Financial Integration and Growth in a Risky World

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Abstract

We revisit the debate on the benefits of financial integration in a two-country neoclassical growth model with aggregate uncertainty. Our framework accounts simultaneously for gains from a more efficient capital allocation and gains from risk sharing—together with their interaction. In our general equilibrium model, risk sharing brought by financial integration has an effect on the steady-state itself, altering convergence gains from capital accumulation. Because we use global numerical methods, we are able to do meaningful welfare comparisons along the transition paths. Allowing for country asymmetries in terms of risk, capital scarcity and size, we find important differences in the effect of financial integration on output, direction of capital flows, consumption and welfare over time and across countries. This opens the door to a richer set of empirical implications than previously considered in the literature.

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Key Words: Financial integration, Capital flows, Risky steady-state, Global Solutions.

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1 Introduction

What should we think about the welfare effects of financial integration? This is one of the perennial questions in international macroeconomics and finance. The usual answer, given by academics and taken up by policy makers, is that financial integration allows for a more efficient allocation of capital and improves risk sharing across countries. To the extent that the policy making world has been actively promoting financial integration, implicit in this answer is that *quantitatively* these gains are large enough to offset any costs associated with integration. So how large are actually the efficiency and risk sharing gains of financial integration? As the literature stands, we cannot answer this question in one go.

In the context of neoclassical growth models, capital flows from capital-abundant to capital-scarce countries and raises welfare as the marginal product of capital is higher in the latter than in the former. Free capital movements thus permit a more efficient global allocation of savings towards their most productive use. But quantitatively, as the calibrations of Gourinchas and Jeanne (2006) show in a deterministic model, those neoclassical gains to international financial integration remain elusive. Even when a country starts off in autarky with a low level of capital, speeding up its transition towards its steady-state by opening the financial account brings small welfare gains. The reason is that the distortion induced by a lack of capital mobility is transitory: the country would have reached its steady-state level of capital regardless of financial openness, albeit at a slower speed. In this framework, only very capital-scarce countries may experience significant gains to financial integration.

In the context of the international risk-sharing literature, which usually does not feature endogenous production, openness to financial flows allows country specific shocks to be diversified away. The debate still rages regarding the magnitude of the gains from risk-sharing (see Cole and Obstfeld (1991), van Wincoop (1994, 1999), Tesar (1995), Lewis (1999, 2000)). In most studies, gains are of second order as financial integration allows a reduction of consumption volatility but does not affect output.[1] Welfare gains are potentially large if the market

[1] A theoretical literature studies the effect of asset trade on efficient specialization and risk taking (Obstfeld (1994a), Acemoglu, and Zilibotti (1997), Martin and Rey (2006). On the empirical side, see Kalemli-Ozcan, Sorensen, and Yosha (2003). We abstract from this channel.

price of risk is high—when asset price data are to be trusted, but remarkably small if computed from consumption data with a reasonable risk aversion parameter (Lucas (1987)).[2] Recent work aims at reconciling the two by relying either on long-run consumption risks (Colacito and Croce (2010), Lewis and Liu (2012)) or rare disasters risks (Martin (2010)). In these contexts, financial integration may bring sizable gains but their magnitude is sensitive to the cross-country correlation of long-run (or disaster) risks. In any case, the framework used is the one of endowment economies, shutting down efficiency gains from capital reallocation.

But assessing efficiency and risk sharing gains separately, using two different types of models, prevents reaching a solid conclusion. Are those two gains substitute or complement? They are surely intertwined as, through precautionary savings, the steady state level of the capital stock depends on the level of risk agents seek to insure (see Aiyagari (1995) in the context of a closed economy). Thus, when capital is allowed to flow across borders, gains from risk-sharing modify the steady-state level of the capital stock and impacts the process of capital reallocation across countries. Financial integration can therefore have a permanent effect on output in a stochastic environment.

In this paper, we study how financial integration affects the growth and welfare of countries in a standard two country version of the stochastic neoclassical growth model. As a well established benchmark in the psyche of the economist profession and of the policy makers, it underpins implicitly the widely heard qualitative claims that financial integration improves capital allocation efficiency and enables risk-sharing across countries. Ironically may be, since they have been very influential in the policy world, those claims have not so far been evaluated in a quantitative version of the model due to the technical difficulties of modelling aggregate uncertainty and production in open economy settings. In our baseline version of the model, the world is made of two heterogeneous countries which are allowed to trade a risk-free bond internationally (incomplete financial markets version of Backus et al. (1992) as in Baxter in Crucini (1995)). Countries produce a single tradable good using capital and labor and face stochastic transitory productivity shocks. Countries are allowed to be asymmetric in three

[2] Large welfare gains driven by realistic asset prices are also hard to reconcile with the observed degree of portfolio home-bias (see Lewis (1999) and Coeurdacier and Rey (2013) for a recent survey).

dimensions: the amount of aggregate risk they are facing, their level of capital at time of integration and their size. This allows us to characterize in a richer way than the previous literature which countries, if any, reap large gains from financial integration.[3] It also opens the door to more precise empirical investigations. We believe our framework is particularly well suited to study the integration of a set of (potentially large) emerging markets that face larger aggregate risk and tend to be on average capital scarce. Importantly, it allows for general equilibrium effects, which we believe can be important since historically liberalization episodes tend to occurs by waves, with a set of countries integrating simultaneously.[4]

Our main findings are that financial integration has very heterogeneous effects depending on the stochastic structure of shocks, the size of countries and their initial degree of capital scarcity. Interestingly the consumption and output profiles of countries undergoing financial integration are very diverse and potentially non monotonic over time. When looking at welfare, we find that financial integration does not bring sizable benefits to any plausibly parameterized country in the context of the neoclassical stochastic growth model, even for the typical emerging country—at most a permanent increase in consumption of 0.5% in our calibration with a moderate degree of risk aversion. The intuition for these results can be summarized as follows. Relatively safe (typically developed) countries have small gains from reducing consumption volatility as calculated in Lucas (1987). They also have small gains due to a more efficient world allocation of capital after integration, as calculated in Gourinchas and Jeanne (2006).[5] Emerging countries face higher levels of uncertainty (Pallage and Robe (2003), Aguiar and Gopinath (2007)) and could have potentially larger gains when they share

[3]In a recent contribution, Fogli and Perri (2014) present a two-country RBC model with aggregate risk but focus on the business cycle implications of (asymmetric) changes in aggregate risk. They provide empirical evidence that differences in aggregate risk are a source of capital flows—as in our framework. Kent (2013) provides a two-country growth model with aggregate risk. The model is solved using perturbation methods so that welfare implications and transition across steady-states are not studied. Angeletos and Panousi (2011) and Corneli (2010) investigate how financial integration can affect the steady-state as well as the transition dynamics in a model with uninsurable idiosyncratic entrepreneurial risk. See also Mendoza et al. (2007, 2008)), Benhima (2013) and Caroll and Jeanne (2013). However, in absence of aggregate risk, they cannot explore the gains from consumption smoothing through international risk sharing. Bai and Zhang (2010) also explores the size of capital flows in a model with idiosyncratic risk and incomplete markets in the form of imperfect spanning and limited commitment.

[4]Most emerging markets opened up to financial markets in the late eighties-early nineties. See Appendix B for liberalization dates of emerging countries.

[5]Hoxha et al. (2013) find higher welfare gains in a model where capital goods are not perfect substitutes.

risk. However, financial integration, by affecting the distribution of risk across countries, also leads to a change in the value of the steady state capital stocks. Unless riskier countries are also capital scarce, they will see capital flowing out and output falling as their precautionary savings are reallocated towards safer (developed) countries and their steady state level of capital stock is lower in the integrated economy. This reallocation of capital reduces their welfare gains from integration. When riskier countries are also significantly capital scarce (as emerging countries in the data), the standard efficiency gains driven by faster convergence are strongly dampened by the reallocation of precautionary savings across countries. Our findings thus qualify in an important way the conventional wisdom that emerging countries should face larger gains from financial integration since they face more volatile business cycles. They also significantly differ from the risk-sharing literature, which would, in the context of endowment economies, typically predicts much higher gains for riskier countries. Our baseline calibration relies on parameters values for risk aversion and levels of risk in line with the business cycles literature but at the expense of counterfactually low risk premia. In an alternative calibration, we show that increasing the market price of risk (increasing risk aversion as in Tallarini (2000) using non-expected recursive utility, Epstein and Zin (1989), Weil (1990)) generates higher welfare gains from financial integration but the same logic applies: gains for volatile emerging countries are dampened by an even stronger capital reallocation towards safer countries.[6] Gains for riskier (emerging) countries remain below 0.5% of permanent consumption. In a world with a higher market price of risk, safer countries actually benefit the most from integration with riskier countries as their permanent increase in consumption reaches 1%. Safer countries sell insurance at higher price and benefit from a larger fall in the world interest rate upon integration. Following the long-run risks literature (Bansal and Yaron (2004), Colacito and Croce (2011, 2013), Lewis and Liu (2012), Nakamura et al. (2014) among others), an extension of our model considers persistent shocks to world productivity growth—allowing us to generate meaningful risk premia without relying on extreme degrees of risk aversion. Our findings are robust to this extension and, if, anything welfare gains are smaller—countries are

[6]In such a context, convergence gains for capital scarce emerging countries are even more severely dampened as the reallocation of precautionary savings dominates.

reluctant to built leveraged positions to limit their exposure to the world long-run risk, which reduces their ability to smooth country specific transitory shocks.

From a methodological point of view, the paper provides an accurate welfare assessment using a 'global solution' for the model along the transition path as well as around the steady-states. Standard approximation methods based on perturbation or log-linearization around deterministic steady-states (see Judd (1998)) are not well suited. First, with incomplete markets, net foreign assets are very persistent and the dynamics of the model can drift away from the point of approximation, casting doubt on the accuracy of the approximation. Second, the steady-state should depend on the risk sharing opportunities of agents due to the presence of precautionary savings so that we should focus on *risky steady-states* and not deterministic ones (Coeurdacier, Rey and Winant (2011)). Because financial integration modifies the ability to smooth shocks, it has a first order effect on the steady-state. Third, global solutions allow for an accurate treatment of non-linearities, when countries are far away from their steady-states or when the utility function has extreme curvature. We build on Kubler and Schmedders (2003) (see also Judd, Kubler and Schmedders (2002)) to develop 'global methods' necessary for the welfare evaluation of financial integration in a two-country stochastic model with incomplete markets.[7] Contrary to standard perturbation methods, we believe the method captures well non-linearities over the state space, and can deal with high risk premia and/or large persistent shocks.

From an empirical perspective, no clear evidence emerges so far from the literature regarding the effect of financial integration on growth and risk sharing. Eichengreen (2002), Kose et al. (2009), Obstfeld (2009) and Jeanne et al. (2012) provide excellent surveys of the hundreds of papers analyzing the effect of financial integration on growth. Overall, we can safely argue that the evidence is mixed, ranging from no effects on growth (Rodrik (1998)) to moderate effects of at most 1% per year following the liberalization of financial flows (see Henry (2003), Bekaert, Harvey and Lundblad (2005) and Quinn and Toyoda (2008) for recent evidence).

[7] The algorithm is based on iteration on the policy function, where the policy function is approximated by products of polynomials over a grid of current state variables. At each stage of the algorithm, optimality and market clearing conditions gives values for prices, quantities and the future states at each point of the grid.

Similarly, empirical results pertaining to the impact of financial integration on risk-sharing across countries are very mixed (Kose et al. (2007)). Our theoretical results show that the effect of financial integration on the growth and the welfare of countries is very heterogeneous (across countries and over time) depending in particular on risk characteristics and a number of other conditioning variables. Such heterogeneity can explain the difficulties of the empirical literature which, by focusing on the *average* effect of financial integration, could not reach a conclusive answer. We also emphasize how taking into account general equilibrium effects can yield different growth and welfare implications compared to the financial integration of a small open economy. The large body of empirical studies implicitly assumes that a small country integrates to the rest of the world independently of others, a fairly strong assumption as groups of (potentially large) countries have historically integrated simultaneously. Our findings may thus help explain why the enormous body of empirical work on financial integration has to some extent failed to produce robust results.[8]

The paper is organized as follows: Section 2 develops our baseline model of financial integration and describes briefly our solution methods. Section 3 presents our main findings regarding the growth impact of financial integration, the dynamics of consumption and net foreign assets in our stochastic environment. Section 4 evaluates quantitatively the welfare benefits of financial integration. Section 5 provides robustness checks and extensions of our findings, performing sensitivity analysis with respect to the specification of shocks—including a long-run risk component, asset market structure and market sizes. Section 7 concludes.

2 A baseline model of financial integration

We consider a two-country neoclassical growth model with aggregate uncertainty. Countries can be asymmetric in three dimensions: the aggregate risk they are facing, their initial level of capital and their size. This allows us to analyze the benefits of financial integration in

[8]The literature on the positive effects of FDI on growth has reached more consensus but does not fit well our context as we abstract from direct effects (or positive externalities) of integration on TFP. See Alfaro et al. (2009), Section 2, for references and recent evidence in Fons-Rosen et al. (2013).

terms of gains from capital accumulation due to capital scarcity together with gains from risk sharing, and study how these gains are distributed across heterogeneous countries.

In our baseline model, we consider an incomplete market set-up where countries are allowed to trade in a riskless bond only. This regime of financial integration is compared to a benchmark model where countries stay under financial autarky. We believe this incomplete markets environment is more realistic since we focus our attention on the liberalization episodes of emerging markets. At the time of their financial integration in late eighties-early nineties, capital flows were mostly driven by intertemporal borrowing and lending (Kraay et al. (2005)).[9] In robustness checks (Section 5), we consider the alternative case of complete markets to provide some upper-bounds of the benefits of integration.

2.1 Set-up

The world is made of two countries $i = \{D, E\}$. D stands for Developed country and E for Emerging in our baseline simulations. There is one good (numeraire) used for investment and consumption. Each country starts with an initial capital stock $k_{i,0}$.

Technologies and capital accumulation. Production in country i uses capital and labor with a Cobb-Douglas production function:

$$y_{i,t} = A_{i,t}\,(k_{i,t})^{\theta}\,(l_{i,t})^{1-\theta} \tag{1}$$

where $A_{i,t}$ is a stochastic level of total factor productivity; $\log(A_{i,t})$ follows an $AR(1)$ process such that $\log(A_{i,t}) = (1-\rho)\log(A_{i,0}) + \rho\log(A_{i,t-1}) + \epsilon_{i,t}$ with $\epsilon_t = \binom{\epsilon_{D,t}}{\epsilon_{E,t}}$ an i.i.d process normally distributed with variance-covariance matrix $\Sigma = \begin{pmatrix} \sigma_D^2 & \zeta\sigma_D\sigma_E \\ \zeta\sigma_D\sigma_E & \sigma_E^2 \end{pmatrix}$. $A_{i,0}$ is the initial level of productivity in each country which proxies in our simulations for country size.[10]

[9] Portfolio equity home bias is also very extreme for emerging markets, even nowadays, as recently pointed out in Coeurdacier and Rey (2013).

[10] Note that increasing the variance of the shocks also imply multiplying the productivity level by a (very) small constant number in our parametrization. This is equivalent to a minor change in country size which does not affect the findings.

The law of motion of the capital stock in each country is:

$$k_{i,t+1} = (1-\delta)k_{i,t} + k_{i,t}\phi\left(\frac{i_{i,t}}{k_{i,t}}\right) \tag{2}$$

where $0 < \delta < 1$ is the depreciation rate of capital and $i_{i,t}$ is gross investment in country i at date t. $\phi(x)$ is an adjustment cost function:

$$\phi(x) = a_1 + a_2\left(\frac{x^{1-\xi}}{1-\xi}\right)$$

with ξ measuring the degree of adjustment costs, a_1 and a_2 chosen such that $\phi(\delta) = \delta$ and $\phi'(\delta) = 1$.[11]

Factor payments. Labour and capital markets are perfectly competitive and inputs are paid their marginal productivity. If $w_{i,t}$ denotes the wage rate in country i and $r_{i,t}$ the rental rate of capital, we have:

$$w_{i,t}l_{i,t} = (1-\theta)\,y_{i,t} \qquad ; \qquad r_{i,t}k_{i,t} = \theta y_{i,t} \tag{3}$$

For simplicity, we normalize population to unity in each country: $l_{i,t} = 1$. Country size is then homogeneous to productivity levels $A_{i,0}$ in our set-up (and not population) but this is irrelevant for our purpose.[12] We also implicitly assume an inelastic labor supply. If anything, this tends to increase the gains from international risk sharing by suppressing a margin of adjustment of households following shocks.

Preferences. Country i is inhabited by a representative household with Epstein-Zin preferences (Epstein and Zin (1989), Weil (1990)) defined recursively as follows:

$$U_{i,t} = \left[(1-\beta)c_{i,t}^{1-\psi} + \beta\left(E_t U_{i,t+1}^{1-\gamma}\right)^{\frac{1-\psi}{1-\gamma}}\right]^{\frac{1}{1-\psi}}. \tag{4}$$

[11] The definition of $\phi(x)$ ensures that in the neighborhood of $i = \delta k$ (replacement of capital), adjustment costs are zero to a first-order. Note also that, for $\xi = 0$ (no adjustment costs), $\phi(x) = x$—implying a standard law of accumulation, while for $\xi \to \infty$, $k_{i,t+1} = k_{i,t}$—corresponding to fixed capital (endowment economy).

[12] More precisely, it is irrelevant for the model dynamics following integration. When computing welfare gains, these gains must be multiplied by $\frac{l_i}{l_j}$ for country j to be expressed in per capita terms.

where $1/\psi$ is the elasticity of intertemporal substitution (EIS) and γ the relative risk aversion. This specification nests the CRRA case when $\psi = \gamma$. This is the case we will first consider. Then, we consider alternative cases where agents are more risk averse than our CRRA baseline, keeping the EIS $1/\psi$ constant: $\gamma \geq \psi$, with γ up to 40.

Budget constraints, household decisions and market clearing conditions. Budget constraints depend on the assets available for savings decisions which is a function of the degree of financial integration. We consider the two following cases in our baseline: (i) financial autarky, (ii) financial integration with a non state-contingent bond only.

The stochastic discount factor in country i is defined as:

$$\mathcal{M}_{i,t+1} = \beta \left(\frac{c_{i,t+1}}{c_{i,t}}\right)^{-\psi} \left(\frac{U_{i,t+1}^{\psi-\gamma}}{\left[E_t\left(U_{i,t+1}^{1-\gamma}\right)\right]^{\frac{\psi-\gamma}{1-\gamma}}}\right). \tag{5}$$

(i) Financial autarky. Under financial autarky, the only vehicle for savings is domestic capital. A household can therefore either consume or invest in domestic capital the revenues from labour and capital. This gives the following household budget constraint:

$$c_{i,t} + i_{i,t} = w_{i,t} + r_{i,t}k_{i,t}.$$

Investment decisions in country i satisfies the following Euler equation:

$$E_t\left[\mathcal{M}_{i,t+1}\left(r_{i,t+1}\phi'_{i,t} + \frac{\phi'_{i,t}}{\phi'_{i,t+1}}\left((1-\delta) + \phi_{i,t+1} - \frac{i_{i,t+1}}{k_{i,t+1}}\phi'_{i,t+1}\right)\right)\right] = 1 \tag{6}$$

where $\phi_{i,t} = \phi\left(\frac{i_{i,t}}{k_{i,t}}\right)$ and $\phi'_{i,t}$ denotes the first derivative of $\phi(x)$ at $x = \left(\frac{i_{i,t}}{k_{i,t}}\right)$.

Abstracting from capital adjustment costs ($\phi(x) = x$), we get the usual Euler equation:

$$E_t\left[\mathcal{M}_{i,t+1}\left(1 + r_{i,t+1} - \delta\right)\right] = 1$$

where $r_{i,t}$ denotes the marginal productivity of capital defined in Eq. (3).

The associated goods market clearing condition in country i is:

$$c_{i,t} + i_{i,t} = y_{i,t}. \tag{7}$$

(ii) Financial integration: bond-only economy. We introduce a riskless international bond whose price at date t is p_t and which delivers one unit of good in the next period. Bonds are in zero net supply. The instantaneous budget constraint at date t in country i in presence of bond trading becomes:

$$c_{i,t} + i_{i,t} = w_{i,t} + r_{i,t}k_{i,t} + b_{i,t-1} - b_{i,t}p_t$$

where $b_{i,t}$ denotes bond purchases at date t by country i. For computational reasons, one needs to bound the state space for bond holdings. We do so by assuming that agents in country i face the following borrowing constraint under financial integration,

$$b_{i,t} \geq \underline{b_i}. \tag{8}$$

The debt limit $\underline{b_i} < 0$ is chosen small enough in our simulations such that the constraint barely affects the path of $b_{i,t}$.[13] The Euler equation for bond holdings in country $i = \{D, E\}$ is:

$$p_t = E_t\left[\mathcal{M}_{i,t+1}\right] + \mu_{i,t} \tag{9}$$

where $\mu_{i,t} \geq 0$ is the Lagrange multiplier associated to the borrowing constraint (Eq. (8)). Household investment decisions satisfies the same Euler equation as before in country $i = \{D, E\}$ (Eq. (6)).

We close the model with goods and bonds markets clearing conditions:

$$b_{D,t} + b_{E,t} = 0 \tag{10}$$

$$c_{D,t} + i_{D,t} + c_{E,t} + i_{E,t} = y_{D,t} + y_{E,t} \tag{11}$$

[13]The numerical applications take full account of occasionally binding constraints. In our simulations, they are seldom binding and have almost no effect on the dynamics (see discussion in Appendix C).

Definition of equilibrium. Under autarky, an equilibrium in a country i is a sequence of consumption and capital stocks $(c_{i,t}; k_{i,t+1})$ such that individual Euler equations for investment decisions are verified (Equation (6)) and goods market clears at all dates (Equation (7)).

Under financial integration, an equilibrium is a sequence of consumption, capital stocks and bond holdings in both countries $(c_{i,t}; k_{i,t+1}; b_{i,t})_{i=\{D,E\}}$ and a sequence of bond prices p_t such that Euler equations for investment decisions are verified in both countries (Equation (6)), Euler equations for bonds together with borrowing constraints are verified in both countries (Equations (8) and (9)), bonds and goods markets clear at all dates (Equations (10) and (11)).

2.2 Solution method

Motivation for a global solution. From a methodological point of view, the paper provides a 'global solution' for the model along the transition path as well as around the steady-states. Standard approximation methods based on perturbation or log-linearization around a deterministic steady-state are not well suited for welfare evaluations. First, with incomplete markets, net foreign assets are extremely persistent (Schmitt-Grohe and Uribe (2003)) and the dynamics of the model can drift away from the point of approximation—casting doubt on the accuracy of the approximation along the transition dynamics. Second, the steady state depends on the risk sharing opportunities of agents due to the presence of precautionary savings so that we should focus on a *risky* steady-state and not a deterministic one as in standard perturbation methods. The risky steady-state is the point where state and choice variables remain unchanged if agents expect future risk but shocks innovations turn out to be zero (Coeurdacier, Rey and Winant (2011), Juillard (2012)). In general, it differs from the deterministic one where agents do not expect any risk in the future. It also differs from the *stochastic* steady-state, which is the state of the economy averaged over an asymptotically stable distribution (Clarida (1987)). Third, standard perturbation methods are found to be less appropriate, when non-linearities are important (e.g. when countries are far away from their steady-state) and/or when countries are asymmetric as in our baseline simulations (see Rabitsch, Stepanchuk and Tsyrennikov (2014)).

Time-iteration algorithm. We solve the model using the time-iteration algorithm (Coleman (1991) and Judd, Kubler and Schmedders (2002)). This algorithm is theoretically appealing since it illustrates computationally a contraction mapping property of rational expectations behaviour. In single agents models its convergence has been proven to be equivalent to value function iteration (Rendahl (2014)). To our knowledge, there is no such proof of convergence in generic two-agent models, with incomplete markets, even as simple as ours. For this reason the time-iteration algorithm can be seen as a substitute to missing theoretical tools in order to investigate the convergence properties of our model.

Model reformulation. Using the net-zero supply condition for bonds (Eq. (10)), we choose to track only bond holdings of the Developed country by setting $b_t = b_{D,t}$ with the constraint $\underline{b_D} = \underline{b} \leq b_t \leq \bar{b} = -\underline{b_E}$. In order to separate conceptually the states from the endogenous controls, we set

$$d_t = b_{t-1}, \tag{12}$$

and define $\mathbf{s_t} = (k_{D,t}, k_{E,t}, d_t)$ the vector of endogenous states.

Our solution approach makes use of first order conditions (Equations (6) and (9)) to solve for unknown policy rules for investment $i_{i,t}$, bond holdings b_t and the bond price p_t. Because of Epstein-Zin preferences, these conditions depend on the utility values $U_{i,t}$. We thus append their definition to our equilibrium conditions, introducing $U^{\star}_{i,t}$ as follows:

$$U^{\star\,1-\gamma}_{i,t} = E_t\left[U^{1-\gamma}_{i,t+1}\right] \tag{13}$$

$$U_{i,t} = \left[(1-\beta)c^{1-\psi}_{i,t} + \beta\left(U^{\star}_{i,t}\right)^{1-\psi}\right]^{\frac{1}{1-\psi}} \tag{14}$$

We rewrite Equation (9) into a single pricing equation,

$$p_t = E_t\left[\lambda_t \mathcal{M}_{D,t+1} + (1-\lambda_t)\mathcal{M}_{E,t+1}\right], \tag{15}$$

with $\lambda_t = \left(\frac{b_t - \underline{b}}{\bar{b} - \underline{b}}\right)$ such that $\lambda_t = 0$ (resp. $\lambda_t = 1$) when country D (resp. E) is constrained. With this formulation, the bond price is always set by a non-constrained country. Lastly, using complementarity notations,[14] one can get rid of the Lagrange multipliers $\mu_{i,t}$ and rewrite Equation (15) together with Equation (8) as follows:

$$E_t\left[\mathcal{M}_{E,t+1}\right] - E_t\left[\mathcal{M}_{D,t+1}\right] \perp \underline{b} \leq b_t \leq \overline{b} \tag{16}$$

We denote by $\mathbf{m_t} = (A_{D,t}, A_{E,t})$ the vector of exogenous productivity processes driving our economy and by $\mathbf{x_t} = (U_{D,t}, U_{E,t}, U^{\star}_{D,t}, U^{\star}_{E,t}, i_{D,t}, i_{E,t}, p_t, b_t)$ the full set of controls. Introducing two smooth functions f (for Equations (6), (13), (14), (15) and (16)) and g (for Equations (2) and (12)), our model is reformulated as follows:

$$\begin{aligned} E_t\left[f(\mathbf{m_t}, \mathbf{s_t}, \mathbf{x_t}, \mathbf{m_{t+1}}, \mathbf{s_{t+1}}, \mathbf{x_{t+1}})\right] \quad &\perp \quad \underline{\mathbf{x}} \leq \mathbf{x_t} \leq \overline{\mathbf{x}}, & (17) \\ \mathbf{s_{t+1}} \quad &= \quad g(\mathbf{m_t}, \mathbf{s_t}, \mathbf{x_t}, \mathbf{m_{t+1}}) & (18) \end{aligned}$$

with boundaries on $\mathbf{x_t}$: $\underline{\mathbf{x}} = (-\infty, -\infty, -\infty, -\infty, -\infty, -\infty, \underline{b})$ and $\overline{\mathbf{x}} = (\infty, \infty, \infty, \infty, \infty, \infty, \overline{b})$.

Numerical implementation. We discretize the bivariate exogenous process of productivity $\mathbf{m_t}$ as a discrete Markov chain with 3×3 states, and choose a compact domain for endogenous states $\mathcal{D} = (k_{D,t} \in [1, 10]) \times (k_{E,t} \in [1, 10]) \times (d_t \in [-5, 5])$, which we discretize using $30 \times 30 \times 30$ points. For each discrete combination $(\mathbf{m_t}, \mathbf{s_t})$, the numerical solution of Eq. (17) and Eq. (18) yields corresponding values for the controls $\mathbf{x_t}$. We use natural cubic splines to interpolate between the grid points. The solution of our problem is a decision rule $\mathbf{x_t} = \varphi(\mathbf{m_t}, \mathbf{s_t})$, continuous with respect to $\mathbf{s_t}$. The relatively high number of grid points (243000) is needed to produce accurate welfare estimates.[15] The implementation of the time-iteration algorithm is further detailed in Appendix C.

[14] For any scalars x, y, a, b, the complementarity condition $y \perp a \leq x \leq b$ is equivalent to say that one of the three following conditions must be met: either $y = 0$, or $y > 0$ and $x = a$, or $y < 0$ and $x = b$. For vectors $\mathbf{x}, \mathbf{y}, \mathbf{a}, \mathbf{b}$, the complementarity condition $(\mathbf{y} \perp \mathbf{a} \leq \mathbf{x} \leq \mathbf{b})$ must hold coordinate by coordinate, i.e. $\mathbf{y_n} \perp \mathbf{a_n} \leq \mathbf{x_n} \leq \mathbf{b_n}$.

[15] The solution runs in approximately 7 hours 40 for the baseline calibration on a 16 core Intel Xeon X5570. This can be reduced to 25 minutes when using the improvement method from Winant (2017).

2.3 Calibration

Our structural parameters, set on yearly basis, are summarized in Tables 1 and 2.

Preferences. We use a standard value for the discount rate β of 0.96 (annual basis). We first consider the CRRA case and set the coefficient of risk aversion γ to 4 (Baseline Low Risk Aversion). Macro models typically use a lower value of 2 while the finance literature uses higher values such as 30 or above to generate meaningful risk premia. Note that with CRRA utility, this pins down the elasticity of intertemporal substitution (EIS) $1/\psi$ to 1/4. Our assumed EIS is in the range of estimates in the literature, towards the lower end of the distribution though.[16] We provide results with a higher EIS in Section 5 but note that a low EIS implies larger benefits from integration, giving the best chances to our calibration to generate large gains. Since the risk aversion coefficient turns out to be a crucial parameter for the quantitative properties of the model and particularly so for the welfare analysis, we also consider higher levels of risk aversion, while keeping the EIS constant to its baseline value of 1/4. To generate reasonable risk premia, we set γ up to 40 in our alternative calibration (Baseline High Risk Aversion).

Technology. The depreciation rate δ and the capital share θ are set to standard values, respectively 8% and 30%. The capital adjustment costs parameter ξ is set to 0.2. In line with the data, this generates a volatility for the rate of investment about 2.5 times higher than the volatility of output.[17]

Countries size and capital scarcity. In our baseline calibrations, we consider countries of equal size—equalizing the initial level of productivity across countries: $A_{D,0} = A_{E,0} = 1$. We do so for two reasons. First, we want to focus on the role played by heterogeneity in risk and/or in the level of capital, neutralizing any effect driven by the size of countries. Second,

[16]Most of the empirical literature surveyed in Campbell (2003) finds estimates of the elasticity of intertemporal substitution between 0.1 and 0.5 (see Hall (1988), Ogaki and Reinhart (1998), Vissing-Jorgensen (2002), and Yogo (2004), Best et al. (2017) among others). The macro and asset pricing literature (discussed in Guvenen (2006)) typically assumes higher values between 0.5 and 1, even though the recent long-run risk asset pricing literature focus on values above unity to match asset prices (Bansal and Yaron (2004)). We investigate those cases in our robustness checks and extensions (Section 5.2).

[17]Standard moments for business cycles and asset prices in our baseline calibrations under autarky and financial integration are reported in Appendix A.1.

Discount rate β	0.96
Elasticity of intertemporal substitution (EIS) $1/\psi$	1/4
Relative risk aversion γ	Low Risk Aversion = 4 High Risk Aversion = 40
Capital share θ	30%
Depreciation rate δ	8%
Capital adjustment costs ξ	0.2
Relative initial productivity $A_{E,0}/A_{D,0}$	1
Relative initial capital scarcity $k_{E,0}/k_{D,0}$	50%

Table 1: Parameters values

we differ from studies focusing on a small open economy as our main focus is not the financial integration of small countries. In late eighties-early nineties, a large set of emerging markets integrated almost simultaneously (see Appendix B for a list of countries and liberalization dates). These countries account for a large share of world GDP, around 50% in 1990,[18] such that general equilibrium effects cannot be neglected. We will however investigate the importance of size for our results in Section 5.

A crucial exogenous parameter for our analysis is the capital stock in both countries at time of integration. In all our baseline experiments, country D starts at its autarky steady-state. Country E is significantly capital-scarce, its initial capital stock being 50% of the initial capital stock of country D. This choice for capital scarcity is well justified regarding the set of emerging markets which opened financially since 1985. Their capital-output ratio at time of opening is on average 62% of the one of (already integrated) developed countries, where capital is measured using a perpetual inventory method. With a usual Cobb-Douglas

[18]The total set of emerging countries liberalizing described in Appendix B accounted in 1990 for 97% of the GDP size of (already integrated) developed countries. If we focus only on emerging countries belonging to the main liberalization wave (between 1988 and 1992), they still account for 83% of the size of (already integrated) developed countries. Note that this sample of countries does not include Russia and Central and Eastern European countries due to lack of data for these countries pre-1990. See Appendix B for details.

production function, this corresponds to a level of capital per efficiency units in emerging markets equal to 52% of the one of developed countries (see Appendix B for details).

Persistence parameter ρ	0.9
Volatility σ_D of shocks in country D	2.5%
Volatility σ_E of shocks in country E	5%
Cross-country correlation of shocks ζ	0

Table 2: Baseline stochastic structure

Stochastic structure. In our baseline simulations, we assume that country E is riskier than country D ($\sigma_D \leq \sigma_E$). Aguiar and Gopinath (2007) provides values for output volatility that are on average twice as large in emerging markets compared to developed countries. In Appendix B, we provide more systematic evidence of the difference in volatility between developed countries and a set of emerging markets which integrated to the world economy since 1985. On an annual basis, the average output growth volatility of these liberalizing emerging markets is 4.9% compared to 2.5% in (already integrated) developed countries. Accordingly, in our baseline calibration, σ_D is set to 2.5% while σ_E is twice as large, set to 5%. Thus, we interpret our baseline experiments as the financial integration of a set of emerging countries to a set of developed countries. The persistence of stochastic shocks ρ is set to 0.9 for both countries.[19] For simplicity, we assume, that productivity shocks are uncorrelated across countries but investigates alternative stochastic structures in Section 5. If anything, such a calibration tends to overstate the gains from financial integration, as the potential for risk sharing is overestimated. The parameters of the variance-covariance matrix Σ of $(\epsilon_{D,t}, \epsilon_{E,t})$ are summarized in Table 2 for our baseline calibration.

[19] Such a persistence parameter for productivity shocks is well within the range of admissible values (standard estimates on an annual basis are usually slightly lower even though not statistically different from 1). A lower value for ρ would reduce further the benefits from integration.

3 Growth and consumption dynamics in a risky world

We turn to the simulations of our model in the baseline calibrations. This section describes the growth and consumption dynamics of countries under integration (compared to autarky) as well as the paths of net foreign assets and world interest rates. These simulations provide a set of predictions regarding the impact of integration on output growth across countries and are helpful in building intuition for the welfare implications developed in Section 4.

3.1 A riskless world: the role of capital scarcity

First, we briefly recall the predictions of the neoclassical growth model with respect to financial integration in a non stochastic environment. In partial equilibrium analyses, countries, modeled as small open economies which display different degrees of capital scarcity, do not impact the world rate of interest when they integrate financially. They will import capital if their autarky interest rate is above the world rate of interest, which will be generally the case if they are capital-scarce emerging markets. Upon integration, their time profile of consumption is perfectly smoothed, investment jumps up so that capital accumulation speeds up. The country borrows internationally to fulfill its optimal consumption and investment plans. Capital flows from 'low marginal product of capital countries' (developed countries) to 'high marginal product countries' (emerging countries). As shown by Gourinchas and Jeanne (2006), financial integration brings welfare gains at it speeds up capital accumulation towards the steady state capital stock, pinned down by the exogenous world rate of interest. Quantitatively, these gains are found to be small (of around 1% and at most 2% increase in permanent consumption for realistic degrees of capital scarcity), a reflection of their transitory nature.

Experiment 1: A riskless world in general equilibrium. Figure 1 shows the dynamics of macro variables in a *non stochastic* environment ($\sigma_D = \sigma_E = 0$).[20] Compared to the experiments in Gourinchas and Jeanne (2006), we relax the small open economy assumption, i.e. the world rate of interest after financial integration is endogenously determined. The

[20]Simulations are performed with an EIS ψ equal to 1/4. The risk aversion is irrelevant in this case.

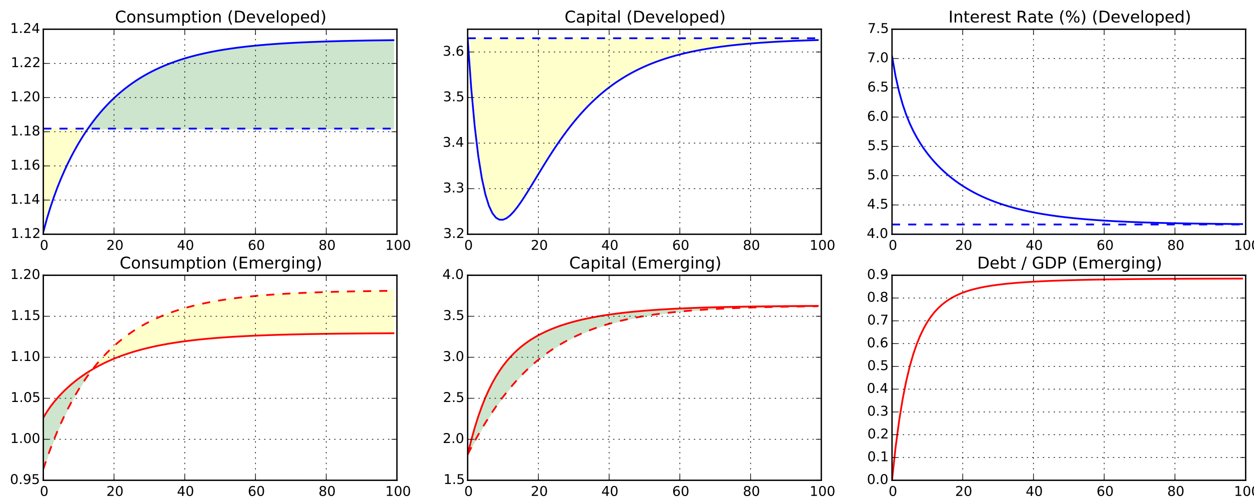

Figure 1: Dynamics along the deterministic path in Experiment 1.

Notes: Parameters of the model are shown in Tables 1 and 2 (risk aversion is irrelevant in the absence of risk). Countries are symmetric except for initial capital stock. The capital scarce country is endowed at the date of integration with 50% of the autarkic steady-state capital stock while the developed economy starts at its steady state. There is no uncertainty. Dotted lines (resp. solid lines) refer to autarky levels (resp. levels under integration).

environment is entirely symmetric except that the emerging country starts off being 50% capital scarce, while the rest of the world (developed country) starts at its autarky steady state. The upper panel of Figure 1 shows the capital and consumption transition paths for the developed country as well as interest rates. The lower panel shows the capital and consumption transition paths for the capital scarce (emerging) country as well as the net external debt over GDP of the emerging country. Dashed lines refer to autarky levels while plain lines refer to levels after integration. The developed country lends to the capital scarce emerging country to finance its capital accumulation: country D cuts consumption and grows at a slower pace under integration, while the emerging country grows faster. Like in the small open economy example, the benefits of integration come from the capacity of the capital scarce economy to borrow in order to speed up capital accumulation and reach faster its steady state level of capital stock. Unlike in the small open economy case, consumption is not constant over time and the debt level is not as high due to the increase of the world interest rate upon integration. In general equilibrium, the increase in output and consumption of the capital scarce economy are dampened by adverse movements of the world interest rate.

3.2 A risky world: capital scarcity and risk sharing effects

We now turn to the richer predictions of the stochastic model, focusing on the interactions between the risk sharing motives and the effect of integration on capital accumulation. To our knowledge, these interactions, which materially affect the predictions of the model with respect to consumption, investment and output have not been studied in the literature.

Risky steady-states. The steady state of the model depends on the risk sharing opportunities of agents due to the presence of precautionary savings (Coeurdacier, Rey and Winant (2011)). As financial integration modifies the ability to smooth shocks, it has a first order effect in the long-run by modifying the steady-state towards which the economy is converging. Under autarky, countries converge to a steady-state described in the first panel of Table 3 in the CRRA case (Baseline low risk aversion with $\gamma = \psi = 4$, Top Panel) and in the Epstein-Zin case (High risk aversion with $\gamma = 40; \psi = 4$, Bottom Panel). The difference in volatility is the only (long-run) asymmetry built in the model. If countries only differ by the level of aggregate risk they are facing, the riskier country E ends up accumulating more capital and producing more output in its autarky steady-state. This is due to the presence of higher precautionary savings in that country. Higher precautionary savings also depress the interest rate in country E. In our framework, risk matters for the steady-state of the economy since higher level of risk drives up savings and foster capital accumulation in the riskier country. Our *risky* steady-state is thus different from the deterministic one. With a low level of risk aversion and risk twice as big in country E, the model generates a fairly small difference in the steady-state level of capital across countries under autarky. As shown in the top panel of Table 3, the riskier country E ends up with a level of capital stock which is 4% higher than the safer country. With a higher level of risk aversion ($\gamma = 40$), precautionary savings increase and differences in autarkic steady-states level of capital are much larger: under autarky, the riskier country ends up having a capital stock 25% higher (bottom panel of Table 3).[21]

[21] The riskier (emerging) country has a higher steady-state capital stock in autarky which might appear counterfactual. This is true *only* at the steady-state for which there is potentially no empirical counterpart. In the data, at time of opening, emerging markets are significantly capital scarce (Appendix B). Under integration, the riskier country has a lower steady-state capital stock.

Low risk aversion ($\gamma = 4$)					
Autarky					
	Capital k	Output y	Riskless rate $1/p - 1$	Risk premium	$\frac{\text{Net foreign assets}}{\text{Output}}$
Country D	3.68	1.48	3.95 %	0.10%	0%
Country E	3.83	1.50	3.32 %	0.39%	0%
Financial integration (bond only)					
	Capital k	Output y	Riskless rate $1/p - 1$	Risk premium	$\frac{\text{Net foreign assets}}{\text{Output}}$
Country D	3.70	1.48	3.90 %	0.10%	-281.2%
Country E	3.66	1.48	3.90 %	0.18%	282.0%
High risk aversion ($\gamma = 40$)					
Autarky					
	Capital k	Output y	Riskless rate $1/p - 1$	Risk premium	$\frac{\text{Net foreign assets}}{\text{Output}}$
Country D	4.10	1.53	2.54 %	0.63%	0%
Country E	5.12	1.63	-0.77 %	2.33%	0%
Financial integration (bond only)					
	Capital k	Output y	Riskless rate $1/p - 1$	Risk premium	$\frac{\text{Net foreign assets}}{\text{Output}}$
Country D	4.40	1.56	1.94 %	0.69%	-206.0%
Country E	4.13	1.53	1.94 %	1.19%	210.0%

Table 3: Risky steady-state values.

Notes: Parameters of the model are shown in Tables 1 and 2. Top panel: Baseline with low relative risk aversion. Bottom panel: Baseline with high relative risk aversion. Countries are symmetric except for risk with $\sigma_D = 2\sigma_E$.

Under financial integration (bond only), the steady-state level of capital converges across countries as the riskless rate is equalized across borders. Note however that in the integrated steady state, capital stocks are not fully equalized across countries. The riskier country E ends up with a permanently lower stock of capital than the safer country D. This is so because the risk premium on capital remains higher in E due to higher volatility. In other words, contrary to autarky, the cost of capital in E is above the one in D: the increase in the riskless rate in E dominates the fall in the risk premium. The difference between the two capital stocks remains however quantitatively very small in this environment with small risk premia.[22] With a high degree of risk aversion ($\gamma = 40$) and a more realistic market price of risk, the difference is more significant and the risky country ends up with a capital stock under integration about 6% lower than the safe country due to higher cost of capital.[23] The reason is that financial integration brings significant risk-sharing opportunities, despite markets remaining incomplete. As both countries can smooth consumption better following productivity shocks, precautionary savings decline and the world steady-state capital stock falls. This largely affects the riskier country which ends up producing less under financial integration than in the autarkic steady-state—the opposite holds for the safer country.[24] The riskier country turns into a net lender in the steady-state as it gets rid of some of his risk by holding a positive net foreign asset position. The safer country is willing to hold that risk by having a leveraged position since it faces a lower amount of aggregate risk on its labor and capital income. Contrary to what is obtained with local approximations around a deterministic steady-state (see Schmitt-Grohe and Uribe (2003)), our global solution pins down a stationary cross-country distribution of wealth. In the long term, there is a stable level of debt associated with the equilibrium world rate of interest. Intuitively, the accumulation of net foreign assets by the riskier country is less attractive once his 'buffer stock' of precautionary savings is reached. An unpleasant feature of our predictions though, is the extreme value for the net foreign asset position once the integrated risky-steady is reached—above 200% of GDP. This calls for two comments. First, welfare gains from integration as computed in Section 4 would be further reduced if the developed country was not allowed—through a stricter borrowing limit—to take such an extreme leveraged position

[22]The risk premium is 0.10% in D and 0.39% in E under autarky, and even smaller under financial integration since consumption becomes smoother.

[23]The risk premium is significant but remains small despite a very high degree of risk aversion. This is a well-known limit of models with production economies—consumption risk is bound to be small since agents can use the domestic capital to smooth consumption (Jermann (1998), Tallarini (2000)). However, as in Tallarini (2000), our calibration with high risk aversion does replicate reasonable market prices of risk (ratio of risk premium to excess returns volatility) of 43% in D and 54% in E under financial integration.

[24]In our baseline calibrations, the level of risk is heterogeneous across countries but when the two countries are equally risky, financial integration still enables them to share their aggregate risk. This reduces the need for precautionary savings in both countries and leads to lower steady state levels of capital stock and output.

to insure the emerging country. Second, our later extension with a worldwide long-run risk component for productivity growth (Section 5) solves for that unrealistic feature of the model, while keeping our results mostly unaffected.

This comparison across steady-states highlights a crucial force that is at play within our model when financial integration takes place: integration enables better risk sharing, which at the same time affects the steady state level of capital stock as precautionary savings adjust. As a result the speed of capital accumulation, associated to the usual neoclassical gains to financial integration, will be altered. We now turn to the description of the transitory dynamics following financial integration in our baseline experiment.

Experiment 2. Growth and capital flows dynamics along the risky path. This experiment corresponds to the financial integration of a large, risky and capital scarce (emerging) country E to a safe (developed) country D and thus resembles the liberalization episode of a set of emerging markets in the late eighties-early nineties. We first stick to the CRRA calibration (Baseline Low risk aversion). In Figure 2, we plot the dynamics of consumption, capital, interest rate and net foreign assets under autarky or following financial integration in period zero. Dynamics of aggregate variables are taken along the path where the realization of innovations are zero. To be consistent with our risky steady state definition, we refer to this path as the *risky path*. Risk is taken into account along that path since agents expect stochastic shocks even though innovations are zero along the path. The upper panel of Figure 2 describes the risky paths of consumption and capital for the (safe) developed economy while the lower panel pertains to the emerging economy.

When a country is capital-scarce and far away from its autarky steady-state, its growth accelerates following financial integration—fostering convergence, like in the non stochastic model. But the key new aspect is that the steady-state towards which the country is converging is *also changing with financial integration* due to new risk-sharing opportunities. Since the growth rate of output depends on how far the country is from its steady-state, two forces are at play: the *capital scarcity* effect and the *risk sharing* effect which alters the desirability of precautionary savings and modifies the country's steady-state upon integration.

For country E, our emerging market style economy which is both capital scarce and volatile, these two forces are conflicting, as displayed in Figure 2. One the one hand, capital scarcity implies faster convergence and faster growth upon financial integration compared to autarky. On the other hand, since the steady-state level of capital stock and output of the riskier country decreases with integration, the country is upon opening *closer* to its steady-state. This implies a lower rate of output growth compared to financial autarky. Which effect dominates at a given date depends on the initial level of capital stock in the country and its distance to its autarky and integration steady-state values. If country E is sufficiently capital scarce as in our baseline experiment (country E's initial capital stock is half of the one in D), the capital scarcity effect dominates initially and financial integration leads to a growth acceleration in the capital scarce country. This acceleration is however muted compared to the deterministic case.[25] As time passes and the capital scarcity effect dissipates, growth slows down and the dominant effect is the risk sharing one: the long-run capital stock of the risky country being smaller in the integrated economy than in autarky, growth is lower under integration than in autarky. Regarding the safer country D which starts at its autarky steady-state, the two same forces are at play: first, its growth rate tends to fall since resources are allocated to the capital scarce country with the highest marginal productivity of capital. Second, the growth rate of country D tends to pick up since it enjoys a higher steady-state level of output as it integrates with a more volatile economy. Due to decreasing marginal productivity of capital, the first effect (capital abundance effect) dominates on impact while the second one (risk sharing effect) dominates when country E is getting closer to its steady-state (see upper panel of Figure 2). Interestingly, both countries exhibit growth and consumption reversals due to financial integration.

When considering international capital flows, similar conflicting forces are at play: on one side, country E tends to have a higher marginal productivity of capital at opening and is willing to borrow internationally to finance capital accumulation. On the other side, E

[25] In another experiment not shown where capital scarcity is less important (country E being 15% away from country D's capital stock), country E is growing at a slower pace compared to autarky at the date of integration.

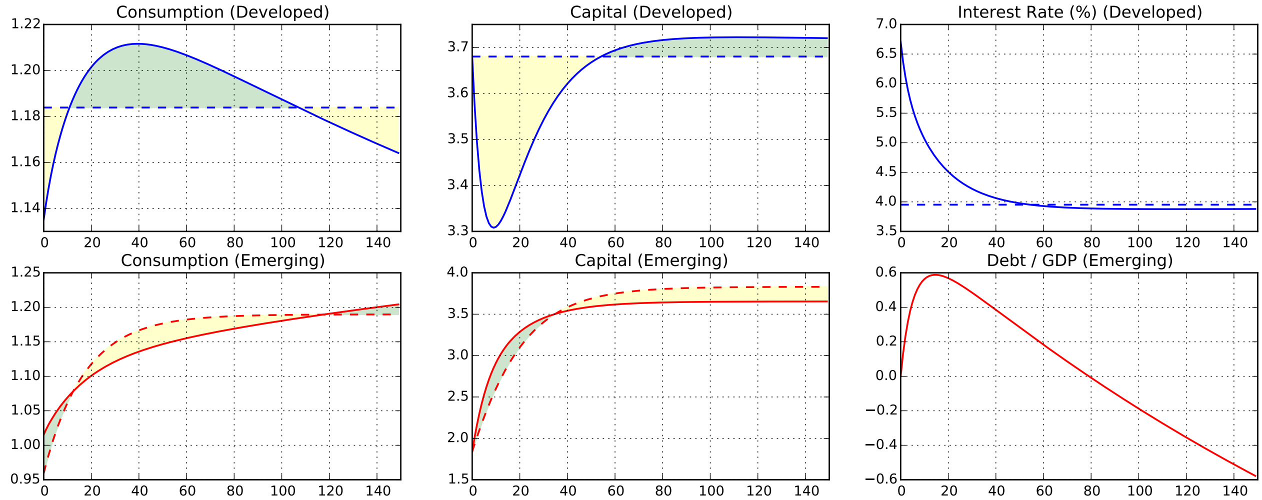

Figure 2: Dynamics along the risky path in Experiment 2 (low risk aversion).
Notes: Parameters of the model are shown in Tables 1 and 2 (baseline with low risk aversion $\gamma = 4$). Countries are asymmetric in terms of risk with $\sigma_E = 2\sigma_D$. Initial capital stock of the risky country E is at 50% of the one in the safe country D. Safe country starts at its autarky steady-state. Dotted lines (resp. solid lines) refer to autarky levels (resp. levels under integration).

is willing to lend internationally for self-insurance due to its higher level of risk. When the country is further away from its autarky steady-state, the capital scarcity effect dominates and country E tends to run current account deficits. As it converges, the risk sharing effect starts to dominate and country E starts running current account surpluses. In the long-run, the intertemporal budget constraint imposes that country E, which ends up as a net lender, runs current account deficit financed by foreign debt payments of country D. Hence, our model exhibits capital flow reversals along the transition path. Quantitatively, in the experiment where country E starts 50% away from its autarky steady-state, it starts running a trade deficit of about 10% of its GDP immediately after opening, then moves into surplus of roughly 3% of GDP (attained after two decades) before moving back again much later into a deficit as country D starts to repay its accumulated foreign debt.

Experiment 3. Growth and capital flows dynamics with high risk aversion. We consider the alternative calibration under non-expected utility, setting the risk aversion γ to the value of 40. The experiment is the exact same one up to the difference in risk aversion and thus market price of risk. One could object that our results in the CRRA case (low risk aversion) rely on counterfactually low risk premia on the capital stock. At this stage, one way to get higher risk premia is to crank up the level of risk aversion up to 40, as commonly done in the macro–finance literature (see Tallarini (2000) among others).

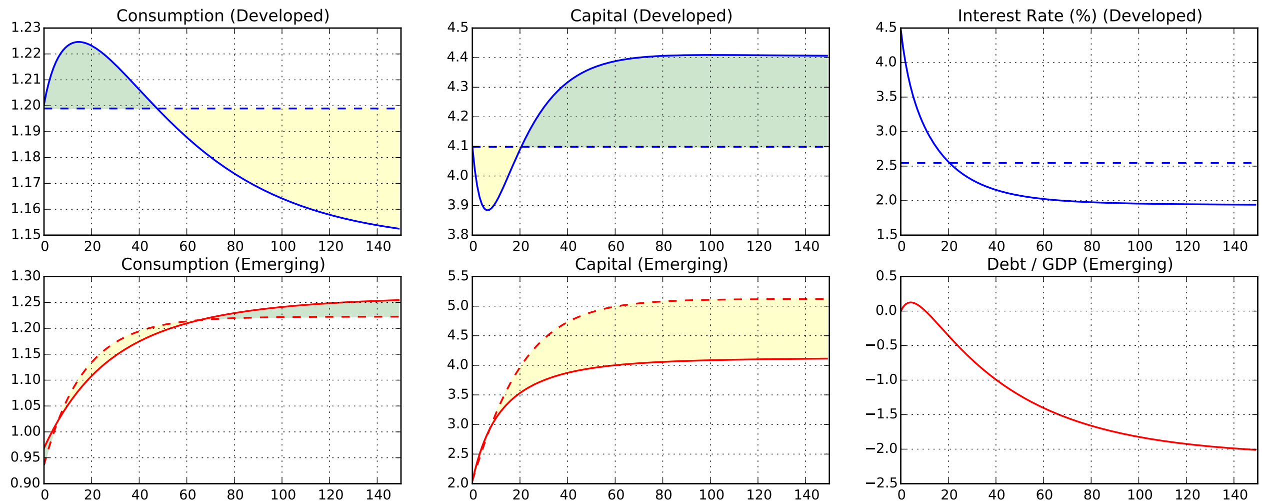

Figure 3: Dynamics along the risky path in Experiment 3 (high risk aversion).
Notes: Parameters of the model shown in Tables 1 and 2 with a high level of risk aversion $\gamma = 40$. Countries are asymmetric in terms of risk with $\sigma_E = 2\sigma_D$. Country D starts at its autarky steady-state. Country E starts with a capital stock equal to 50% of the one in country D.

Figure 3 shows the main variables of interest following integration. The dynamic is quantitatively altered compared to the previous experiment with a low risk aversion but our main message goes through. Intuitions are the same and we insist on the differences in terms of output growth dynamics. Since countries care more about risk, the effects driven by the reallocation of precautionary savings are quantitatively amplified compared to the effects due to capital scarcity. Capital still moves away from the capital-abundant country D upon integration, which grows initially at a smaller pace. However, this reallocation of capital due to the capital scarcity of E is severely muted and quickly dominated by the risk-sharing effect. As precautionary savings are reallocated away from the risky country towards the safe country, D grows at a much faster pace later on, ending with a significantly higher capital stock.[26] Capital flows towards D finance a long-lasting consumption and investment boom in

[26]The permanent difference in the long-run capital stock between autarky and integration is quantitatively important when risk premia are large: under integration, country D ends up with capital stock 6% larger while E ends up with a capital stock 20% smaller.

the country. To the opposite, despite its low initial capital stock, the output growth of the emerging market is barely affected by integration on impact (compared to autarky) and over time, as the risk sharing effect dominates, the risky country turns into a capital exporter and starts growing at a slower pace than it would have under autarky. In other words, following integration, the risky country is very quickly close to its steady-state and growth slows down significantly.

We believe our experiment illustrates the heterogenous effects of financial integration on output growth, heterogenous effects across countries as well as over time. Depending on the degree of heterogeneity across countries in capital scarcity and in the level of aggregate risk, output responses to financial integration can be fairly different. The empirical literature has been mostly trying to identify an *average* effect of financial integration on growth, leaving aside potential heterogeneous responses (see Kose et al. (2009) for a survey). Empirical estimates vary significantly across studies (across countries samples and sample periods) and tend to be fairly low—at most 1% increase in growth on impact. Our theory might provide a rationale for such heterogeneity in the empirical estimates. Moreover, the empirical literature often tries to identify the growth impact of the financial integration of an individual emerging country. While taken individually, most emerging countries are small, they have historically integrated over the same time frame—making it likely that their integration impacted the world interest rate. With adverse general equilibrium movements of world interest rates as in our experiments, the growth effects of financial integration can be severely dampened—which could also partially explain the low growth effects of financial integration found in the data.

4 Welfare analysis

As the previous section shows, if riskier countries are also capital scarce at opening, the effect on output will be ambiguous, depending on two conflicting forces, the standard efficiency gains and the reallocation of precautionary savings towards the safer country. Efficiency gains are reduced since the riskier country will end up with a permanent fall in steady-state output and thus the convergence-gap, essential for these efficiency gains, falls at time of integration. Our findings thus qualify the conventional wisdom that risky and capital scarce emerging countries should face large gains from financial integration. In this section we present quantitative estimates of the welfare gains of financial integration.

Definition of welfare gains. As standard, we express welfare gains in terms of equivalent increase in permanent consumption compared to autarky. For a given asset market structure (A for autarky or FI for financial integration), let us define the permanent certainty equivalent level of consumption $\overline{c_i}^j$ in country $i = \{D, E\}$ in regime $j = \{A; FI\}$ such that: $U^j_{i,0}(\overline{c_i}^j) = E_0(U^j_{i,0})$, where $U^j_{i,0}$ is the utility defined recursively in Equation (4) in regime $j = \{A; FI\}$ and $\overline{c_i}^j$ is constant consumption path providing the same expected utility. The welfare gains from financial integration in country i, in % increase of permanent consumption, are equal to $\frac{\overline{c_i}^{FI} - \overline{c_i}^A}{\overline{c_i}^A}$.

4.1 Welfare analysis with constant relative risk aversion

We start by quantifying the welfare gains in our baseline case with low risk aversion and expected-utility.

Results. Table 4 (upper panel) provides a summary of the findings with CRRA utility (Baseline Low Risk aversion). The Baseline corresponds to Experiment 2, where country E is riskier and capital scarce initially. The 'no capital scarcity' case corresponds to a case where the emerging country starts with the same level of capital as the developed country (only risk asymmetry). For comparison purposes, keeping all other parameters identical, we also provide results for a case with symmetric (developed) countries (symmetric risk $\sigma_D = \sigma_E = 2.5\%$ and identical initial autarky steady-state capital stock, line 3 of Table 4), for endowment economies (infinite capital adjustment costs $\xi \to \infty$, line 4 of Table 4) and for the riskless world model ($\sigma_D = \sigma_E = 0$, line 5 of Table 4). In the latter case, E starts off being capital-scarce ($k_{E,0}$ is 50% of the initial (steady-state) capital stock of D). Thus, it has to be compared to the capital scarce experiment with aggregate risk (Baseline, line 1 of Table 4).

First and foremost, in our stochastic model with production, gains from financial integration are remarkably small for each country: less than half-a-percent of permanent consumption, and thus despite the presence of both types of gains, efficiency gains coming from capital reallocation (from capital-abundant countries towards capital scarce countries) and gains from better risk-sharing. This is so because, in our baseline experiment, gains from efficient capital reallocation are reduced by the need to share risks efficiently. In other words, gains from efficient reallocation of capital and gains from risk-sharing are roughly speaking substitutes, which makes it very unlikely to observe large gains from financial integration for any country. The intuition goes as follows: the riskier country benefits the most from better consumption smoothing, but self-insurance requires capital to reallocate away from that country. This goes against what standard neoclassical efficiency gains would require as the riskier country is also capital scarce. Reciprocally, as the country is initially very capital scarce in our baseline, by importing capital initially for efficiency reasons, the risky country cannot self-insure optimally, reducing its gains from risk-sharing along the transition.

In these experiments with asymmetric risk, the risky country has to pay a price for better insurance, which benefits the safer one—in the form of higher consumption for a while following integration. Thus, perhaps counter-intuitively, gains from better risk sharing are relatively equally shared, even though the safer country ends up with an almost unchanged volatility of consumption under financial integration (see Appendix A.1 for business cycles moments).[27] Welfare gains are also relatively equally shared when the risky country is initially capital scarce: the emerging country benefits more from the efficient reallocation of capital but this entails welfare costs as the country has to bear more risk along the transition to build up its capital stock. In a world with symmetric (but low) risk, aggregate welfare gains are even smaller compared to the case with asymmetric risk: this should not come at a surprise since similar countries have less incentives to reallocate capital and risk—significantly lowering the 'gains from trade'.

[27] Two conflicting forces under integration are at play to determine the steady-state volatility of consumption of D: D can smooth better transitory shocks through the bond market but D is also holding more risk on average (leveraged position). The former effect dominates slightly in our calibration with a low risk aversion, while the latter effect dominates with a high risk aversion.

		Country D	Country E
CRRA Utility	Baseline (Exp. 2)	0.37%	0.48%
Low risk aversion	No capital scarcity	0.26%	0.23%
	Symmetric	0.09%	0.09%
	Endowment	0.60%	0.56%
	Riskless world (Exp. 1)	0.29%	0.37%
Non-Expected Utility	Baseline (Exp. 3)	0.85%	0.20%
High risk aversion	Endowment	1.79%	0.43%

Table 4: Welfare gains of financial integration.

Notes: Gains expressed in % equivalent increase of permanent consumption. Parameters of the model are shown in Tables 1 and 2 (low risk aversion with $\gamma = 4$ and high risk aversion with $\gamma = 40$). For the benchmark and 'no capital scarcity' cases, $\sigma_E = 2\sigma_D = 5\%$. For the 'symmetric' case: $\sigma_D = \sigma_E = 2.5\%$ and both countries start at their autarky steady state capital stock. In the riskless world and in the benchmark cases, country E is capital scarce (50% of the developed country capital stock) at date 0. In the endowment case, both countries have the same initial size and adjustment costs to capital are infinite.

Comparison with alternative models. The welfare gains from integration are significantly higher when considering endowment economies, even if there are no efficiency gains due to efficient capital reallocation.[28] With endogenous production, gains from risk-sharing are significantly smaller for both countries because capital can be used in the autarky regime to smooth stochastic shocks. One could still argue that our gains from risk sharing are low because market price of risk is too low in our economy, a criticism we tackle in Section 4.2.

Our findings call for another important comment when comparing to Gourinchas and Jeanne (2006). In their exercise, Gourinchas and Jeanne (2006) neglects potential losses due to capital reallocation since they focus on a small open economy. Hence, in their set-up, the capital reallocation is not slowed down by a raise in the world interest rate: the capital abundant country is exporting capital abroad without decreasing its own capital stock. Even in the non-stochastic case, the general equilibrium forces in our model already reduce the

[28] This experiment corresponds to the ones run in the international risk-sharing literature (see van Wincoop (1999) and Lewis (1999) among others for references).

welfare gains due to efficient capital reallocation to an order of magnitude smaller. While in partial equilibrium, welfare gains of the capital scarce economy amount to a 1.30% increase of permanent consumption, in general equilibrium, the emerging country gains 0.37% of consumption and the rest of the world 0.29% only. Not taking into account adverse changes of interest rates leads to an overestimation of the neoclassical gains from financial integration. Moreover, with aggregate risk, if the capital scarce country is also riskier, the reallocation of capital due to the change in the steady-state partially offsets the standard efficiency gains, keeping welfare gains for country E fairly low.

Timing of the welfare gains. In these experiments, countries can extract most of the welfare benefits in the earlier periods (if not all and then pay in later periods), or to the opposite, suffer in the earlier periods for larger gains in the far future. Abstracting from capital scarcity, gains are front loaded by the safer country which enjoys a consumption boom initially. Gains in the medium-run (first two decades following integration) are thus potentially much larger than the overall gains. Far in the future, the country faces more volatile consumption due to its leveraged position and cuts its consumption to pay back the initial debt. The opposite holds for the riskier country which significantly cuts consumption in the medium-run for better self-insurance far in the future. Holding risk constant across countries, welfare gains are front loaded by capital scarce economies. Therefore, in our benchmark experiment, two forces are at play: on one side, the capital scarcity effect generates medium-run consumption gains (resp. losses) for country E (resp. country D). On the other side, the reallocation of precautionary savings towards the safer country generates medium-run gains (resp. losses) for country D (resp. country E). When simulating the consumption paths, we find that on average both effects tend to offset each other and both countries have fairly small consumption gains in the first twenty years following integration.

4.2 Welfare analysis with non-expected utility

Our baseline calibration with CRRA utility generates very low risk premia. We now compute welfare gains with recursive utility as defined in Equation (4), cranking up the degree of risk aversion to generate more realistic risk premia. Other parameters are kept to their baseline values (Tables 1 and 2). Estimates of the welfare gains in our baseline financial integration experiment with a high risk aversion ($\gamma = 40$) are shown in Table 4 (bottom panel). To isolate the effect driven by the price of risk, Figure 4 shows the welfare gains as a function of the degree of risk aversion γ when countries are asymmetric in terms of risk but start with the same level of capital ($\frac{k_{E,0}}{k_{D,0}} = 1$).

Aggregate welfare gains. First, not surprisingly, overall welfare gains from financial integration (i.e the average of the gains across countries) are increasing in the degree of risk aversion. This is so because international risk sharing is more valued with higher risk aversion. However, despite a much higher market price of risk (a 2.3% risk premium in autarky in the risky country under autarky, resp. 0.6% in the safe country, about six times higher than in our baseline with low risk aversion), the welfare gains remain quite small, with an average across countries barely above 0.5%. As in the CRRA case, they are also remarkably lower in production economies than in endowment economies—despite gains from more efficient capital reallocation.

Distribution of welfare gains. Regarding the distribution of gains across countries, they are remarkably low for the risky country for any level of risk aversion (always below 0.5%). With higher degree of risk aversion, the increase in aggregate welfare gains masks an important heterogeneity across countries. Welfare gains are very unevenly shared between the safe and the risky country: the higher the degree of risk aversion, the more the safe country benefits from financial integration compared to the risky country (see Figure 4 in the absence of capital scarcity in E). In the extreme case of $\gamma = 40$, welfare gains in the safe country are getting close to 1% of permanent consumption (versus 0.37% in the baseline experiment with $\gamma = 4$). The emerging risky country has actually lower gains when $\gamma = 40$ (only 0.20% of permanent consumption) compared to our baseline with $\gamma = 4$ (0.48%). The intuition for this result goes as follows: the safe country has the technology that both countries prefer, i.e. a less risky production function. Comparative advantage logic would predict that the safe country benefits more from trading. The higher the risk aversion the more agents value the safest technology, increasing thereby the wealth of the safe country. From the perspective of the risky country, it benefits more from risk sharing if more risk averse but the costs of reallocating risk are also much higher: insurance is more expensive in this case and the world interest rates is much lower upon integration (see Figure 3).

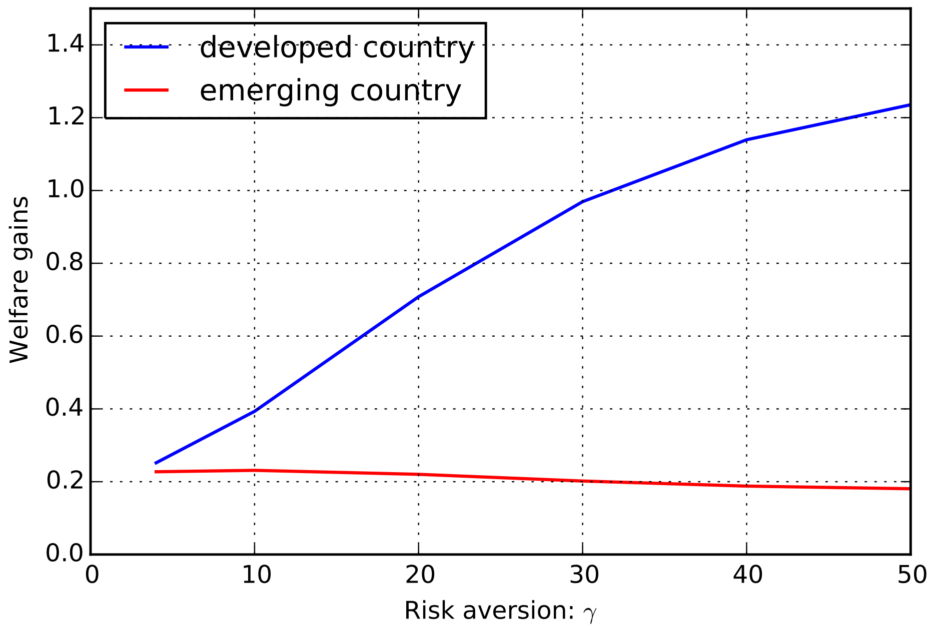

Figure 4: Welfare analysis of financial integration with higher degree of risk aversion (Epstein-Zin preferences).
Notes: Gains are expressed in % equivalent increase of permanent consumption as a function of the degree of risk aversion γ. Countries starts off with the same capital stock (no capital scarcity) $\frac{k_{E,0}}{k_{D,0}} = 1$. Individuals have Epstein-Zin preferences with an elasticity of intertemporal substitution $1/\psi = 0.25$ and a risk aversion $\gamma \geq 4$. Other parameters of the model are kept identical to the ones in Tables 1 and 2.

4.3 Sensitivity and Accuracy

The role of capital scarcity. We investigate how the overall welfare gains from integration depend on the initial relative endowment in capital. Figure 5 shows the welfare benefits for both countries in our baseline with low risk aversion ($\gamma = 4$, upper panel) and for high risk aversion ($\gamma = 40$, lower panel) as a function of the relative initial capital stocks ($\frac{k_{E,0}}{k_{D,0}}$).

In our baseline with CRRA utility (upper panel), the curves exhibit a clear U-shape since large ex-ante differences in capital stocks increases benefits from efficient capital reallocation. For most values of relative capital stock, the safer country benefits more from integration but

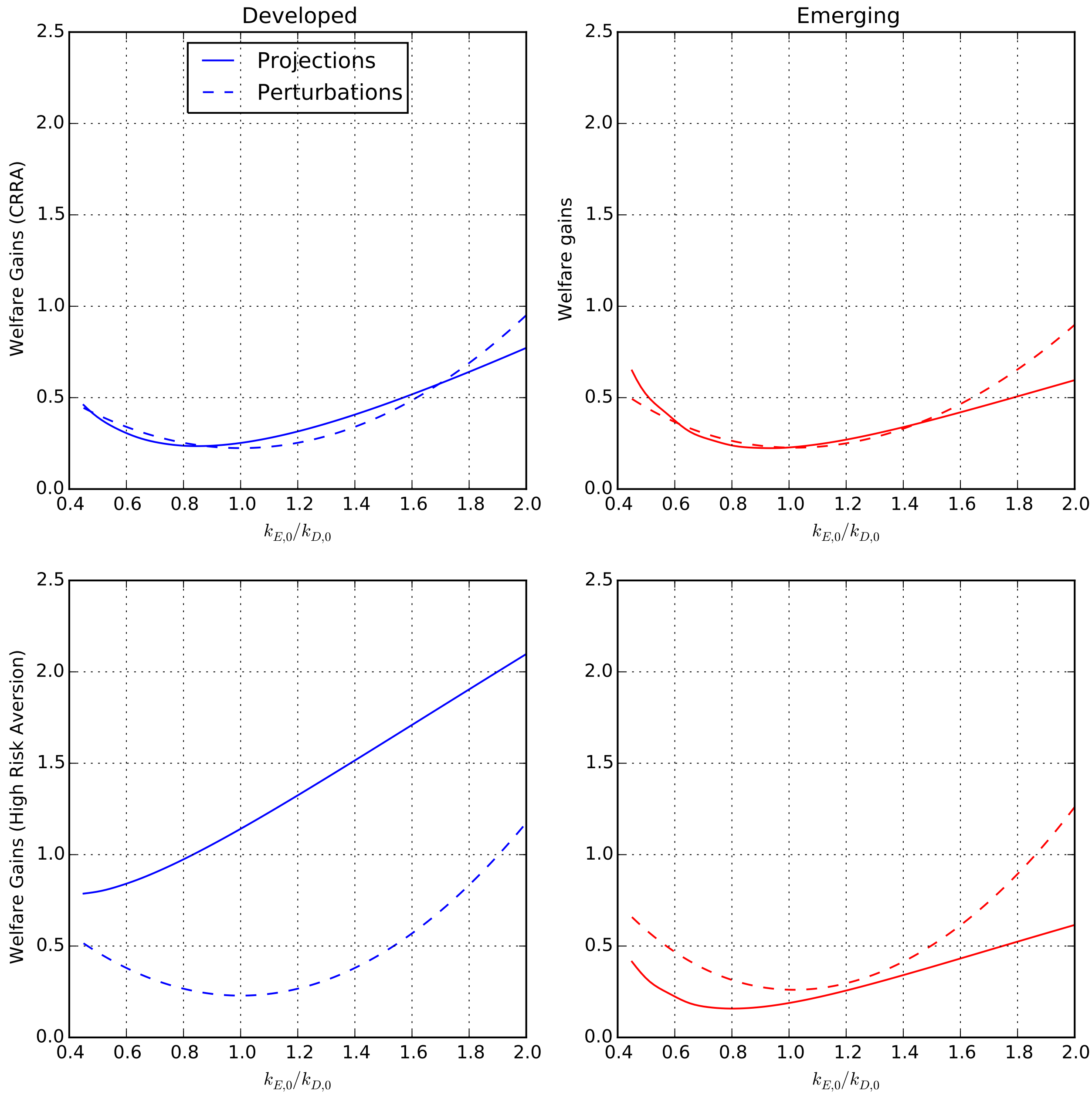

Figure 5: Welfare analysis of financial integration. Sensitivity for different degrees of capital scarcity.

Notes: Gains are expressed in % equivalent increase of permanent consumption as a function of initial relative capital stock ($\frac{k_{E,0}}{k_{D,0}}$). The upper panel correspond to our baseline calibration with CRRA utility ($\gamma = \psi = 4$). The lower panel corresponds to Epstein-Zin preferences with high risk aversion ($\gamma = 40$ and $\psi = 4$). Other parameters of the model are kept identical to the ones shown in Tables 1 and 2. Dotted lines are welfare estimates using second-order perturbation methods.

the difference is small quantitatively for a low risk aversion. In our alternative calibration with a high risk aversion (lower panel), as explained above, the safer country extracts a much larger share of the benefits. The risky country benefits less from integration when risk aversion is high, even if capital scarce since the dominant force driving the capital allocation across countries is the reallocation of precautionary savings. This makes it much less likely that financial opening will increase the stock of capital of the riskier country, thus despite a lower level of capital ex-ante. For the safer country, welfare gains are larger but the shape of the curve is also modified. The minimum is shifted to the left and the slope is now much steeper: when the safe country starts with an initially low level of capital, the additional gains from integration are larger. Contrary to the risky country, when the safe country is initially capital scarce, the reallocation of precautionary savings and the reallocation of capital for efficiency reasons are complementing each other. They both imply capital to flow towards the safe country. With a high degree of risk aversion, the larger reallocation of precautionary savings away from the risky country accelerates the convergence of the safe country when capital scarce, boosting its gains from financial integration. This case is the mirror of our low gains result for the risky country: for a capital scarce and risky country, capital flows are the outcome of two conflicting forces (precautionary savings vs. allocation efficiency)—implying low welfare gains. With a safe and capital scarce country, these two forces are moving capital in the same direction, reinforcing each other.

Global methods vs. perturbations. Figure 5 also shows the welfare gains estimated using a standard *second-order* perturbation method around the deterministic steady-state. Not surprisingly, the perturbation method gives results similar to our global method when none of the country is significantly capital scarce and when the degree of risk aversion is low. For these parameters values, the model does not drift to far away from the approximation point and curvature in the utility is small enough to guarantee a minimal effect of non-linearities.[29] Perturbation methods are however quite inaccurate when risk aversion is set to a high value and/or one country is significantly capital scarce.

[29]Note that when countries start with similar level of capital (or slightly higher in the emerging country), welfare gains are significantly underestimated for the safe country using the perturbation method, even with a low risk aversion. The perturbation methods does not capture well that the minimum level of gains for the safe country is shifted to the left (compared to the deterministic case) due to the reallocation of precautionary savings (see Figure 5)). To the opposite, when one country is very capital scarce and/or precautionary savings matter more (high risk aversion), the perturbation methods provides very different (inaccurate) estimates of the welfare gains. This is the combination of two effects: non-linearities are more important with significant capital scarcity or high curvature in the utility function and the risky steady-state is further away from the deterministic one.

5 Robustness Checks and Extensions

We perform a wide range of robustness checks regarding the stochastic process governing the shocks, the financial asset market structure (assuming complete markets), the size of countries. In particular, we provide an extension of our model with a world long-run risk component to be able to generate a significant market price of risk without extreme values for the risk aversion. Our main findings still hold: financial integration does not bring sizable welfare gains, in particular for riskier emerging economies where benefits do not exceed 1% of permanent consumption for realistic parameters values. Only a small, capital scarce and very safe country can extract significant welfare gains when integrating to riskier countries.

5.1 Alternative specifications of transitory risk

We investigate the robustness of our findings with respect to the stochastic structure in the baseline model of Section 2. We compute the welfare gains for different levels of volatility in the risky country σ_E and different correlation ζ of productivity shocks across countries (assumed to be zero in the baseline).

Data. For comparison, in our sample of emerging countries integrating to the world economy, the volatility of output ranges from 2.1% (Spain) to 8.7% (Jordan) (see Appendix B for details). The correlation of output growth between emerging countries and the sample of developed countries (already integrated) varies across regions, ranging from close to zero in Asia and Middle-East to 0.6 for Southern Europe countries.[30] The average (GDP-weighted) correlation across all liberalizing emerging markets is equal to 0.20 (see Appendix B for details on the correlation structure of GDP growth across countries). This suggests that potential gains from financial integration are slightly overestimated in our baseline calibration.

Results. In the following simulations, all parameters but the volatility σ_E and correlation ζ are kept to their baseline values (see Tables 1 and 2) in the low risk aversion case (Experiment 2). We also provide results when both countries start at the same level of capital ('No capital

[30] Abstracting from Southern Europe, the correlation of output growth a given region of emerging markets with developed countries is always between 0 and 0.35.

	Baseline						No capital scarcity					
	$\zeta = 0$		$\zeta = 0.25$		$\zeta = 0.5$		$\zeta = 0$		$\zeta = 0.25$		$\zeta = 0.5$	
	D	E	D	E	D	E	D	E	D	E	D	E
(Symmetric) $\sigma_E = 2.5\%$	0.37%	0.49%	0.35%	0.47%	0.36%	0.47%	0.09%	0.09%	0.07%	0.07%	0.05%	0.05%
(Baseline) $\sigma_E = 5\%$	0.38%	0.49%	0.30%	0.41%	0.29%	0.40%	0.26%	0.23%	0.15%	0.14%	0.07%	0.07%
$\sigma_E = 10\%$	0.94%	0.75%	0.47%	0.41%	0.19%	0.24%	1.32%	1.02%	0.73%	0.60%	0.21%	0.18%

Table 5: Welfare gains from financial integration with alternative stochastic structures for transitory shocks.

Notes: Welfare gains from financial integration are expressed in % equivalent of permanent consumption. Apart from σ_E and ζ, parameters of the model are set to their baseline values in Tables 1 and 2 with risk aversion equal to its low value ($\gamma = 4$). In the 'No capital scarcity' experiment, both countries start with the same level of capital corresponding to the autarkic steady-state in D.

scarcity', right panel) to isolate better the role of risk sharing. Welfare gains from integration with alternative stochastic structures of transitory shocks are displayed in Table 5.

Higher correlation of shocks ζ reduces the gains from integration, limiting the ability of countries to share risks internationally. Abstracting from capital scarcity, gains fall quickly with the level of correlation and are quite negligible with $\zeta = 0.5$ when countries start at the same initial level of capital (right panel). If one considers a correlation ζ of 0.25 and a volatility σ_E of 5%, very close to the empirical average across liberalizing emerging markets (see Appendix B), gains from financial integration amounts to 0.30% in D and 0.41% in E in our baseline experiment (resp. roughly 0.15% in both countries if they start at the same level of capital). Larger asymmetry in aggregate risk across countries mostly increases the welfare gains for both countries but the safe country benefits more—similarly to our calibration with a higher market price of risk through higher risk aversion.[31] As country E gets riskier, its precautionary demand for safe assets at opening increases, which benefits more to the safe country.

[31]Higher risk asymmetry increases welfare gains for both countries in the absence of capital scarcity (right panel of Table 5). With E capital scarce (left panel), results are ambiguous at higher level of correlation ζ (e.g. third column for $\zeta = 0.5$): in this case, the direction of capital flows due to capital scarcity implies less efficient risk sharing along the transition—the riskier country, attracting capital, is less able to self-insure along the transition. Consumption smoothing is even more limited if shocks are more correlated. The combination of high correlation and capital scarcity can generate lower gains despite higher risk asymmetry.

5.2 Extension with long-run world productivity risk

Our production economies feature low risk premia, unless assuming extreme values for the risk aversion. Another unpleasant prediction of the model is very high net foreign asset positions in the long-run risky steady-state, which is reached after a very long transition of at least a century. To remedy these limitations, we add persistent shocks to *world* productivity growth, following the long-run risk literature (Bansal and Yaron (2004)).

Set-up with a long-run world productivity risk. We specify a common *world* component instead of country-specific long-run risks, partly for technical reasons—otherwise countries dynamics would not be stationary in an incomplete markets model, but this choice is also motivated by empirical evidence. Country-specific long-run risks are found very highly correlated across countries using asset prices data (Lewis and Liu (2012)) or consumption data (Colacito and Croce (2011), Nakamura et al. (2014)).[32] Our framework is thus broadly in line with previous empirical findings, which point towards a fairly low cross-country correlation of transitory risk and a very high correlation of persistent risk.

In our extended version of the model, stochastic total factor productivity $A_{i,t}$ in country i can be decomposed into a transitory country-specific component $a_{i,t}$ and a persistent world component $a_{W,t}$, such that: $A_{i,t} = a_{W,t}a_{i,t}$, where $\log(a_{i,t})$ follows an AR(1) process as defined in Section 2.1—with our baseline calibration of transitory shocks (Table 2). The long-run component $a_{W,t}$ is such that the world is hit by persistent world TFP growth shocks: $\log(\frac{a_{W,t+1}}{a_{W,t}}) = \rho_W \log(\frac{a_{W,t}}{a_{W,t-1}}) + \epsilon_{W,t}$, with $\epsilon_{W,t}$ an i.i.d process normally distributed with volatility σ_W—$\epsilon_{W,t}$ is assumed to be uncorrelated with transitory shocks. In our baseline with long-run risk (LRR), we use the following values for the persistence ρ_W and volatility parameter σ_W: $\rho_W = 0.999$ and $\sigma_W = 8\% \cdot \sigma_D = 4\% \cdot \sigma_E = 0.002$—a calibration close to Colacito and Croce (2011, 2013) or Lewis and Liu (2012).[33] As well known in the literature, for long-run risks

[32] Nakamura et al. (2014) find that world (highly persistent) growth rate shocks are crucial to match cross-country consumption data over a long-time period. They disentangle country-specific and world growth shocks and find the latter to be twice as persistent and thus more crucial for asset pricing.

[33] In those papers, the ratio of volatility between long run and short run shocks is small, between 4% and 10% depending on the calibration and the country. Our calibration features slightly more persistent risk as countries considered in those papers — US/UK/Canada — are among the ones with the lowest variability of consumption in our sample. To reduce the state-space, persistent shocks to world productivity growth

to matter for asset prices, the elasticity of intertemporal substitution $1/\psi$ needs to be above unity. In our simulations, we set $1/\psi$ to 2. With persistent risk, our model does not require such a high risk aversion to generate significant risk premia and we set γ to 10, as in Lewis and Liu (2012).

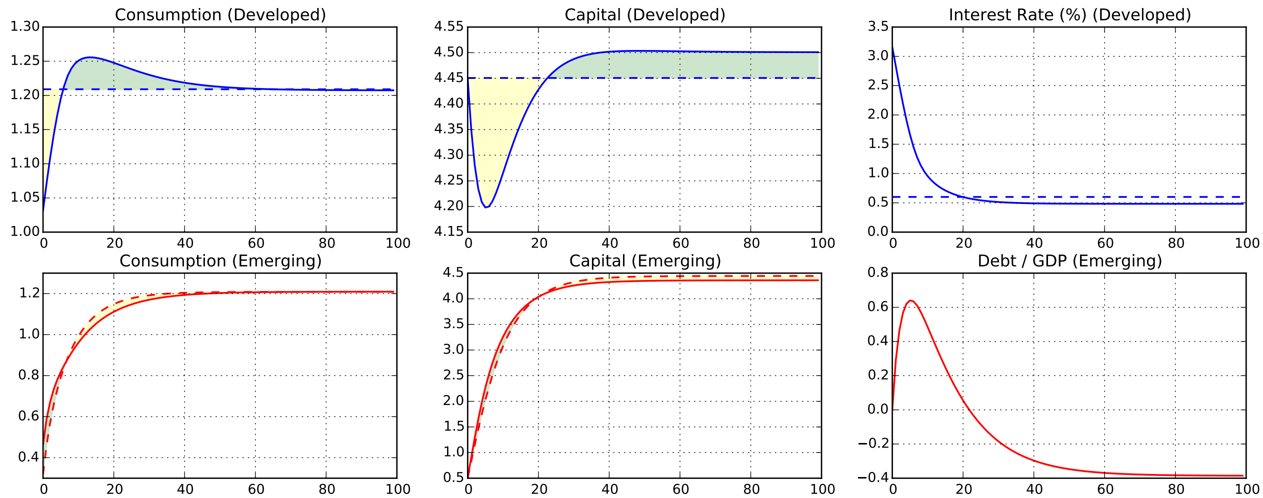

Figure 6: Dynamics along the risky path in presence of long-run world productivity risk. Notes: Preferences are such that $1/\psi = 2$ and $\gamma = 10$—other parameters are identical to the baseline calibration in Table 1. Parameters of the model for transitory risk are shown in Table 2. Persistence (resp. volatility) of long-run world productivity risk ρ_W (resp. σ_W) is set to 0.999 (resp. 0.002). Country D starts at its autarky steady-state. Country E starts with a capital stock equal to 50% of the one in country D.

Results. We start by describing the risky-steady state under autarky and financial integration in our economies with a world long-run risk. Results are shown in Table A.2 in Appendix A.1. As expected, our version with long-run risk generates significantly higher risk premia (together with a reasonably low riskfree rate): the risk premium under integration is 1.98% (resp. 2.22%) in the developed country (resp. emerging country).[34] Moreover, the introduction of long-run risk modifies the long-term distribution of wealth: the safe country still ends up as a debtor but the net foreign asset position is an order of magnitude smaller (-38% in the risky-steady state compared to multiple of GDPs, see Table 3).[35] The safe country is willing

are approximated by a three-states Markov chain with the same persistence and volatility—similarly to our transitory shocks. See details in the online Appendix C.

[34]See Appendix A.1. Our baseline LRR calibration still falls short of observed risk premia compared to the LRR literature as our model features production economies.

[35]This lower level of net foreign assets is due to the presence of long-run risk and not to alternative values

to borrow from the riskier one but any leveraged position implies a higher exposure to the (non-diversifiable) world-long run risk. In order to optimally allocate the exposure to the long-run risk component, countries have more compressed net foreign asset positions. For the same reasons, countries will be less willing to borrow and lend to smooth transitory shocks, implying also a more compressed distribution of net foreign assets (see Figure A.1 in Appendix A.1). Thus, our extension with a world persistent risk generates more realistic asset prices together with more realistic net foreign asset positions.

The dynamics of the main aggregate variables following integration are qualitatively unchanged (see Figure 6) but the lower magnitude of capital flows reduces significantly the quantitative impact of financial integration (compared to autarky)—limiting in particular its growth impact. Regarding the welfare effect of financial integration, the same logic applies: as the reallocation of transitory risk in the long-run as well as the ability to smooth transitory shocks are limited by the presence of a world long-run risk, welfare gains are very small, significantly smaller than in our baseline calibrations with transitory shocks only: for both countries, the gains are below a 0.1% increase in permanent consumption in our baseline LRR calibration (see Appendix A.2).

5.3 The role of financial markets structure

In our baseline incomplete markets model, international risk-sharing is limited due to the absence of state-contingent claims. We go to the extreme case of complete financial markets (perfect risk-sharing) as a robustness check. This provides a useful upper-bound of the gains from financial integration.

Solution under complete markets. To solve the model under complete markets, we assume that the world economy consists in only one fictitious agent whose preferences are identical to those of each country (either CRRA with a coefficient γ in our baseline or Epstein-Zin preferences with coefficients ψ and γ). This agent invests optimally in both countries, maximizing

for the preference parameters—our economy with identical preferences but no LRR behave similarly as in the previous simulations.

its intertemporal utility subject to the law of capital accumulation (Equation (2)) and the resource constraints (Equation (11)).

Let us denote c_t^{CM} her consumption. With complete markets and symmetric preferences, each country i is consuming a constant fraction λ_i of the world consumption at all dates, with $\lambda_D + \lambda_E = 1$:

$$c_{i,t}^{CM} = \lambda_i c_t^{CM}$$

These fractions are allocated according to initial wealth at time of integration, which depends on initial state variables, the capital stock and the productivity level. The wealth $W_{i,t}$ of country i, a claim on total output net of investment, is defined by the recursive equation: $W_{i,t} = (y_{i,t} - i_{i,t}) + E_t\{\mathcal{M}_{t+1} W_{i,t+1}\}$, with $\mathcal{M}_{t+1}$ the stochastic discount factor common to both countries under financial integration (defined in Equation (5)), which is also the stochastic discount factor of the fictitious representative agent. The initial consumption share λ_i in country i at date of integration ($t = 0$) is equal to $\frac{W_{i,0}}{W_{D,0}+W_{E,0}}$.

Following our notations, we denote by $\overline{c}^{CM}$ the welfare of the representative (fictitious) agent in terms of permanent consumption equivalent. The homogeneity of preferences implies: $\overline{c_i}^{CM} = \lambda_i \overline{c}^{CM}$, where $\overline{c_i}^{CM}$ denotes the welfare in terms of permanent consumption equivalent in country i after integration under complete markets. The welfare increase from financial integration under complete markets follows immediately by computing $\frac{\overline{c_i}^{CM} - \overline{c_i}^{A}}{\overline{c_i}^{A}}$ for country i.

Welfare analysis under complete markets. Figure 7 shows the welfare benefits from financial integration under complete the markets (solid line) as a function of the relative initial capital stocks ($\frac{k_{E,0}}{k_{D,0}}$) in our baseline calibration with CRRA utility (top panel) and with non-expected utility and a high degree of risk aversion ($\gamma = 40$, bottom panel). The welfare gains are compared to our baseline model with incomplete markets (dotted line).

In the calibration with CRRA preferences (Low risk aversion), welfare gains under complete markets are significantly higher than under incomplete markets, roughly doubling in magnitude. They do remain small, about 1% of permanent consumption. With a high risk aversion (bottom panel of Figure 7), the welfare benefits of completing the markets are signif-

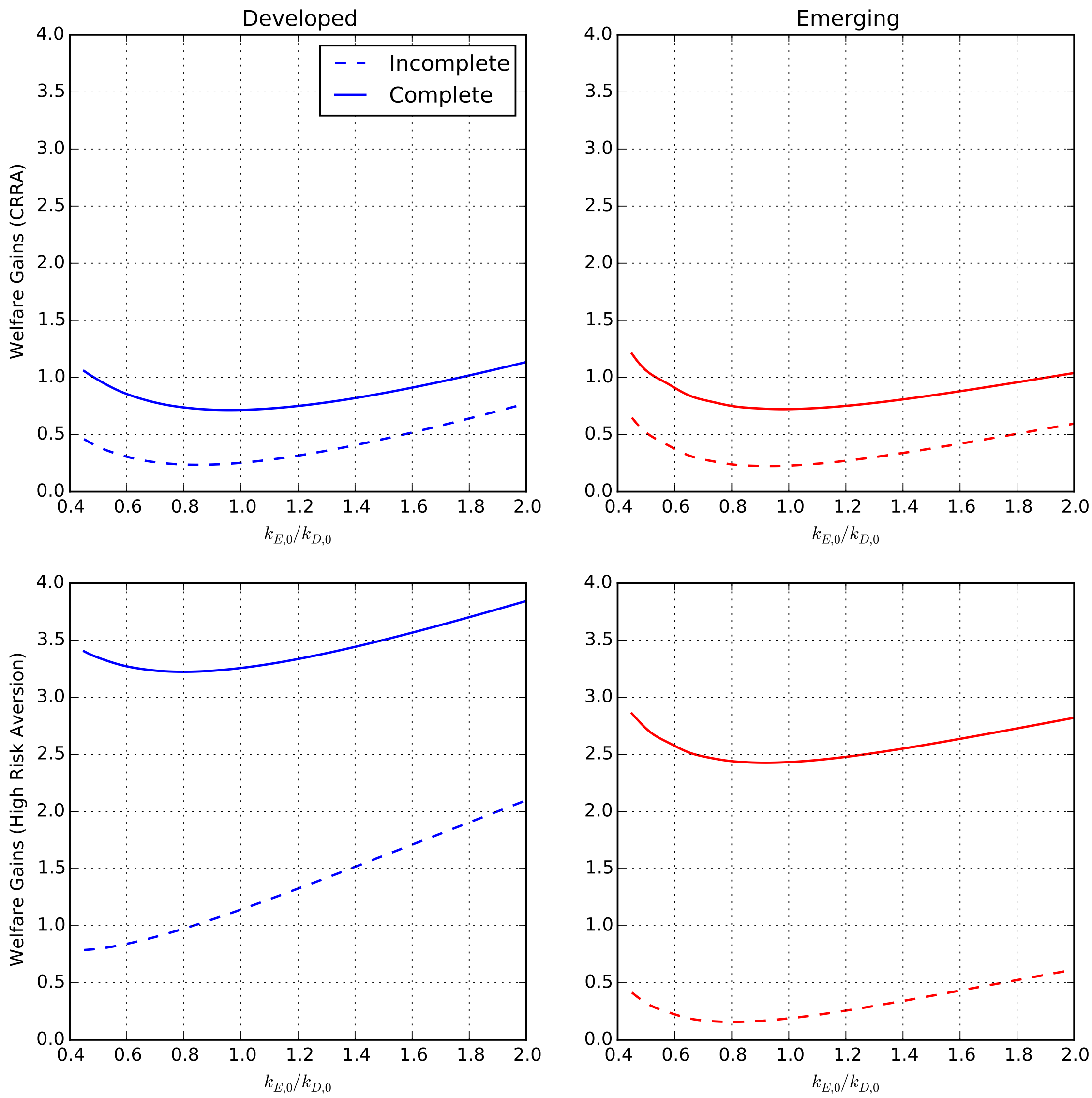

Figure 7: Welfare analysis of financial integration. Robustness with alternative financial markets structure.

Notes: Gains are expressed in % equivalent of permanent consumption as a function of initial relative capital stock ($\frac{k_{E,0}}{k_{D,0}}$). The solid line shows the welfare gains under complete financial markets. The dotted line corresponds to our baseline case with incomplete markets (bond-only). The upper panel corresponds to our baseline calibration with CRRA utility ($\gamma = \psi = 4$). The lower panel corresponds to Epstein-Zin utility with high risk aversion ($\psi = 4$ and $\gamma = 40$). Parameters are kept identical to the ones shown in Tables 1 and 2.

icantly higher. Depending on the level of initial capital stock and depending on the country, the gains are roughly three to five times larger than in the model with incomplete markets. In this case, completing the markets has a significant welfare impact since agents are extremely risk averse to consumption fluctuations.[36]

[36]This result might come as a surprise as bond-only integration is known to deliver similar outcomes to complete markets unless shocks are almost permanent (Baxter and Crucini (1995), Kollmann (1996)). In

When countries start off with similar initial capital stock, they amount to about 3.25% of permanent consumption in the safe country and 2.4% in the risky one—resp. 3.35% and 2.73% in our baseline with E significantly capital scarce. This is arguably a loose upper bound of the welfare gains that can be achieved—risk aversion being very high and financial markets complete. The magnitude of the gains has changed but regarding the shape of the curves and the distribution of the gains across countries, our results go through qualitatively. In particular, the gains are unevenly distributed across countries when risk aversion is high, but less so compared to the incomplete markets model. With incomplete markets, the safer asset issued by country D is more valuable since the country E is less able to smooth consumption.

5.4 The role of countries size

Welfare gains with small countries. So far our experiments rely on countries of equal sizes, focusing on the integration of a set of potentially large emerging countries. We consider it as a reasonable baseline to understand the recent liberalizing wave where large emerging markets, accounting for almost 50% of world GDP, did integrate financially at similar dates. However, one could argue that some emerging countries did integrate at earlier (resp. later) dates and were a much smaller share of world GDP—such as Southern Europe countries (resp. some Middle East countries).[37] From a theoretical perspective, investigating the importance of countries size in assessing the welfare benefits of integration provides an upper bound of the potential gains and allows comparisons with papers focusing on the case of small open economies (e.g. Gourinchas and Jeanne (2006), Obstfeld (1994b)).

Our baseline calibration might understate the welfare gains from financial integration of smaller open economy. Smaller countries have less impact on the world interest rate and are less negatively affected by adverse movements in the world interest rate, both at opening and

our framework, business cycle implications around the steady-states are also very similar (see Appendix A.1) but welfare benefits are fairly different due to the low frequency changes in the consumption profiles upon integration.

[37] In our sample, Spain, Portugal and Greece integrated financially in the mid-eighties slightly before the main wave of liberalization in Latin America and Asia in the early nineties. Countries such as Oman and Saudi Arabia integrated financially in the late nineties (1999).

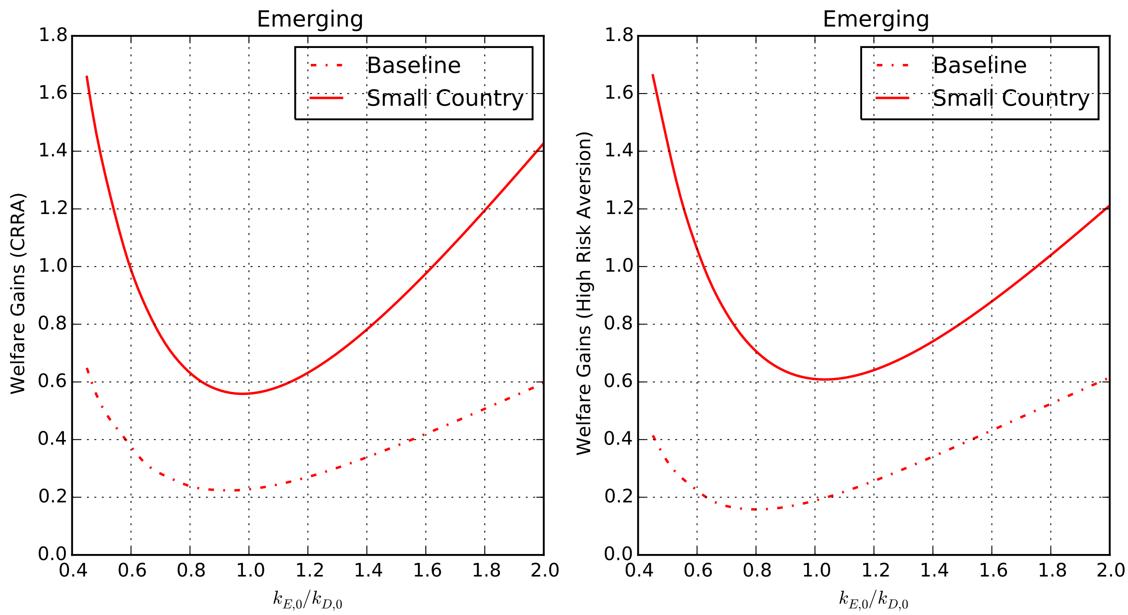

Figure 8: Welfare analysis of financial integration with a small country E.
Notes: Welfare gains are expressed in % equivalent of permanent consumption as a function of initial relative capital stock ($\frac{k_{E,0}}{k_{D,0}}$). The left (resp. right) panel corresponds to low risk aversion (resp. high risk aversion). Parameters of the model are shown in Tables 1 and 2 apart from relative productivity ($\frac{A_{E,0}}{A_{D,0}}$). Financial integration is a bond-only economy. The solid line shows the welfare gains with a country E ten times smaller than D: $A_{E,0}/A_{D,0} = 10\%$. The dotted line corresponds to our baseline case of symmetric initial productivity.

when hit by a productivity shock. To investigate the importance of size, we explore the case of different initial (steady-state) relative productivity across-countries $\left(\frac{A_{E,0}}{A_{D,0}}\right)$. We assume that country E is of a smaller size being on average ten times less productive: $\frac{A_{E,0}}{A_{D,0}} = \frac{1}{10}$. All other parameters values are kept identical to our baseline experiment (Tables 1 and 2). Welfare gains from integration (% of permanent consumption) are shown in Figure 8 for the risky country of small size for different values of the relative initial capital stocks ($\frac{k_{E,0}}{k_{D,0}}$). The results in the baseline case of symmetric initial size/productivity (dotted line) are shown for comparison purposes.[38]

Not surprisingly, market size matters for the distribution of the gains and country E benefits more from financial integration if smaller—the converse holds for the large country D. Interest rates move more favorably for country E following financial integration: E is now lending at higher rates, very close to the autarky interest rate of country D. Similarly, when country E is willing to lend more following a productivity shock, interest rates do not fall and

[38]The large country D is not shown as for such a large size difference, welfare gains are negligible in D.

the country can smooth consumption at a better price. The overall welfare gains (average across countries weighted by size) remain small. And even in this case where gains fall almost entirely on country E, they do not exceed a 1.5% increase of permanent consumption for realistic degrees of capital scarcity. They are of the same order of magnitude than in the riskless case despite additional gains from risk sharing: as in the baseline experiment with equal sizes, the reallocation of precautionary savings away from E dampens significantly the potential gains from capital scarcity.[39]

6 Conclusion

Intuitions about the gains from financial integration are implicitly based on neoclassical growth model. We fill an important gap in the theoretical literature that has either focused on deterministic efficiency gains in production economies or gains from international risk sharing, but neglecting the adjustment of production factors across countries. We provide an integrated framework where one can study the standard neoclassical efficiency gains together with gains from risk sharing and investigate how they interact. Using a general equilibrium model featuring aggregate risk, potentially asymmetric across countries, and endogenous capital accumulation, we show that the welfare gains from financial integration are small, at most a couple percentage points even in the most favourable cases where risk premia are high. This is so despite the possibility of having neoclassical efficiency gains arising from capital scarcity and gains from international risk sharing.

A key feature of our finding is that riskier countries while benefiting more from risk sharing will also reallocate precautionary savings towards the safer countries, boosting its capital accumulation. This has two important implications. First, it qualifies the conventional wisdom that riskier countries should have large gains from financial integration. In reality, safer (developed) country are benefiting more from their integration with riskier (emerging) countries. This is so because they sell insurance at a high price, even more so if risk aversion and risk

[39] With such a size for E, results in the deterministic case are quantitatively very close to the small open economy experiment performed in Gourinchas and Jeanne (2006).

premia are high. Second, it also qualifies the standard predictions linking financial integration and growth. In our framework, the predictions are much more complex and financial integration has heterogeneous effects on growth depending on the degree of capital scarcity, the level of risk and the size of countries. Financial integration potentially reduces growth in emerging markets compared to autarky if their level of aggregate risk is high compared to developed countries (or if market price of risk is high), thus despite being welfare enhancing. If emerging markets are sufficiently capital scarce at opening, financial integration accelerates growth in the short-run but slows it down at longer horizons. This potential heterogeneous output responses across countries and across time following financial integration can partially explain why the empirical literature has had difficulties to find robust results across countries and time-periods. Our results also open the door for a new empirical investigation regarding the growth effect of financial integration.

Finally, we focused on previous liberalization episodes where a group of large emerging countries integrated over a short time period. We emphasized how general equilibrium effects were detrimental in that case, reducing significantly the gains compared to the case where only one small country is integrating. This also potentially challenges the way the growth benefits of integration have been identified empirically as the literature implicitly assumes that the growth impact of integration is independent across countries. From a theoretical perspective, this has the flavour of a pecuniary externality. Individually, benefits of integration can outweigh significantly the costs but correlated behaviour where all emerging countries simultaneously integrate reduces significantly the gains due to adverse price movements. A full-fledged theory of endogenous financial integration with multiple countries is beyond the scope of the paper and left for future work.

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A Additional results

A.1 Business cycles and asset prices moments

Baseline calibrations. Table A.1 summarizes the basic business cycles and asset prices moments in our baseline calibrations (High and Low Risk Aversion).

Baseline Low Risk Aversion

	Financial Autarky		Financial Integration		Complete markets	
	D	E	D	E	D	E
Standard deviations of:						
Output	1.54%	3.02%	1.52%	3.15%	1.53%	3.09%
Consumption	0.93%	1.81%	0.87%	1.49%	1.74%	1.74%
Investment	3.97%	7.80%	3.56%	6.81%	3.55%	6.69%
Net exports over GDP	0	0	0.70%	0.71%	1.26%	1.28%
Asset prices[a]						
Riskless rate (Steady-state)	3.95 %	3.32 %	3.90 %	3.90 %	3.97 %	3.97 %
Risk premium	0.10%	0.39%	0.10%	0.18%	0.07%	0.15%
Volatility excess returns	1.56%	3.10%	1.41%	2.71%	1.41%	2.71%
Market price of risk	6.6%	12.5%	6.2%	10.0%	5.1%	6.6%

Baseline High Risk Aversion

	Financial Autarky		Financial Integration		Complete markets	
	D	E	D	E	D	E
Standard deviations of:						
Output	1.53%	3.03%	1.54%	3.07%	1.52%	3.05%
Consumption	0.83%	1.36%	0.93%	1.00%	1.56%	1.56%
Investment	4.13%	8.18%	4.08%	5.49%	3.66%	6.20%
Net exports over GDP	0	0	1.21%	1.22%	1.28%	1.27%
Asset prices						
Riskless rate	2.54 %	-0.77 %	1.94 %	1.94 %	2.27 %	2.27 %
Risk premium	0.63%	2.33%	0.69%	1.19%	0.48%	0.96%
Volatility excess returns	1.59%	3.21%	1.48%	2.37%	1.40%	2.54%
Market price of risk	38.1%	67.3%	43.1%	54.1%	35.3%	38.7%

Table A.1: Business cycles and asset prices moments.

Notes: Business cycle moments and asset prices moments are obtained by averaging the statistics over 1000 successive runs, each one lasting 150 periods. Business cycles series are detrended using Hodrick-Prescott filter with $\lambda = 6.25$. Parameters of the model are set to their baseline values in Tables 1 and 2.

Extension with world long-run risk. Table A.2 shows the risky-steady states, business cycles and asset prices moment in our baseline calibration with persistent shocks to world productivity growth.

Long-Run Risk Model

	Financial Autarky		Financial Integration		Complete markets	
	D	E	D	E	D	E
Risky steady state						
Capital	4.45	4.44	4.50	4.36	4.50	4.40
$\frac{\text{Netforeign assets}}{\text{Output}}$	0	0	-38.2%	-38.6%	/	/
Standard deviations of:						
Output	1.53%	3.09%	1.54%	3.11%	1.54%	3.14%
Consumption	1.18%	2.44%	1.14%	2.00%	2.53%	2.53%
Investment	2.93%	5.52%	2.87%	5.19%	2.84%	5.23%
Net exports over GDP	0	0	0.48%	0.60%	1.44%	1.66%
Asset prices						
Riskless rate	0.60 %	0.23 %	0.48 %	0.48 %	0.58 %	0.58 %
Risk premium	1.95%	2.33%	1.98%	2.22%	1.89%	2.05%
Volatility excess returns	1.19%	2.33%	1.16%	2.21%	1.14%	2.22%
Market price of risk	168.9%	102.1%	173.5%	103.2%	171.4%	93.9%

Table A.2: Risky steady-states, business cycles and asset prices moments with a world long-run risk.

Notes: Business cycle moments and asset prices moments are obtained by averaging the statistics over 1000 successive runs, each one lasting 150 periods. Business cycles series are detrended using Hodrick-Prescott filter with $\lambda = 6.25$. Parameters of the model except EIS $(1/\psi)$ and risk aversion (γ) are set to their baseline values (Table 1). With LRR, $1/\psi = 2$ and $\gamma = 10$. On the top of transitory shocks (Table 2), the model includes persistent shocks to world productivity growth with persistence $\rho_W = 0.999$ and volatility $\sigma_W = 0.002$.

Distribution of net foreign assets. Figure A.1 shows simulation paths for the net foreign asset position of the emerging country together with its ergodic distribution in our baseline calibrations.

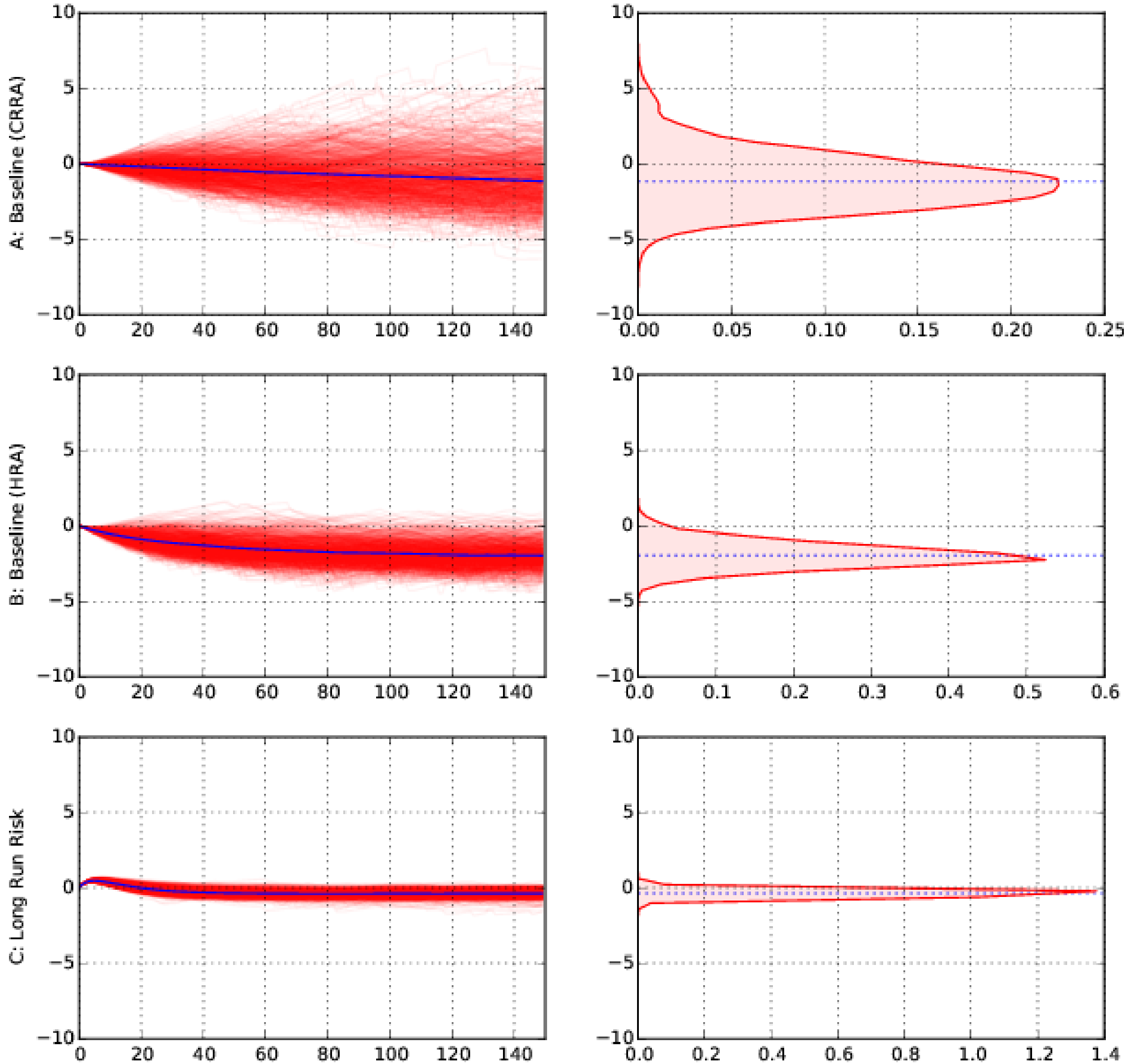

Figure A.1: Simulations and ergodic distribution of Net Foreign Debt over GDP (country E). Baseline Calibrations.

Notes: Panel A and B corresponds to our baseline calibrations without long-run risk (Low and High Risk Aversion). Parameters of the model are shown in Tables 1 and 2. Panel C corresponds to our calibration with a world long-run risk, with EIS $1/\psi = 2$ and risk aversion $\gamma = 10$. Under the long-run risk calibrations, parameters of the model except EIS $(1/\psi)$ and risk aversion (γ) are set to their baseline values in Tables 1. On the top of transitory shocks described in Table 2, the long-run risk version of the model includes persistent shocks to world productivity growth with persistence $\rho_W = 0.999$ and volatilities $\sigma_W = 0.002$.

A.2 Welfare gains from financial integration

Welfare gains. Table A.3 summarizes the welfare gains across the main calibrations used in the paper. Figure A.2 summarizes the gains under the same baseline calibrations as a function of the initial relative level of capital stock.

		Financial Integration		Complete markets	
		D	E	D	E
Baseline	$1/\psi = 1/4$				
Low Risk Aversion	$\gamma = 4$	0.37%	0.48%	0.95%	1.02%
High Risk Aversion	$\gamma = 40$	0.85%	0.20%	3.26%	2.54%
Deterministic	$\sigma_D = \sigma_E = 0$	0.29%	0.37%	0.29%	0.37%
Long-run risk (LRR)	$1/\psi = 2$ and $\gamma = 10$				
Baseline LRR volatility	$\sigma_W = 0.002$; $\rho_W = 0.999$	0.063%	0.051%	1.29%	1.48%
No Long Run Risk	$\sigma_W = 0$	0.108%	0.120%	1.30%	1.34%

Table A.3: Welfare gains of financial integration under various calibrations.

Notes: Gains expressed in equivalent increase of permanent consumption. Under the baseline calibrations, parameters of the model are shown in Tables 1 and 2. Under the long-run risk calibrations, parameters of the model except EIS $(1/\psi)$ and risk aversion (γ) are set to their baseline values in Tables 1. On the top of transitory shocks described in Table 2, the long-run risk version of the model includes persistent shocks to world productivity growth with persistence ρ_W and volatilities σ_W.

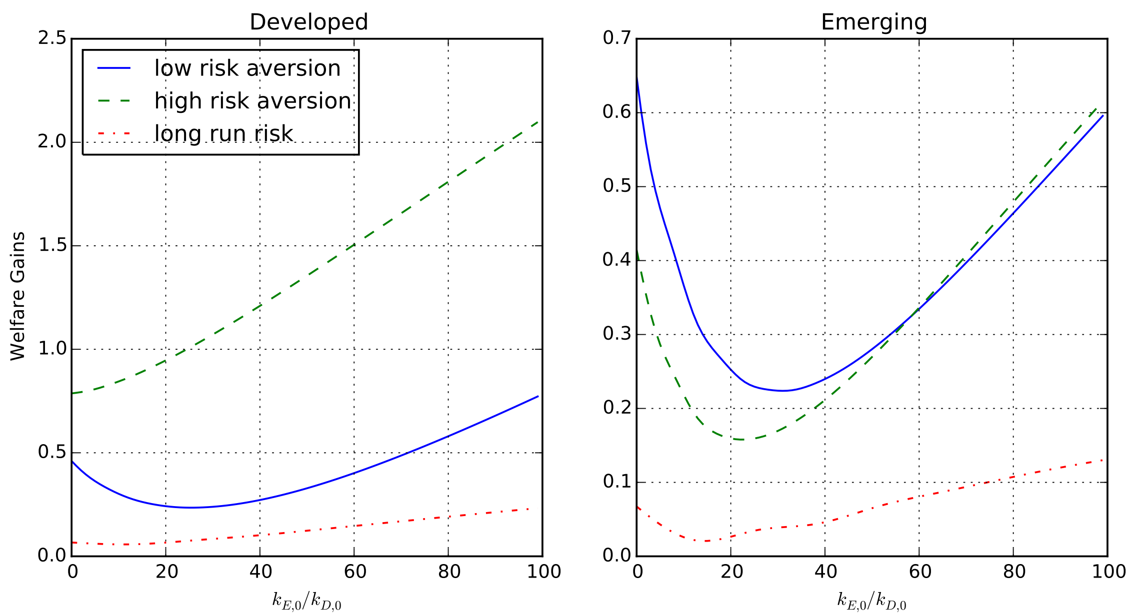

Figure A.2: Welfare analysis of financial integration across alternative calibrations for different degrees of capital scarcity.

Notes: Gains are expressed in equivalent of permanent consumption as a function of initial relative capital stock $(\frac{k_{E,0}}{k_{D,0}})$. Solid lines correspond to our baseline calibrations without long-run risk (High and Low Risk Aversion). Parameters of the model are shown in Tables 1 and 2. Dashed lines correspond to our calibration with a world long-run risk, with EIS $1/\psi = 2$ and risk aversion $\gamma = 10$. Under the long-run risk calibrations, parameters of the model except EIS $(1/\psi)$ and risk aversion (γ) are set to their baseline values in Tables 1. On the top of transitory shocks described in Table 2, the long-run risk version of the model includes persistent shocks to world productivity growth with persistence $\rho_W = 0.999$ and volatilities $\sigma_W = 0.002$.

B Data

B.1 Data sources and countries sample

Data sources.

Capital account liberalization dates: Bekaert et al. (2005).

GDP, Investment, GDP per capita: Penn World Tables. Sample period varies across countries depending on data availability (1950-2009 for developed countries, later starting date for most emerging markets but not later than 1975).

Sample of countries. 15 always financially opened developed countries. 40 liberalizing emerging markets (integration date $\geq$ 1985). Emerging markets do not include countries from Central and Eastern Europe due to lack of data before 1990.

Developed countries (already financially integrated in 1985).

Australia, Austria, Canada, Denmark, Finland, France, Germany, Ireland, Japan, Italy, Netherlands, Sweden, Switzerland, United Kingdom, United States.

Emerging countries (by geographical zone, integration date in parenthesis).

Southern Europe: Greece (1987), Israel (1993), Malta (1992), Portugal (1986), Spain (1985), Turkey (1989).

Latin America: Argentina (1989), Brazil (1991), Chile (1992), Colombia (1991), Ecuador(1994), Jamaica (1991), Mexico (1989), Peru (1992), Trinidad and Tobago (1997), Venezuela (1990).

Asia: Bangladesh (1991), China (1991),[40] India (1992), Indonesia (1989), Malaysia (1988), Pakistan (1991), Philippines (1991), South Korea (1992), Sri Lanka (1991), Thailand (1987).

Middle-East: Egypt (1992), Jordan (1995), Oman (1999), Saudi Arabia (1999).

Africa: Botswana (1990), Ghana (1993), Ivory Coast (1995), Kenya (1995), Mauritius (1994), Morocco (1988), Nigeria (1995), Tunisia (1995), South Africa (1996), Tunisia (1995), Zimbabwe (1993).

[40] According to the definition of Bekaert et al. (2005), China remains closed over the period considered. According to other indicators of financial integration, the country can be considered as opened starting 1991 (see Bekaert et al. (2005) for a discussion). We do include China in our sample.

Countries sizes. Table B.1 shows the PPP adjusted share of world GDP of each group of countries in 1990. World GDP is made of our set of 55 countries (15 developed financially opened and 40 liberalizing emerging markets). For comparison purposes, the US accounts in 1990 for 21.3% of the world GDP we consider.

Zone	Developed	Southern Europe	Latin America	Asia	Middle East	Africa	All Emerging
Share of World GDP	51.4%	5.6%	12.9%	26.7%	1.6%	2.1%	48.6%

Table B.1: Contribution to world GDP of group of countries in 1990.
Notes: Data from Penn World Tables. PPP adjusted GDP in 1990. World is made of our sample of 55 countries (15 developed countries and 40 emerging liberalizing countries). See Section B.1 for the sample of countries.

B.2 Output growth volatility and correlation

Volatility of output growth. We compute the volatility of annual real GDP per capita for each country in the sample over the period 1975-1995 (PPP adjusted). This corresponds largely to the time period before and around the integration date of the emerging markets considered. Volatility computed over a longer time frame gives very similar results. Figure B.1 reports the volatility for each group of countries (arithmetic or GDP-weighted average across countries belonging to the group). The (arithmetic) averaged volatility of output growth across liberalizing emerging countries is 4.9% compared to 2.5% in developed countries, in line with our baseline calibration.[41]

Correlation of output growth with developed countries. For any given country, we also compute the correlation of annual real GDP growth per capita in the country with the group of (already integrated) developed countries over the period 1975-2010.[42] We compute

[41] We display simple arithmetic averages and GDP-weighted (using 1990 PPP GDPs) averages. Both are very similar quantitatively although the GDP-weighted averages tend to be smaller (except for Asia) since larger countries tend to be less volatile. Importantly, the ratio of volatilities between developed and emerging markets is very similar across the two measures.

[42] We used a longer time frame to compute correlations for a better accuracy of our estimates but results are very similar when considering the period 1975-1995. The real GDP growth rate of developed country is weighted sum of the GDP growth rates of each country, where weights correspond to the size of countries.

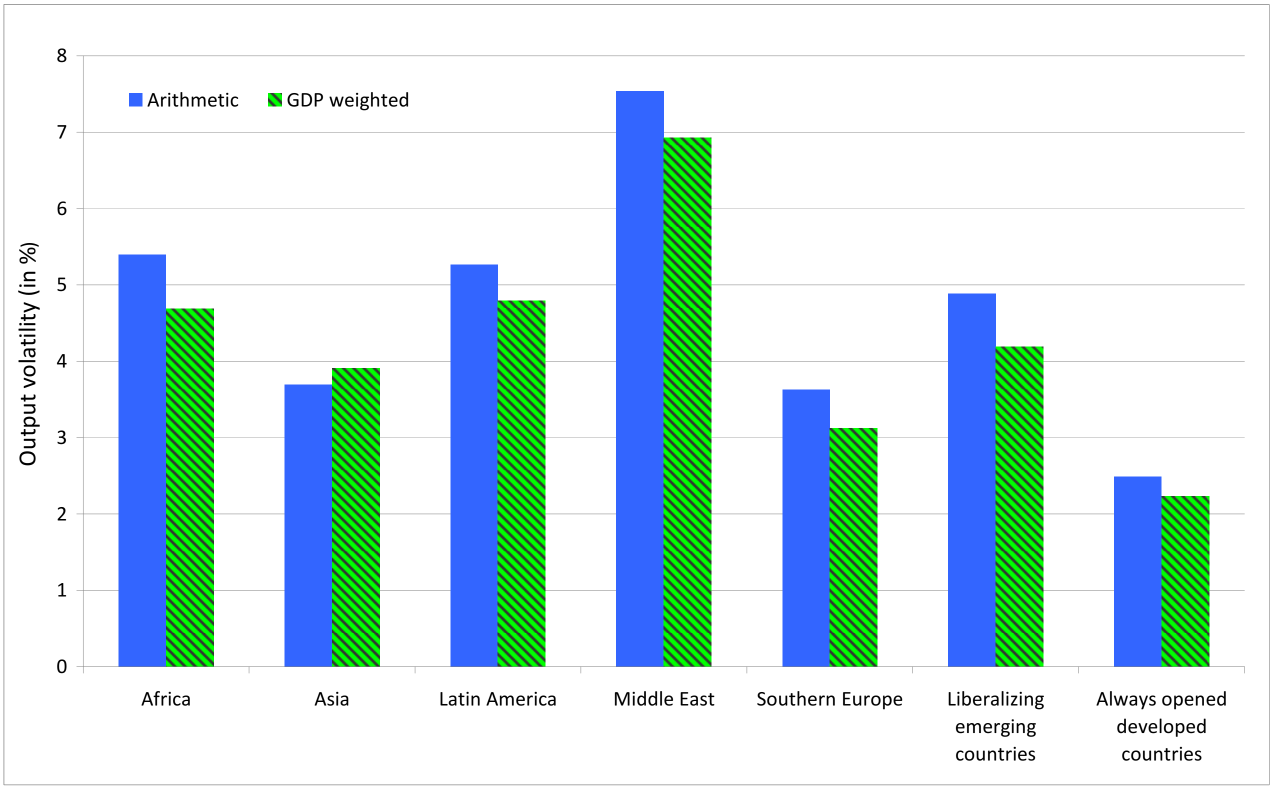

Figure B.1: Volatility of annual real output growth per capita across countries (1975-1995). Notes: Penn World Tables. Volatility of annual real output growth per capita for each country is computed over the period 1975-1995. Volatility of each group of countries is a sample average (arithmetic or GDP-weighted) of the volatility of each country in the group as defined in Section B.1. GDP weights are based on PPP GDP in 1990.

Zone	Southern Europe	Latin America	Asia	Middle East	Africa	All Emerging
Correlation with developed (Arithmetic mean)	0.53	0.14	0.10	0.00	0.27	0.21
Correlation with developed (GDP-weighted mean)	0.60	0.22	−0.01	0.06	0.35	0.14

Table B.2: Correlation of annual real output growth with the sample of (already integrated) developed countries (1975-2010).
Notes: Penn World Tables. The correlation of annual real output growth per capita for each country is computed over the period 1975-2010. Real per capita GDP growth of the sample of (already integrated) developed countries is a GDP-weighted average of the growth of countries in the sample. The correlation for each group of countries is a sample average (arithmetic or GDP-weighted) of the correlation of each country in the group as defined in Section B.1. GDP weights are based on PPP GDP in 1990.

the arithmetic and the GDP-weighted means in a given group of countries (region or whole sample of liberalizing countries). Results are shown in Table B.2. Our baseline assume zero correlation while the correlation is between 0 and 0.25 for all groups but Southern Europe, which is significantly higher. Thus, If anything, we overestimate slightly the gains from financial integration in our baseline.

B.3 Capital scarcity

Definitions. Consider a country i with the following production function at date t:

$$Y_{i,t} = A_{i,t} \left(K_{i,t}\right)^{\theta} \left(L_{i,t}\right)^{1-\theta}$$

where $K_{i,t}$ denotes the capital stock, $A_{i,t}$ the country TFP and $L_{i,t}$ the labour supply.

Capital-output ratio $(\frac{K}{Y})_{i,t}$ is then a monotonic transformation of capital per efficiency units $k_{i,t} = \frac{K_{i,t}}{A_{i,t}^{1/(1-\theta)} L_{i,t}}$:

$$(\frac{K}{Y})_{i,t} = (\frac{K_{i,t}}{A_{i,t}^{1/(1-\theta)} L_{i,t}})^{1-\theta} = k_{i,t}^{1-\theta}$$

Thus capital per efficiency units $k_{i,t}$ can easily be recovered from capital-output ratio as follows:

$$k_{i,t} = \left[(\frac{K}{Y})_{i,t} \right]^{1/(1-\theta)} \tag{19}$$

$k_{i,t}$ is the empirical counterpart of the capital stock in the model of Section 2.

Capital stocks. We compute the stock of capital $K_{i,t}$ of country i at date t using the perpetual inventory method with a depreciation rate of $\delta = 8\%$ per year (see Hall and Jones (1999)). The initial value capital stock at date t_0 is defined as:

$$\frac{\text{Investment rate at } t_0}{\delta + g_{t_0}},$$

where g_{t_0} is the average geometric growth rate of investment over the ten years preceding t_0.

The initial period t_0 considered depends on data availability for a given country. For developed countries, we use 1960, for emerging markets, we use generally 1970 and at the latest 1980. Results are quite insensitive to the use of a common initial date if anterior to 1980.

We compute the capital-output ratio $(\frac{K}{Y})_{i,t}$ at date t in country i defined as $K_{i,t}$ divided by GDP of that year (all expressed in constant 2005 USD). $k_{i,t}$ is then defined according to Equation (19) with $\theta = 0.3$. The capital-output ratio of the sample of developed countries (already integrated in 1985) is the GDP-weighted average of capital-output ratios in these countries. Their capital per efficiency unit k_t^* is defined according to Equation (19) with $\theta = 0.3$.

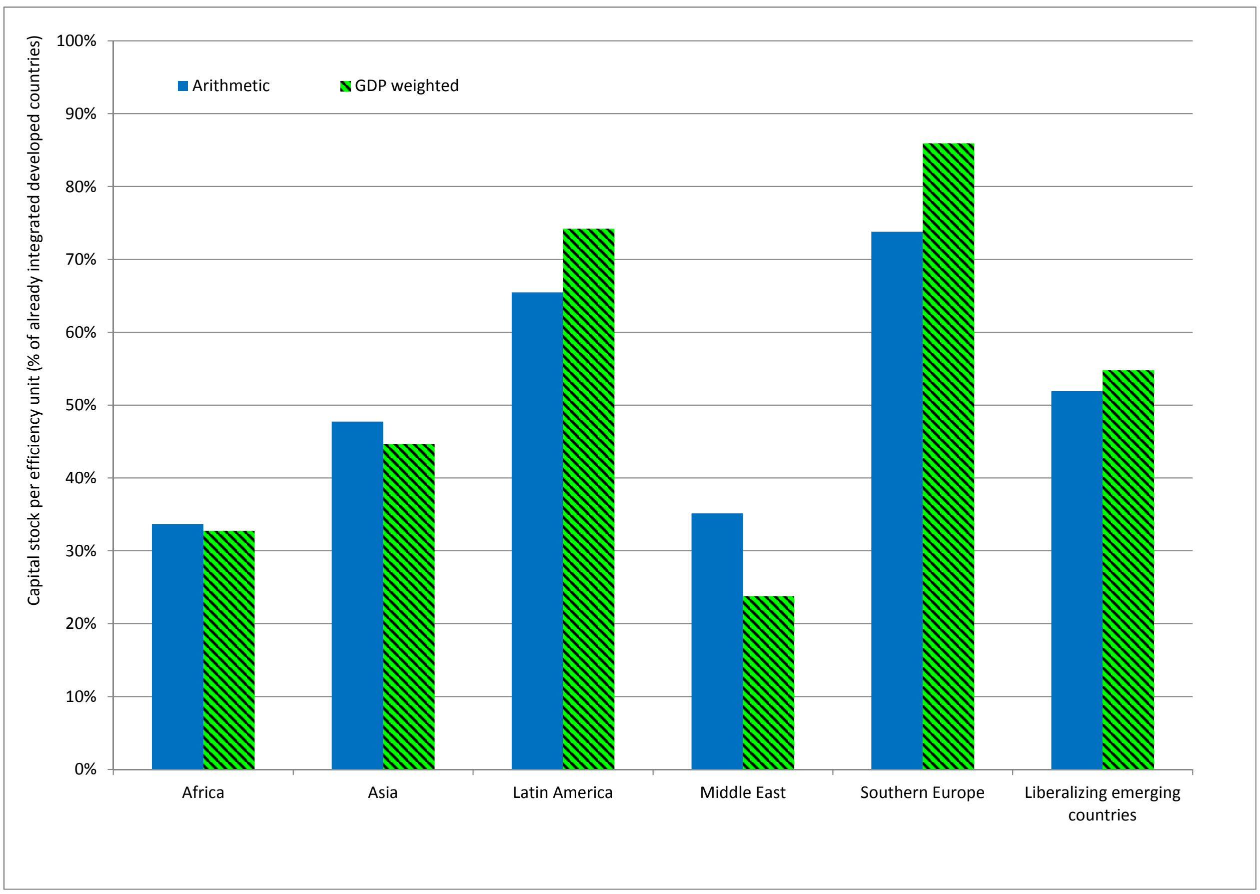

Figure B.2: Degree of 'capital scarcity' at time of opening across emerging liberalizing countries.

Notes: Penn World Tables. Capital scarcity of a given region at time of opening is the average (arithmetic or GDP-weighted) across countries i in the region of $k_{i,t_i}/k^*_{t_i}$. k_{i,t_i} (resp. $k^*_{t_i}$) denotes the capital per efficiency units in country i (resp. the set of developed countries) at time of opening. The sample of countries is described in Section B.1. GDP weights for the average scarcity across countries in a group are based on 1990 GDPs.

Capital scarcity at date of financial opening. Consider an emerging country i integrating financially at date t_i with the sample of developed country $(*)$. We measure 'capital scarcity' *at opening* by the following ratio:

$$\text{`capital scarcity'}(i, t_i) = \frac{k_{i,t_i}}{k^*_{t_i}}$$

A ratio smaller than 1 indicated that at time of opening, country i has a lower capital stock per-efficiency unit than developed countries. Note that the use the word scarcity is a bit of a language abuse since in a stochastic environment as ours, country i can have a higher capital stock than developed countries and still be below its own autarky steady-state.

We measure the average capital scarcity at time of opening of a considered group of countries by computing the arithmetic average of $k_{i,t_i}/k^*_{t_0}$ across countries i belonging to the group (region or set of emerging liberalizing countries).[43] Figure B.2 reports the degree of capital scarcity at time of opening for each group of countries. At time of opening, liberalizing emerging countries have on average a capital stock very close to 50% of the one of developed countries, in line with our baseline calibration. There is some heterogeneity though with Southern Europe being much more capital abundant at opening than Asia or Middle-Eastern countries.

[43] GDP-weighted (using GDPs in 1990) averages gives very similar results quantitatively.

C Numerical methods

Model description. The model's equations are reformulated as follows (see Section 2.2):

$$E_t \left[f \left(\mathbf{m_t}, \mathbf{s_t}, \mathbf{x_t}, \mathbf{m_{t+1}}, \mathbf{s_{t+1}}, \mathbf{x_{t+1}} \right) \right] \quad \perp \quad \underline{\mathbf{x}} \leq \mathbf{x_t} \leq \overline{\mathbf{x}} \tag{C.1}$$

$$\mathbf{s_{t+1}} \quad = \quad g(\mathbf{m_t}, \mathbf{s_t}, \mathbf{x_t}, \mathbf{m_{t+1}}) \tag{C.2}$$

where $\mathbf{m_t}$ is a vector of exogenous Markov processes, $\mathbf{s_t}$ the vector of endogenous states and $\mathbf{x_t}$ the vector of controls to be determined, constrained to lie within $[\underline{\mathbf{x}}, \overline{\mathbf{x}}]$. The solution satisfies at all dates $\mathbf{x_t} = \varphi(\mathbf{m_t}, \mathbf{s_t})$ where φ is the unknown decision rule to solve for. The algorithm described in the next paragraphs, and our implementation in Python, is independent from the precise formulation of the model. We describe the model in a text file, using the conventions set by the Dolo software, freely available online.[44] Section 2.2 shows how to cast the baseline model into functions f and g. The reference set of equations for the other model variables (autarky, complete markets, endowments, long-run risk) are included in the companion code and its online documentation.[45]

Removing occasionally binding constraints. Mixed complementarity problem (Eq. C.1) could be solved using a specialized nonlinear complementarity solver.[46] We choose instead to follow the simple approach of reformulating the slackness conditions as smooth functions and solve the resulting system using a regular nonlinear solver. Recall that $v \perp a \leq x \leq b$ is by definition equivalent to $|\min(x-a, v)| + |\min(b-x, -v)| = 0$. Using the Fischer-Burmeister function $\varphi^B(a,b) = a + b - \sqrt{a^2+b^2}$, one can check that the complementarity condition is equivalent to $\varphi^B(b-x, -\varphi^B(x-a, v)) = 0$ which is a smooth function of f. In the following sections, we assume, without loss of generality, that f and g are differentiable functions which incorporate occasionally binding constraints.

Discretizing the exogenous process. We discretize the joint AR(1) process of the pro-

[44] Dolo is released under a BSD license at `https://github.com/EconForge/dolo`. The solution method, initially developed for this paper, is now merged in Dolo library with other solution algorithms and can be applied to any model satisfying the same specification.

[45] Companion code and documentation for this paper available at `https://bitbucket.org/albop/finint/`

[46] For a commercial complementarity solver see PATH: `http://pages.cs.wisc.edu/~ferris/path.html`

ductivity shocks as a finite Markov chain. For this purpose, we perform a Cholesky decomposition of the random innovations $\epsilon_{D,t}, \epsilon_{E,t}$. This gives us a lower tridiagonal matrix Ω and two independent i.i.d. Gaussian noises $(\epsilon'_{D,t}, \epsilon'_{E,t})$ whose joint process is defined by a diagonal covariance matrix Σ_d such that $\Sigma_d = \Omega\Omega'$. Let us define:

$$\begin{pmatrix} \log(A'_{D,t}) \\ \log(A'_{E,t}) \end{pmatrix} = \Omega \begin{pmatrix} \log(A_{D,t}) \\ \log(A_{E,t}) \end{pmatrix}$$

Since the autocorrelation coefficient for $\log(A_{D,t})$ and $\log(A_{E,t})$ is ρ, the processes $\log(A'_{D,t})$ and $\log(A'_{E,t})$ are two independent unidimensional AR(1) processes with autocorrelation ρ and conditional variance given by the diagonal elements of Σ_d. We discretize each of them as a three states Markov chain, using the method from Rouwenhorst (1995). We choose the free coefficients so that the resulting Markov chain has the exact same autocorrelation and asymptotic variance as the original continuous process. The discretized process is a series of $N_m = 9$ vectors of two elements: $(\mathbf{m_i})_{i\in[1,N_m]}$ and a matrix of weights $(p_{ij})_{i,j\in[1,N_m]^2}$ such that p_{ij} is the conditional probability of reaching state j from state i.

Discretizing the endogenous state-space. First, we need to choosing boundaries for the domain containing the continuous states k_D, k_E and d. We study the capital over a wide enough interval, so that we can simulate economies starting with a significant capital scarcity while capturing the potentially larger capital stocks under autarky or integration. We set the same bounds for both countries $[k_{\min}, k_{\max}] = [1, 10]$.[47] Consistent with the borrowing constraints we restrict $-\bar{b} \leq d \leq \bar{b}$ where $\bar{b}$ denotes the exogenous debt limit. We take $\bar{b} = 10$. As we do not want our solution to be dependent on an arbitrary $\bar{b}$, we perform robustness checks with higher/lower debt limits. Using 30 points along each dimension, we discretize the state-space as a list of points $\mathbf{S} = (\mathbf{s_n})_{n\in[1,N_s]}$ where each of the $N_s = 30 \times 30$ elements is a different set of coordinates in the state space.

Decision rules. The numerical solution of the problem for each realization $\mathbf{m_i}$ of the Markov

[47] For comparison, in our baseline simulation, in country D the steady-state stocks of capital are respectively 2.32, 2.92, and 3.68 respectively, when the productivity shocks stays constant at its lower, medium and high level. Country E starts with 50% of the steady-state autarky capital stock in D. As a result, in our simulations, capital always stays within the boundaries.

chain is a matrix $\mathbf{X_i} = (\mathbf{x_{in}})_{n\in[1,N_s]}$ whose elements are vectors with $n_x = 8$ coordinates. We also set $\mathbf{X} = (\mathbf{X_i})_{i\in[1,N_m]}$. For any exogenous value $\mathbf{m_j}$ and any state $\mathbf{s}$, possibly outside of the grid, we approximate the solution $\varphi(\mathbf{m_j}, \mathbf{s})$ with an interpolation scheme $\mathcal{I}$ such that $\varphi(\mathbf{m_j}, \mathbf{s}) \approx \mathcal{I}(\mathbf{s}, \mathbf{X_j})$. Given the nature of our welfare comparison exercise, we trade speed for precision and use natural cubic splines to interpolate the decision rule.[48] At each iteration step, we store the filtered coefficients to avoid recomputing them for multiple evaluations of $\mathcal{I}(s, \mathbf{X_j})$ with the same $\mathbf{X_j}$.

Time iterations. To check optimality conditions, we compute:

$$\sum_{j\in[1,N_m]} p_{ij} f(\mathbf{m_i}, \mathbf{s_n}, \mathbf{x_{in}}, \mathbf{m_j}, \mathbf{s_{inj}}, \mathbf{x_{inj}})$$

$$\mathbf{s_{inj}} = g(\mathbf{m_i}, \mathbf{s_n}, \mathbf{x_{in}}, \mathbf{m_j})$$

$$\mathbf{x_{inj}} = \mathcal{I}(\mathbf{s_{inj}}, \tilde{\mathbf{X}}_\mathbf{j})$$

where $(\mathbf{m_i}, \mathbf{s_n})$ is a discretized state today and $\mathbf{x_{in}}$ the control taken in that state. The state attained with the exogenous realization $\mathbf{m_j}$ is denoted by $\mathbf{s_{inj}}$. The decision taken in tomorrow's state $(\mathbf{m_{jn}}, \mathbf{s_{inj}})$ is $\mathbf{x_{inj}}$ according to the rule $\tilde{\mathbf{X}}_\mathbf{j}$.

These optimality conditions can be vectorized with respect n. For any $i \in [1, N_m]$ we define the residual function for exogenous realization $\mathbf{m_i}$ today at all grid points $\mathbf{S}$

$$\Phi_i(\mathbf{X_i}, \tilde{\mathbf{X}}) = \sum_{j\in[1,N_m]} p_{ij} f(\mathbf{m_i}, \mathbf{S}, \mathbf{X_i}, \mathbf{m_j}, \mathbf{S_{ij}}, \mathbf{X_{ij}}) \tag{C.3}$$

$$\mathbf{S_{ij}} = g(\mathbf{m_i}, \mathbf{S}, \mathbf{X_i}, \mathbf{m_j}) \tag{C.4}$$

$$\mathbf{X_{ij}} = \mathcal{I}(\mathbf{S_{ij}}, \tilde{\mathbf{X}}_\mathbf{j}) \tag{C.5}$$

where $\mathbf{S_{ij}}$ is the list of points reached with the exogenous realization $\mathbf{m_j}$ and $\mathbf{X_{ij}}$ the corresponding controls.

[48]Interpolation code available as a separate library at `https://github.com/EconForge/interpolation.py`.

Given a termination criterium $\epsilon_\eta > 0$, the time-iteration algorithm works as follows:

- Choose an initial guess for the controls $\mathbf{X^0}$
- At step k given an initial guess $\mathbf{X^k}$
  - assume future controls are given by the preceding step $\tilde{\mathbf{X}} = \mathbf{X^k}$
  - For each i in $[1, N_m]$
    - find the zero $\mathbf{X_i^{k+1}}$ of $\mathbf{X_i} \to \Phi_i(\mathbf{X_i}, \tilde{\mathbf{X}})$
  - Define new set of controls $\mathbf{X^{k+1}} = (\mathbf{X_i^{k+1}})_{i\in[1,N_m]}$
  - Compute successive approximation errors $\eta_{k+1} = \left|\left|\mathbf{X^{k+1}} - \mathbf{X^k}\right|\right|_\infty$ and ratio $\lambda_{k+1} = \frac{\eta_{k+1}}{\eta_k}$
  - If $\eta_{k+1} < \epsilon_\eta$, solution has converged. Otherwise, start again with $k \leftarrow k + 1$.

We choose $\epsilon_\eta = 10^{-7}$. As the simulation go, we make sure there is a $\lambda < 1$ and a rank K such that $\forall k > K, \lambda_k \leq \lambda < 1$. In the baseline we find that λ_k converges towards 0.959, which is a necessary condition for the model to be well defined (see Winant (2017) for details). After the solution has converged to $\overline{\mathbf{X}}$, we also check that the final residuals (computed with $\overline{\mathbf{X}} = \mathbf{X^k} = \mathbf{X^{k+1}}$) are smaller than $\epsilon = 10^{-6}$.

Up-to-date computer code, with its complete documentation, is available on the websites of the authors.