendix A: Robustness results

The first robustness check consists in using Christiano and Fitzgerald's (2003) asymmetric bandpass filter as an alternative to the Hodrick-Prescott filter (footnote 19 in Gourinchas and Rey (2007)). The second robustness check looks at the detrending of the returns on gross assets, gross liabilities and the growth rate of household financial wealth (footnote 21). The last robustness check looks construct the weights for nxa separately for the Bretton Woods and post Bretton Woods period (footnote 23).

1.1 Christiano and Fitzgerald's (2003) asymmetric filter.

Figure 1 reports the trends constructed with Christiano and Fitzgerald's (2003) asymmetric filter.[1] The residual terms $\epsilon_t^{z,CF}$ are very similar to those obtained in our benchmark estimation (see figure 2, pp46 in Gourinchas and Rey 2007).

Given the trends, we obtain the following estimates of the weights (compare to p21 of Gourinchas and Rey 2007):

$$\mu^{a,CF} = 8.98;\ \mu^{l,CF} = 7.98;\ \mu^{x,CF} = -9.84;\ \mu^{m,CF} = -10.84;\ \rho^{CF} = 0.94$$

and construct nxa_t^{CF} as (compare to p22 of Gourinchas and Rey 2007):

$$nxa_t^{CF} = 0.91\epsilon_t^{a,CF} - 0.81\epsilon_t^{l,CF} + \epsilon_t^{x,CF} - 1.10\epsilon_t^{m,CF}$$

Figure 2 reports nxa^{CF}. Comparing with figure 4 of Gourinchas and Rey (2007), it is clear that the two approaches give a very similar account of cyclical imbalances.[2]

1.1.1 VAR decomposition

Table 1 column 1 reports the VAR decomposition. The overall fit of the VAR is also reported on figure 2. The fit of the VAR remains excellent, with the same unconditional contribution of the return component (0.27) and slightly lower overall fit (0.86 vs. 0.91). We cannot reject that the restriction on the VAR is satisfied (p-value of 0.96).

1.1.2 In Sample Forecasts

Table 2 reports the results from the in-sample long horizon regression. Comparing the results to table 3 of Gourinchas and Rey (2007), we find most of the results unchanged.

[1] We construct the filter assuming (a) no unit root; (b) a linear trend and (c) extracting all frequencies between 2 and 200 quarters.

[2] Christiano and Fitzgerald's (2003) asymetric filter gives a slightly larger weight on the detrended components of gross external assets and gross external liabilities,

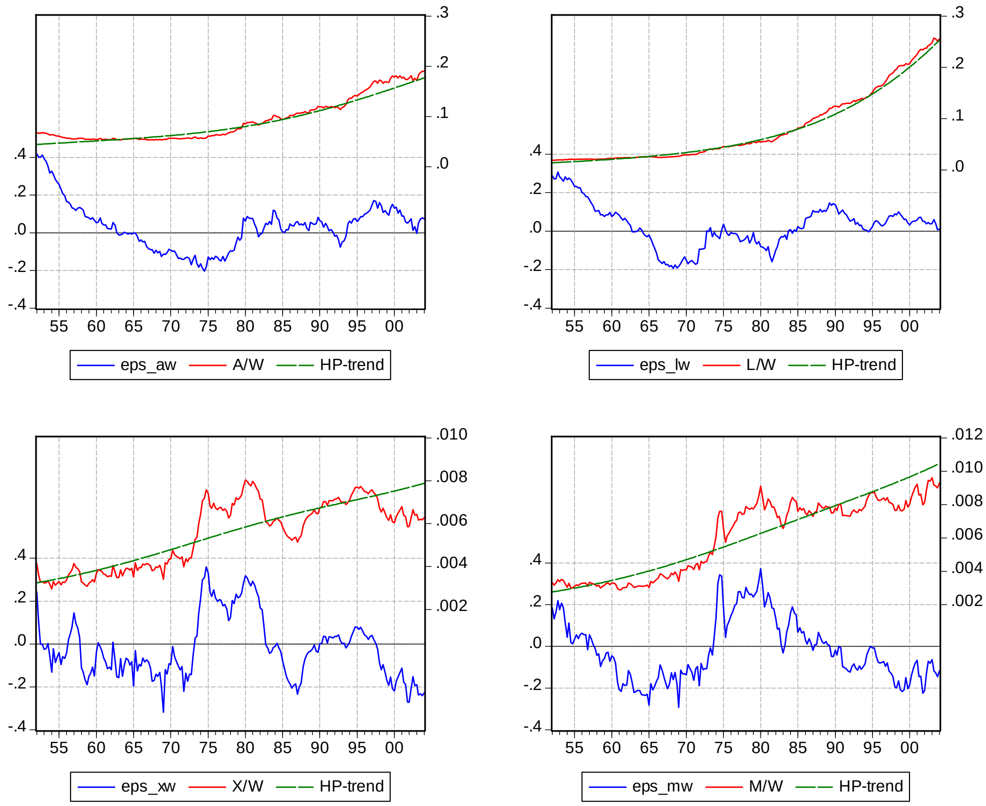

Figure 1: Detrending with Christiano-Fitzgerald asymmetric filter.

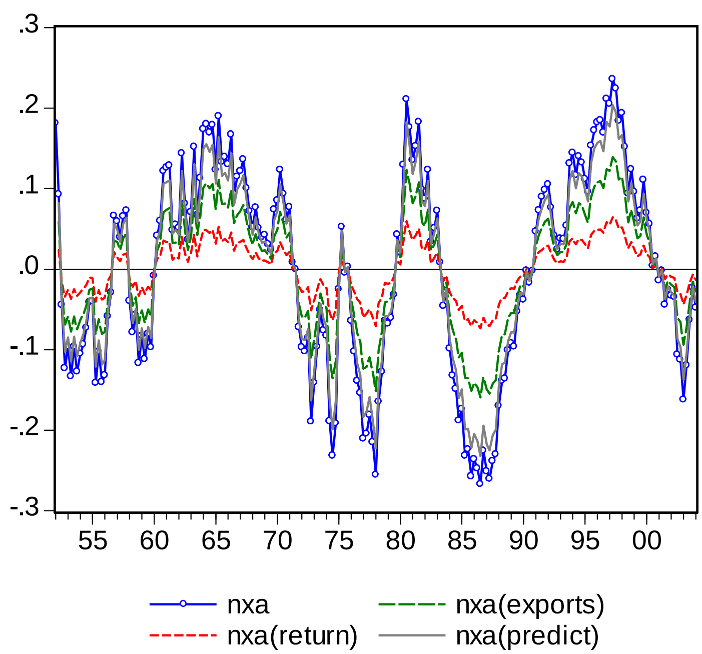

Figure 2: VAR decomposition, Christiano-Fitzgerald asymmetric filter.

#		1 Christiano-Fitzgerald	2 Detrended Returns	3 Bretton Woods	4 post BW
1	$\beta_{\Delta nx}$	0.59	0.64	0.67	0.62
2	β_r	0.27	0.27	0.25	0.25
	of which:				
3	β_a	0.20	0.21	0.20	0.19
4	β_l	0.07	0.06	0.05	0.06
5	Total (lines 1+2)	0.86	0.91	0.92	0.87
6	μ_a	8.98	8.49	-9.30	4.93
7	χ^2	0.28	0.10	0.24	0.20
8	p-val	0.96	0.99	0.97	0.97

Table 1: Unconditional Variance Decomposition for nxa for various discount rates. Sample: 1952:1 to 2004:1. The sum of coefficients $\beta_a + \beta_l$ is not exactly equal to β_r due to numerical rounding in the VAR estimation.

Trend: Christiano and Fitzgerald's (2003) asymmetric filter								
	Forecast Horizon (quarters)							
	1	2	3	4	8	12	16	24
Real Total Net Portfolio Return $r_{t,k}$								
nxa	**-0.36**	**-0.36**	**-0.36**	**-0.34**	**-0.23**	**-0.15**	**-0.11**	**-0.04**
	(0.07)	(0.05)	(0.05)	(0.05)	(0.03)	(0.03)	(0.02)	(0.02)
$\bar{R}^2(1)$	[0.10]	[0.17]	[0.23]	[0.26]	[0.21]	[0.14]	[0.09]	[0.02]
$\bar{R}^2(2)$	[0.13]	[0.23]	[0.32]	[0.36]	[0.32]	[0.21]	[0.15]	[0.11]
Real Total Excess Equity Return $r^{ae}_{t,k} - r^{le}_{t,k}$								
nxa	**-0.13**	**-0.12**	**-0.12**	**-0.11**	**-0.06**	**-0.03**	-0.02	0.00
	(0.03)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.07]	[0.12]	[0.16]	[0.17]	[0.10]	[0.03]	[0.01]	[0.00]
$\bar{R}^2$ (2)	[0.10]	[0.19]	[0.27]	[0.30]	[0.24]	[0.13]	[0.09]	[0.17]
Net Export growth $\Delta nx_{t,k}$								
nxa	**-0.08**	**-0.07**	**-0.07**	**-0.07**	**-0.06**	**-0.06**	**-0.05**	**-0.06**
	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)
$\bar{R}^2$ (1)	[0.05]	[0.09]	[0.13]	[0.17]	[0.30]	[0.42]	[0.51]	[0.56]
$\bar{R}^2$ (2)	[0.04]	[0.08]	[0.12]	[0.17]	[0.37]	[0.56]	[0.67]	[0.80]
FDI-weighted effective nominal rate of depreciation $\Delta e_{t,k}$								
nxa	**-0.08**	**-0.07**	**-0.07**	**-0.07**	**-0.06**	**-0.05**	**-0.04**	**-0.02**
	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.08]	[0.15]	[0.25]	[0.29]	[0.38]	[0.38]	[0.32]	[0.11]
$\bar{R}^2$ (2)	[0.10]	[0.21]	[0.35]	[0.41]	[0.53]	[0.56]	[0.55]	[0.37]

Table 2: Long Horizon Regressions, Portfolio Returns on lagged nxa or ^{a}, ^{l}, ^{x} and ^{m}.. 1952:1 to 2004:1 (1973:1 to 2004:1 for exchange rate). Newey-West robust standard errors in parenthesis with $k-1$ Bartlett window. Adjusted R^2 in brackets.

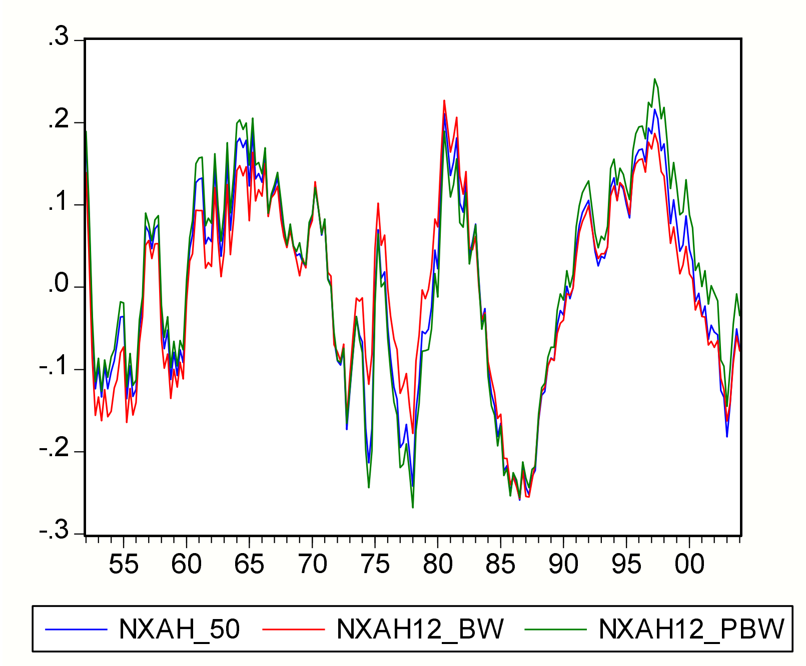

Figure 3: nxa estimated using weights for BW and post BW period (imposes a common ρ).

1.2 Detrended returns and wealth.

This robustness check construct the returns on gross assets and gross liabilities r^i_{t+1} as $\ln\left(R^i_{t+1}/\bar{R}^i_{t+1}\right)$ and $\epsilon^{\Delta w}_{t+1} = \ln\left(\Gamma_{t+1}/\bar{\Gamma}_{t+1}\right)$. This does not affect the construction of nxa, but influences the measurement of r_t and Δnx_t. As we argued in the paper (see 18), we cannot reject the equality between $\bar{R}^i_{t+1}$ or $\bar{\Gamma}_{t+1}$ and the sample mean of the corresponding series. Accordingly, it is not surprising that the results are virtually unchanged. Table 1 column 2 reports the variance decomposition. The in-sample forecasts are unchanged.

1.3 Different weights

We construct the weights for the two different subsamples using our HP filter estimates. We obtain:[3]

$$\begin{aligned}
\mu^a_{BW} &= -9.30;\ \mu^x_{BW} = 13.84 \\
\mu^a_{PBW} &= 4.93;\ \mu^x_{PBW} = -5.74
\end{aligned}$$

These estimates capture the fact that the trade balance was positive in the Bretton Wood era, and negative in the post Bretton Wood period ($\mu^x, \mu^m < 0$).

Table 1 columns 3 and 4 report the variance decomposition using the weights from the two periods. Again, we find that the results are very similar to our benchmark estimates. Figure 3 reports our benchmark estimate of nxa (nxa^{50}) as well as the Bretton Woods (nxa^{BW}_{12}) and post Bretton Wood estimates (nxa^{PBW}_{12}).

Tables 3 and 4 report the in-sample forecasts for the two different weights. The results are very similar to the benchmark estimates.

2 Appendix B: Linearization around trends.

This appendix establishes the conditions of validity of the linearization around trends described in the paper in section 2.1.

[3] Our approach here is similar to the out-of-sample estimation of the paper (see section 3.6): we impose a constant discount factor equal to its steady state value $\rho = 0.95$ to recover the weights.

Bretton Woods weights								
	Forecast Horizon (quarters)							
	1	2	3	4	8	12	16	24
	Real Total Net Portfolio Return $r_{t,k}$							
nxa	**-0.43**	**-0.43**	**-0.44**	**-0.41**	**-0.29**	**-0.19**	**-0.13**	-0.05
	(0.09)	(0.09)	(0.09)	(0.09)	(0.08)	(0.07)	(0.07)	(0.05)
$\bar{R}^2(1)$	[0.08]	[0.14]	[0.20]	[0.22]	[0.19]	[0.12]	[0.08]	[0.01]
$\bar{R}^2(2)$	[0.12]	[0.21]	[0.30]	[0.35]	[0.37]	[0.28]	[0.21]	[0.15]
	Real Total Excess Equity Return $r^{ae}_{t,k} - r^{le}_{t,k}$							
nxa	**-0.15**	**-0.15**	**-0.14**	**-0.13**	**-0.07**	-0.04	-0.02	0.01
	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
$\bar{R}^2$ (1)	[0.08]	[0.14]	[0.19]	[0.20]	[0.12]	[0.04]	[0.02]	[0.00]
$\bar{R}^2$ (2)	[0.10]	[0.19]	[0.26]	[0.30]	[0.24]	[0.13]	[0.09]	[0.17]
	Net Export growth $\Delta nx_{t,k}$							
nxa	**-0.08**	**-0.08**	**-0.08**	**-0.08**	**-0.07**	**-0.07**	**-0.06**	**-0.05**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.05]	[0.10]	[0.15]	[0.20]	[0.39]	[0.52]	[0.62]	[0.68]
$\bar{R}^2$ (2)	[0.05]	[0.10]	[0.14]	[0.19]	[0.40]	[0.56]	[0.67]	[0.79]
	FDI-weighted effective nominal rate of depreciation $\Delta e_{t,k}$							
nxa	**-0.10**	**-0.09**	**-0.09**	**-0.09**	**-0.08**	**-0.07**	**-0.05**	**-0.03**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.10]	[0.19]	[0.31]	[0.36]	[0.48]	[0.49]	[0.43]	[0.19]
$\bar{R}^2$ (2)	[0.10]	[0.21]	[0.35]	[0.41]	[0.53]	[0.56]	[0.55]	[0.37]

Table 3: Long Horizon Regressions, Portfolio Returns on lagged nxa or ϵ^a, ϵ^l, ϵ^x and ϵ^m. Bretton Woods weights. 1952:1 to 2004:1 (1973:1 to 2004:1 for exchange rate). Newey-West robust standard errors in parenthesis with $k-1$ Bartlett window. Adjusted R^2 in brackets.

Post Bretton Woods weights								
	Forecast Horizon (quarters)							
	1	2	3	4	8	12	16	24
	Real Total Net Portfolio Return $r_{t,k}$							
nxa	**-0.19**	**-0.18**	**-0.18**	**-0.17**	**-0.11**	**-0.07**	**-0.05**	-0.02
	(0.04)	(0.04)	(0.04)	(0.04)	(0.03)	(0.03)	(0.02)	(0.02)
$\bar{R}^2(1)$	[0.09]	[0.16]	[0.22]	[0.25]	[0.20]	[0.12]	[0.09]	[0.03]
$\bar{R}^2(2)$	[0.13]	[0.23]	[0.31]	[0.34]	[0.28]	[0.17]	[0.12]	[0.10]
	Real Total Excess Equity Return $r^{ae}_{t,k} - r^{le}_{t,k}$							
nxa	**-0.12**	**-0.12**	**-0.11**	**-0.10**	**-0.06**	-0.03	-0.02	0.01
	(0.03)	(0.03)	(0.03)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.07]	[0.12]	[0.16]	[0.17]	[0.09]	[0.04]	[0.01]	[0.00]
$\bar{R}^2$ (2)	[0.10]	[0.19]	[0.26]	[0.30]	[0.24]	[0.13]	[0.09]	[0.17]
	Net Export growth $\Delta nx_{t,k}$							
nxa	**-0.07**	**-0.07**	**-0.07**	**-0.07**	**-0.06**	**-0.05**	**-0.05**	**-0.04**
	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.04]	[0.09]	[0.12]	[0.17]	[0.27]	[0.39]	[0.47]	[0.52]
$\bar{R}^2$ (2)	[0.03]	[0.07]	[0.11]	[0.16]	[0.36]	[0.55]	[0.67]	[0.79]
	FDI-weighted effective nominal rate of depreciation $\Delta e_{t,k}$							
nxa	**-0.08**	**-0.07**	**-0.07**	**-0.06**	**-0.06**	**-0.05**	**-0.03**	-0.02
	(0.02)	(0.02)	(0.01)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)
$\bar{R}^2$ (1)	[0.07]	[0.13]	[0.22]	[0.25]	[0.32]	[0.33]	[0.26]	[0.09]
$\bar{R}^2$ (2)	[0.10]	[0.21]	[0.35]	[0.41]	[0.53]	[0.56]	[0.55]	[0.37]

Table 4: Long Horizon Regressions, Portfolio Returns on lagged nxa or ϵ^a, ϵ^l, ϵ^x and ϵ^m. Post Bretton Wood weights. 1952:1 to 2004:1 (1973:1 to 2004:1 for exchange rate). Newey-West robust standard errors in parenthesis with $k-1$ Bartlett window. Adjusted R^2 in brackets.

The key assumption is assumption 1, p9: that the deterministic and stochastic economies should remain close to one another. At first glance it might appear implausible that it should be satisfied. Indeed, consider a small open economy facing a constant risk-free interest rate. In presence of uncertainty, standard precautionary effects tilt the consumption profile and lead to a faster growth rate of consumption than in the deterministic case. But this simple example assumes that the interest rate remains constant. If instead the interest rate is determined by the demand for and supply of capital, precautionary savings will depress equilibrium interest rates and bring consumption growth back in line with the long run growth rate of the economy. In fact, the decline in interest rates will push equilibrium interest rates down just enough so that steady state consumption growth remains unchanged, and the stochastic economy will remain relatively close to its deterministic counterpart.

A similar argument underlies the use of linearization methods around a deterministic steady state in modern business cycle theory. Although uncertainty affects individual behavior, in general equilibrium prices and asset returns will adjust so as to keep the economy close to its deterministic steady state when the conditions for a saddle-path equilibrium are satisfied.

In this appendix, we extend this argument and show that in a wide class of models with a deterministic trend, the stochastic economy will remain close to its deterministic counterpart (that is, Assumption1 holds). Since the point we are trying to illustrate –the quality of a budget constraint measured in deviations from trend– is quite general, we abstract from many of the specifics of our paper and explore a simple but widely-used class of models: the neoclassical stochastic growth model.

We consider a version of the stochastic growth model with a distortion in the accumulation of capital (a capital wedge). The non-stationarity is induced by a deterministic change in this capital wedge over time. We characterize numerically the exact solution both to the deterministic model and to its stochastic counterpart. We then check the quality of the linearization of the budget constraint (i.e. the quality of both Assumption 1 and lemma 1 in the paper).

Let us summarize the assumptions of the exercise. Consider a closed economy with a representative agent whose preferences over sequences of consumption are defined as $U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$. The flow utility $u(c)$ exhibits constant relative risk aversion: $u(c) = c^{1-\gamma}/(1-\gamma)$ with $\gamma > 0$. Output per worker y_t is determined by a Cobb-Douglas production function: $y_t = k_t^{\alpha} A_t^{1-\alpha}$ where A_t is a -potentially stochastic- productivity term. In the stochastic economy, $A_t = A_0 g^t \epsilon_t$ where ϵ is log-normally distributed and i.i.d. and g is the constant trend growth rate of productivity. For future reference, define $\bar{A}_t = A_0 g^t$ the trend productivity growth, so that $A_t = \bar{A}_t \epsilon_t$. In the deterministic economy, $A_t = \bar{A}_t$.

The source of the non-stationary dynamics in the economy is a distortion on capital (a capital wedge). Assume that capital owners receive only a fraction $(1-\tau_t)$ of the gross returns to capital at time t. The sequence of capital wedges $\{\tau_t\}_{\tau \geq 0}$ is known in advance: a fraction x of the initial capital wedge τ_0 is eliminated linearly over T periods:

$$\begin{aligned} \tau_t &= \tau_0 \left(1 - x\frac{t}{T}\right) \text{ for } t \leq T \\ \tau_t &= \tau_\infty \equiv \tau_0 (1-x) \text{ for } t \geq T \end{aligned}$$

For simplicity, we also assume that the 'revenues' $z_t = \tau_t \left(1 - \delta + \alpha k_t^{\alpha-1} A_t^{1-\alpha}\right) k_t$ generated by the capital wedge are rebated in lump sum fashion to the representative household (i.e. we focus purely on the distortive aspect of the capital wedge, not its aggregative effects).

Measured appropriately, the budget constraint faced by the representative household is:[4]

$$W_{t+1} = R_{t+1} (W_t + yl_t - c_t) \tag{1}$$

[4] To be specific, W_t denotes financial wealth at the beginning of period t while k_t denotes the capital stock at the end of period $t-1$.

where W_t denotes household's financial wealth at the beginning of period t, yl_t represents non-financial income (equal to wages w_t plus transfers z_t), and R_{t+1} is the after-tax return to capital:

$$\begin{aligned} R_{t+1} &= (1-\tau_{t+1})\left[1-\delta+\alpha k_{t+1}^{\alpha-1}A_{t+1}^{1-\alpha}\right] \\ w_t &= (1-\alpha)\,k_t^{\alpha}A_t^{1-\alpha} \end{aligned}$$

The next section specifies how we construct the exact solution to both the stochastic and deterministic problems. Equipped with these solutions, we now evaluate the quality of the approximation of the budget constraint around its trend. We follow the steps of the paper and normalize everything by domestic wealth W_t. The budget constraint (1) becomes:

$$\Gamma_{t+1} = R_{t+1}\left(1+y\hat{l}_t-\hat{c}_t\right) \tag{2}$$

where $\Gamma_{t+1} = W_{t+1}/W_t$ is the growth rate of domestic wealth and $\hat{Z}_t = Z_t/W_t$. Note the analogy with the external constraint in our paper (equation 2, p9 in the paper). As before, denote $\bar{Z}_t = Z_t/W_t$ in the deterministic model, $\bar{R}_{t+1}$ the associated after tax return to capital and $\bar{\Gamma}_{t+1}$ the trend growth rate of wealth in the deterministic economy. Consider now the deviations from the deterministic economy:

$$\begin{aligned} \epsilon_t^{z} &= \ln\left(\hat{Z}_t/\bar{Z}_t\right) \\ \hat{r}_{t+1} &= \ln\left(R_{t+1}/\bar{R}_{t+1}\right) \\ \epsilon_{t+1}^{\Delta w} &= \ln\left(\Gamma_{t+1}/\bar{\Gamma}_{t+1}\right) \end{aligned}$$

Under the equivalent of assumption 1 above ($|\epsilon_t^{z}|, |\hat{r}_{t+1}|, \left|\epsilon_{t+1}^{\Delta w}\right| \ll 1$) one can derive the budget constraint in deviation from trends (the equivalent of lemma 1 in the paper):

$$\epsilon_{t+1}^{\Delta w} \approx \hat{r}_{t+1} + \mu_t^{y}\epsilon_t^{y} - \mu_t^{c}\epsilon_t^{c} \tag{3}$$

where $\mu_t^{y} = y\bar{l}_t/\left(1+y\bar{l}_t-\bar{c}_t\right)$ and $\mu_t^{c} = \bar{c}_t/\left(1+y\bar{l}_t-\bar{c}_t\right)$.

How accurate is this approximation? To answer this question, define the approximation error ρ_t as the difference between the right hand side and the left hand side of (3):

$$\rho_{t+1} = \epsilon_{t+1}^{\Delta w} - \hat{r}_{t+1} - \mu_t^{y}\epsilon_t^{y} + \mu_t^{c}\epsilon_t^{c}$$

We check both that $|\epsilon_t^{z}|, |\hat{r}_{t+1}|, \left|\epsilon_{t+1}^{\Delta w}\right| \ll 1$ (assumption 1) and, more importantly for us, that the approximation error remains small: $\left|\rho_{t+1}\right| << 1$ (lemma 1).

The parameters we adopt are reported in Table 5. A period is a quarter. The capital share α is set to 0.3. The annual discount factor is set to 0.96, which yields a quarterly discount factor of 0.9898. The gross growth rate of labor-augmenting productivity is set to 1.012, which yields a quarterly productivity growth of $g = 1.0029$. Finally, we assume a depreciation rate of 6% annually, equivalent to 1.535% quarterly. Preferences are logarithmic ($\gamma = 1$) and the initial capital wedge is set to 0.05 for $t \leq 0$, yielding a capital-output ratio of 0.91. Starting at $t = 0$, we assume a gradual and predictable transition to $\tau_{\infty} = 0$ after 50 years (implying a final steady state capital output ratio of 2.62). Along this transition, the consumption-wealth and the non-financial income-wealth ratios decrease from 0.25 (resp. 0.24) to 0.08 (resp. 0.07). This calibration is standard. Our results do not depend on the particular values of these parameters, provided we stay in a reasonable range.

We report below the result from two simulations. In the first simulation, we assume that the standard deviation of (log) quarterly productivity shocks every period is 0.1: $\sigma_{\ln\epsilon}(1) = 0.1$. This implies that the standard deviation of shocks to quarterly output represent roughly 7% ($0.1\,(1-\alpha)$) of trend output, which

Table 5: Parameter Values

Parameters	α	β	γ	δ	g	$\sigma_{\ln\epsilon}(1)$	$\sigma_{\ln\epsilon}(2)$
Value	0.3	0.9898	1	1.35%	1.003	0.01	0.1

is already on the high side (see Cooley and Prescott 1995). In the second simulation, we increase the standard deviation of shocks by a factor of 7 ($\sigma_{\ln\epsilon}(2) = 0.7$) and re-check the quality of the linearized budget constraint.

Figure 4 reports the results from a typical simulation with realistic income shocks. The top left panel of figure 4 reports the deviation of the consumption-income ratio from the deterministic economy (i.e. $\epsilon_t^c = \ln(\hat{c}_t/\bar{c}_t)$). Consumption smoothing implies very small consumption fluctuations: the standard deviation of ϵ_t^c is 1.24% against 6.73% for ϵ_t^y, reported in the top right panel. The bottom left panel reports the deviation of the growth-adjusted after tax interest rate ($\hat{r}_{t+1} - \epsilon_{t+1}^{\Delta w}$). We see very small fluctuations in this growth-adjusted real interest rate (the s.d. is 0.92%). Taken together, this three graphs indicate that the conditions for assumption 1 are satisfied in the stochastic growth model calibrated to reasonable productivity shocks.[5]

The bottom right panel reports the approximation errors ρ_{t+1}. It is immediate that these approximation errors are extremely small, as expected under lemma 1: we find a standard deviation of ρ_{t+1} of only 0.04% of trend wealth! In fact, the approximation error ρ_t is an many orders of magnitude smaller than the deviations $\epsilon_t^{\tilde{z}}$ or $\hat{r}_{t+1}$. Hence, this provides an example where the conditions for the approximation of the budget constraint around the deterministic economy (assumption 1) are satisfied and lemma 1 provides an excellent approximation to the dynamic budget constraint, around its deterministic counterpart.

Our second simulation, reported in figure 5 assumes unrealistically large shocks to output. We do this to generate fluctuations in $\epsilon_t^{\tilde{z}}$ comparable to what we observe in figure 2 of our paper. We now have roughly 4 times larger deviations of consumption and income (the standard deviations of ϵ_t^c and ϵ_t^y are 4.75% and 26.74% respectively), so the conditions of assumption 1 are less well satisfied. However, the bottom right panel of figure 5 indicates that the linearization around the trend budget constraint remains very accurate: the approximation error ρ_{t+1} has a standard deviation of only 0.62%, again, many orders of magnitude smaller than the innovations to consumption and income. Hence, even for large shocks, the linearization around the deterministic economy provides a very accurate characterization.

What explains the quality of our approximation? It is a general equilibrium feature of the economy we consider. Uncertainty creates a precautionary motive and leads to additional accumulation of capital. In turn, this lowers the equilibrium rate of return on capital, giving households incentives to unwind their capital holdings back to the deterministic steady state. The decline in equilibrium interest rates is sufficiently small that the approximation error on the growth adjusted interest rate (lower left panel) remains small too.

This simple example also gives us a hint of the circumstances under which assumption 1 would be inappropriate. Under certainty equivalence, for instance, the stochastic economy would exhibit a random walk in consumption and a unit root in wealth that would push it permanently away from its deterministic counterpart. In a small open economy facing an exogenous interest rate, uncertainty would tilt consumption profiles and change consumption trends. While these are important special cases, they are special cases nonetheless: the certainty equivalent model faces strong rejection in the data; the small open economy would not remain small if consumption kept growing faster than world income.... In the general case, we believe that the conditions for assumption 1 are likely to be satisfied and lemma 1 is likely to hold.[6]

[5] We assume for simplicity that the productivity shocks are i.i.d. Assuming serially correlated productivity shocks with the same conditional variance of shocks would increase the unconditional variance of output relative to trend. However, our results are robust to this change as the next simulation makes clear (where we increase 10 fold the variance of productivity shocks).

[6] If we had to venture, we would conjecture, but have not proven, that assumption 1 obtains more generally in models that satisfy the conditions for a saddle-path equilibrium, i.e. where the number of non-predetermined variables equals the number of eigenvalues of the linearized system that lie outside the unit circle. These are the conditions that guarantee the validity of a linearization around a steady state in standard models, and by extension, they should garantee that the economy does not stray too far from the deterministic economy. These conditions are satisfied in a wide variety of macroeconomic models.

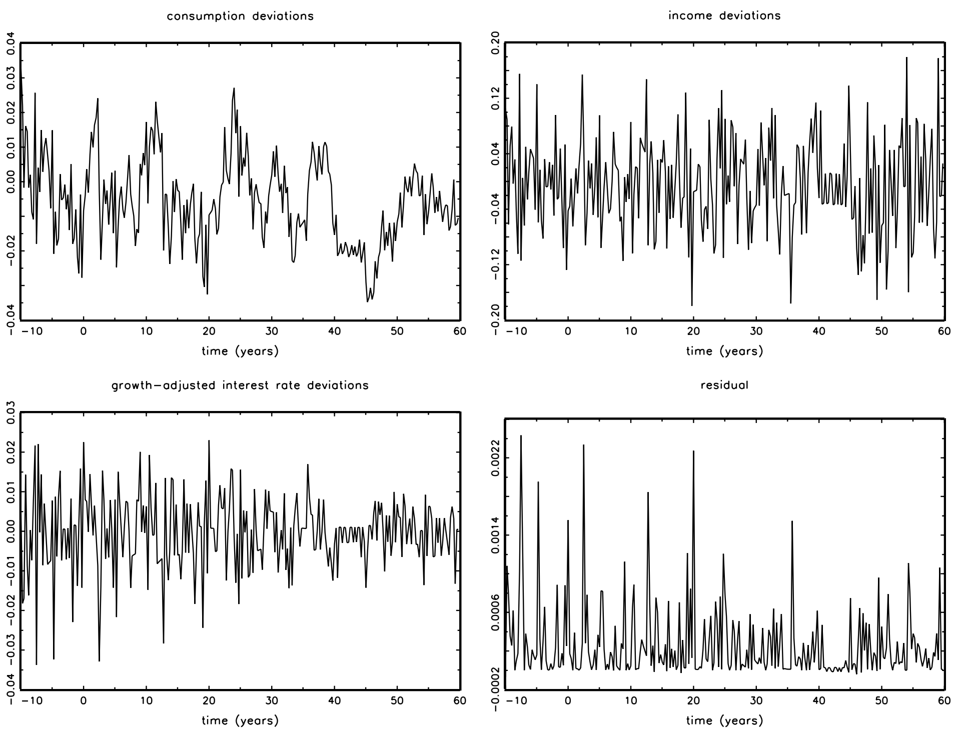

Figure 4: Simulation with regular shocks

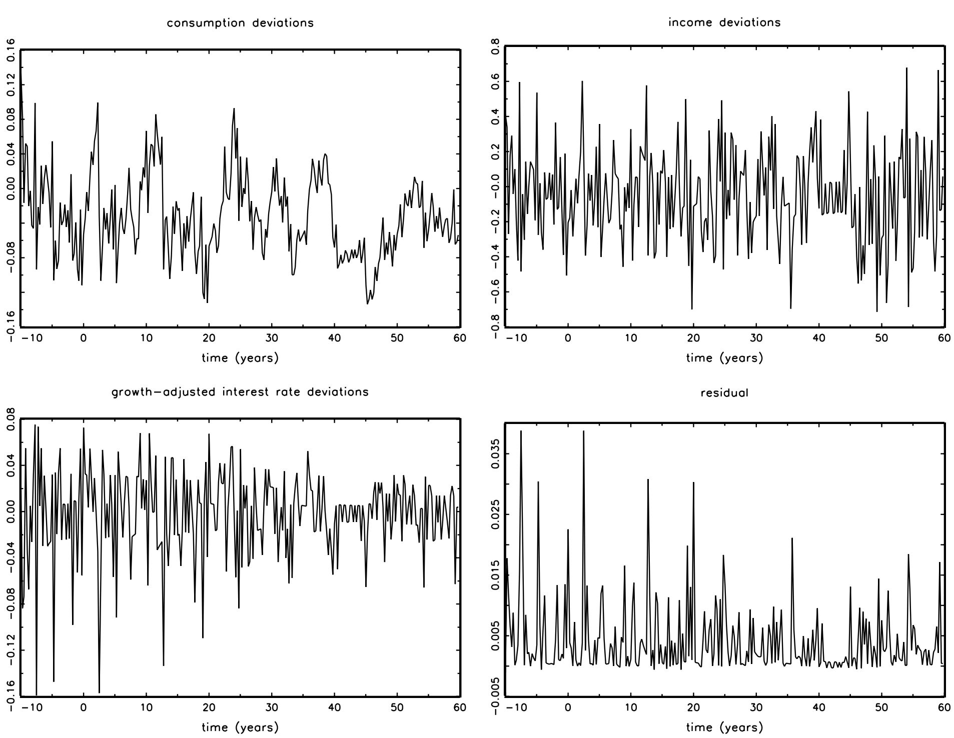

Figure 5: Simulation with large shocks.

In sum, the conditions for assumption 1 are satisfied in a wide class of models. Under these conditions, the budget constraint linearized around the deterministic economy proves to be very accurate. In fact, we find that lemma 1 remains accurate even when the shocks are unrealistically large. We believe that we have provided a simple but 'sensible model' where the necessary conditions for the trend linearization of the budget constraint holds, as you requested in your letter.

The final step is to connect our empirical implementation to the results from proposition 1 (p14). To do so, we need to estimate the trends $\bar{Z}$, $\bar{R}_{t+1}$ and $\bar{\Gamma}_{t+1}$ of the deterministic economy. Of course, these trends are not directly observable. However, under the conditions for lemma 1, we know that they are not far from the data itself. Our approach in the paper is to detrend the stochastic variables $\hat{Z}$, R and Γ using a very low frequency HP-filter. A legitimate question is whether the trends obtained using this method are a good approximation of the deterministic trend of our economy. To examine this question, we compare the trends of the deterministic economy to the HP-filtered trends of the stochastic economy in our simulated model.

Figure 6 reports the deterministic trends and the HP-filter trends of the consumption-wealth ratio (top left panel), income-wealth ratio (top right panel) and the return to capital (bottom left panel). Figure 7 reports the cyclical components $\epsilon_t^{\tilde{z}}$ as well as the approximation error of the budget constraint ρ_{t+1}in deviation from the true model and from the HP-filter, for the case with large productivity shocks. The HP filter is set, as in our paper, to filter out only very low frequencies (i.e. we filter out cycles of more than 50 years).

The figure highlights that using our HP-filter provides a very accurate approximation to the true but unobserved deterministic trends. We see small differences between the deterministic model and the HP-filter around $t = 0$ and $t = 50$, when there is a change in the trends. But these approximation errors remain very small, as indicated by figure 7. The correlation between the two error terms is 0.85, and the approximation error obtained using the HP filter has a standard deviation of 0.63% relative to trend wealth! We conclude that our empirical implementation is very likely to provide accurate estimates of the deterministic trends.

3 Details on the exact solution method

This appendix details how we construct the exact solution to the stochastic and deterministic growth model with transition. First, we normalize all variables by $\bar{A}_t$. Denote $\tilde{x} = x/\bar{A}$ for any variable x and define $\bar{R} = \beta^{-1}g^{\gamma}$. $\bar{R}$ represents the natural real interest rate in the economy that would obtain in the absence of capital wedge $(\tau = 0)$. We can rewrite the problem as

$$\begin{aligned}
v\left(\tilde{k}_0, \epsilon_0\right) &\equiv \frac{U_0}{\bar{A}_0} = \max_{\left\{\tilde{c}_t, \tilde{k}_{t+1}\right\}} E_0 \sum_{t=0} \left(\frac{g}{\bar{R}}\right)^t u\left(\tilde{c}_t\right) \\
\text{s.t. } \tilde{k}_{t+1} &= \left[R_t \tilde{k}_t + \tilde{w}_t + \tilde{z}_t - \tilde{c}_t\right] \frac{1}{g} \\
\tilde{z}_t &= \tau_t R_t \tilde{k}_t / \left(1 - \tau_t\right) \\
R_t &= \left[1 - \delta + \alpha \tilde{k}_t^{\alpha-1} \epsilon_t^{1-\alpha}\right] \left(1 - \tau_t\right) \\
\tilde{w}_t &= \left(1 - \alpha\right) \tilde{k}_t^{\alpha} \epsilon_t^{1-\alpha} \\
& \tilde{k}_0, \epsilon_0 \text{ given}
\end{aligned}$$

The first-order conditions for consumption (Euler equation) is:

$$u'\left(\tilde{c}_t\right) = E_t\left[u'\left(\tilde{c}_{t+1}\right) \frac{R_{t+1}}{\bar{R}}\right]$$

After time T, the problem is time invariant with a capital wedge τ_∞. The optimal consumption rule $\tilde{c}\left(\tilde{k}, \epsilon\right)$ satisfies the following functional equation

$$u'\left(\tilde{c}\left(\tilde{k}, \epsilon\right)\right) = E\left[u'\left(\tilde{c}\left(\tilde{k}', \epsilon'\right)\right) \frac{R'}{\bar{R}}\right]$$

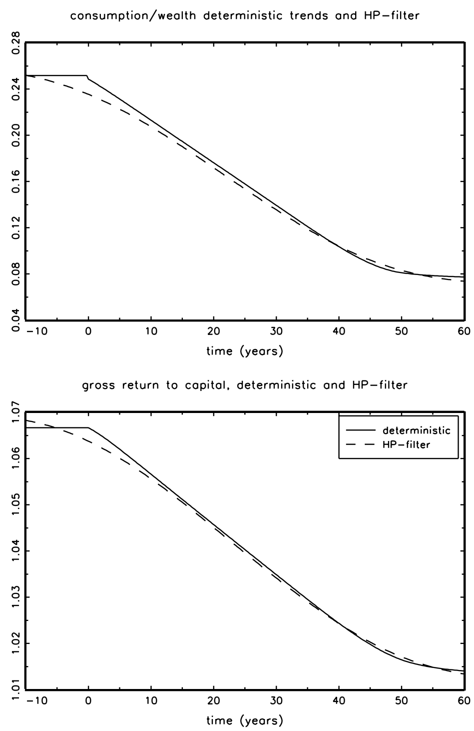

Figure 6: A comparison of the true deterministic trends and the HP-filter in the neoclassical growth model.

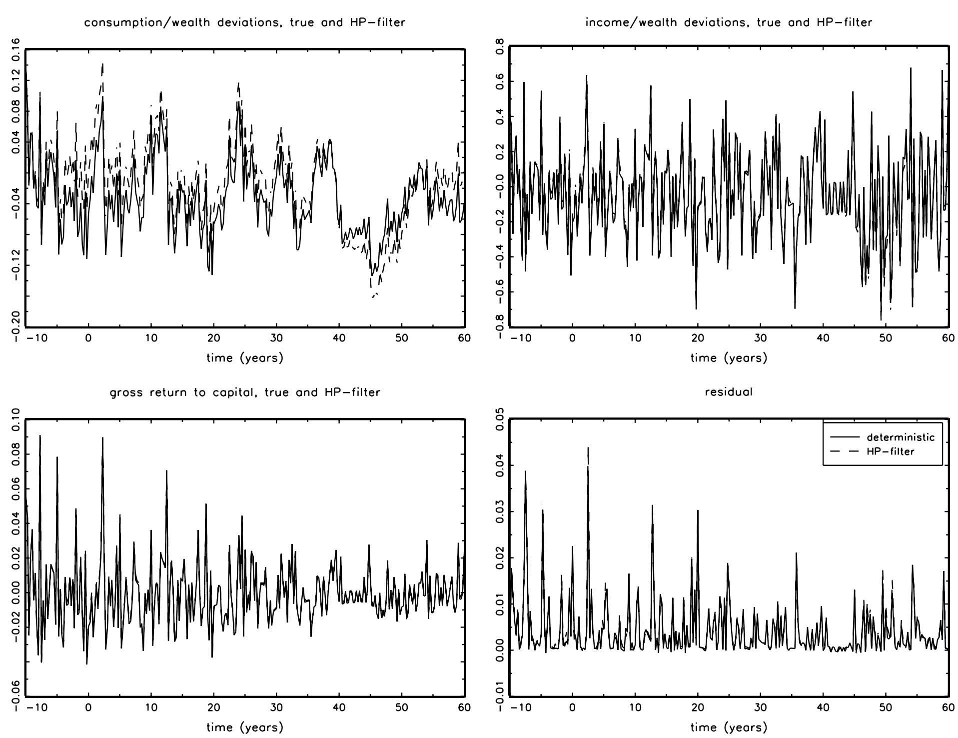

Figure 7: A comparison of the cyclical components from the true model and the HP-filter.

where "$'$" denotes next period's values. $\tilde{k}'$ and R' satisfy:

$$\begin{aligned} \tilde{k}' &= \left[\tilde{k}^{\alpha}\epsilon^{1-\alpha} + (1-\delta)\,\tilde{k} - \tilde{c}\left(\tilde{k},\epsilon\right)\right]\frac{1}{g} \\ R' &= \left[1-\delta+\alpha\tilde{k}'^{\alpha-1}\epsilon'^{1-\alpha}\right](1-\tau_{\infty}) \end{aligned}$$

where we substituted the equilibrium values for $\tilde{w}$, $\tilde{z}$. We construct the solution to this functional equation by iterating the Euler equation over a grid $\left\{\tilde{k},\epsilon\right\}$, using Gauss-Hermite quadratures for the productivity shocks with 21 nodes (see Gourinchas and Parker (2002) for details).

Before time T, we solve recursively for the consumption rule $\tilde{c}_t\left(\tilde{k},\epsilon\right)$ as the solution to the following set of functional equations:

$$u'\left(\tilde{c}_t\left(\tilde{k},\epsilon\right)\right) = E\left[u'\left(\tilde{c}_{t+1}\left(\tilde{k}',\epsilon'\right)\right)\frac{R'}{\bar{R}}\right]$$

where $\tilde{k}'$ is defined as before and R' satisfies:

$$R' = \left[1-\delta+\alpha\tilde{k}'^{\alpha-1}\epsilon'^{1-\alpha}\right](1-\tau_{t+1})$$

This method provides us with an exact solution to the deterministic and stochastic problems.

References

Christiano, Lawrence J. and Terry J. Fitzgerald, "The Band Pass Filter," *International Economic Review*, May 2003, *44* (2), 435–465.

Cooley, Thomas and Edward Prescott, "Economic Growth and Business Cycles," in Thomas Cooley, ed., *Frontiers of Business Cycle Research*, Princeton University Press, 1995.

Gourinchas, Pierre-Olivier and Hélène Rey, "International Financial Adjustment," *Journal of Political Economy*, 2007. forthcoming.

— **and Jonathan A. Parker**, "Consumption Over the Life Cycle," *Econometrica*, January 2002, *70* (1), 47–89.