Global Real Rates: A Secular Approach*

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Abstract

The current environment is characterized by low real rates and by policy rates close to or at their effective lower bound in all major financial areas. We analyze these unusual economic conditions from a secular perspective using data on aggregate consumption, wealth and asset returns. Our present-value approach decomposes fluctuations in the global consumption-to-wealth ratio over long periods of time and show that this ratio anticipates future movements of the global real rate of interest. Our analysis identifies two historical episodes where the consumption-to-wealth ratio declined rapidly below its historical average: in the late 1920s and again in the mid 2000s. Each episode was followed by a severe global financial crisis and depressed real rates for an extended period of time. Our empirical estimates suggest that the world real rate of interest is likely to remain low or negative for an extended period of time.

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1 Introduction

The current macroeconomic environment remains a serious source of worry for policymakers and of puzzlement for academic economists. Global real rates have been trending down since the 1980s are at historical lows across advanced economies, both at the short and long end of the term structure. Policy rates are close to or at their effective lower bound in all major financial areas. Figures 1 and 2 report the nominal policy rates and long yields for the U.S., the Eurozone, the U.K. and Japan since 1980. Large amounts of wealth are invested at zero or negative yields.[1]

Despite the aggressive global monetary policy treatment administered, levels of economic activity have remained weak across the advanced world strongly suggesting that the *natural interest rate*, i.e. the real interest rate at which the global economy would be able to reach its potential output, remains below *observed* global real interest rates. Far from being overly accommodating, current levels of monetary stimulus may well be insufficiently aggressive because of the Zero Lower Bound constraint on policy rates.[2]

Understanding whether natural rates are indeed low, for how much longer, and the source of their decline has become a first-order macroeconomic question. More generally, understanding what drives movements in real rates in the long run is one of the most intriguing questions in macroeconomics. In a celebrated speech given at the IMF in 2013, five years after the onset of the Global Financial Crisis, Summers (2015) ventured that we may have entered an age of '*secular stagnation*', i.e. and era where output remains chronically below its potential, or equivalently real rates remain above their natural rate. Not coincidentally, the secular stagnation hypothesis was first voiced by Hansen (1939), ten years after the onset of the Great Depression. The fundamental reasons behind this '*secular stagnation*' largely remain to be elucidated but several hypotheses have been put forward: a global savings glut (Bernanke (2005)), i.e. a rise in desired savings due to the fast growth of emerging market economies with relatively underdeveloped financial sectors; a decline in investment rates due to a lack of investment opportunities, potentially because of a technological slowdown (Gordon (2012)); a decline in the relative price of investment goods such as machine and robots, which depresses the level of investment; a decline in the rate of population

[1] According to FitchRatings (2017), the total amount of fixed-rate sovereign debt trading at negative yields was \$9.1 trillion as of March 2017, after having reached \$11.7 trillion in June 2016.

[2] Most central banks also deployed nonconventional monetary policy mostly in the form of asset purchases, or forward guidance. While the evidence suggests these policies have contributed to stabilize the economy, they may not have been sufficient to raise the natural rate above actual rates.

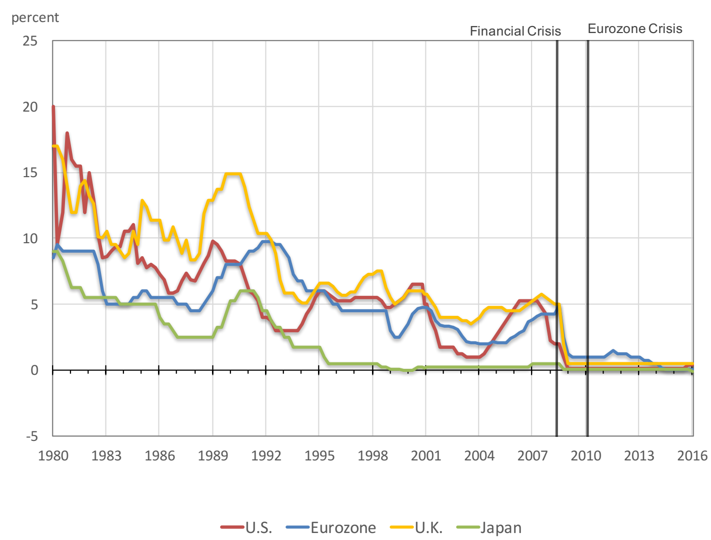

Figure 1: **Policy Rates, 1980-2016**. Sources: U.S.: Federal Funds Official Target Rate; Eurozone: until Dec. 1998, Germany's Lombard Rate. After 1998, ECB Marginal Rate of Refinancing Operations; U.K.: Bank of England Base Lending Rate; Japan: Bank of Japan Target Call Rate. Data from Global Financial Database.

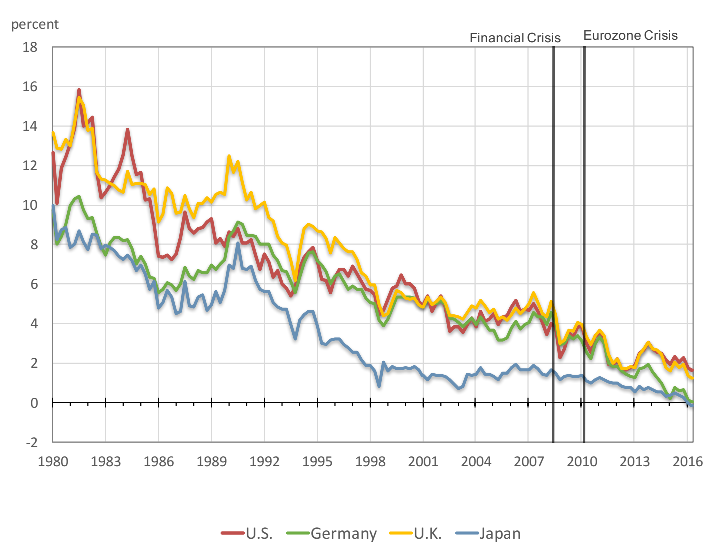

Figure 2: **Long yields. 1980-2016**. Sources: U.S.: 10-year bond constant maturity rate; Germany: 10-year benchmark bond; U.K.: 10-year government bond yield; Japan: 10-year government bond yield. Data from Global Financial Database

growth; an increase in the demand for safe assets (Caballero et al. (2015)).

This paper is an empirical contribution to this debate. Using an almost a-theoretical empirical approach inspired by the finance literature but applied to long run historical data, we show that low-frequency movements in the consumption-to-wealth ratio are key to understand movements in the global real interest rate. We find that, as in other historical periods, most notably in the 1930s, low real rates are the likely outcome of an extended and on-going process of deleveraging that creates a '*scarcity of safe assets*.' As a by product of our analysis, we are also able to make some low frequency forecasts of the real rate.

1.1 Review of the Literature

This is a placeholder for a literature review. It will include:

- papers on estimation of the natural rate: Laubach and Williams (2016, 2003); Hamilton et al. (2015); Pescatori and Turunen (2015); Del Negro et al. (2017); Farooqui (2016); Barro and Sala-i Martin (1990)
- papers on secular stagnation: Eggertsson et al. (2015); Eggertsson and Mehrotra (2014); Caballero et al. (2015); Summers (2015); Hansen (1939); Sajedi and Thwaites (2016)
- papers on the present value approach: Campbell and Shiller (1991); Lettau and Ludvigson (2001); Gourinchas and Rey (2007)

2 Dynamics of Global Real Interest rates

This section, presents a present value model that connects low frequency movements in aggregate consumption, wealth and asset returns.

2.1 The Global Budget Constraint: Some Elements of Theory

We are interested in understanding the determinants of the global safe real interest rate. With integrated capital markets, the relevant unit of analysis is the global (i.e. world) resource constraint. Denote the beginning-of-period global *total* private wealth $\bar{W}_t$, composed of global private wealth

W_t and global human wealth H_t.[3] Total private wealth evolves over time according to:

$$\bar{W}_{t+1} = \bar{R}_{t+1}(\bar{W}_t - C_t). \tag{1}$$

In equation (1), C_t denotes global private consumption expenditures and $\bar{R}_{t+1}$ the gross return on total private wealth between periods t and $t+1$.[4] All variables are expressed in real terms. Equation (1) is simply an accounting identity that holds period-by-period. We add some structure to this identity by observing that, in most theoretical models, households aim to smooth consumption and stabilize the consumption-to-wealth ratio.[5] If the average propensity to consume out of wealth is stationary, equation (1) can be log-linearized around its steady state value $C/\bar{W} \equiv 1-\rho_w$, where $\rho_w < 1$.[6] Denote Δ the difference operator so that $\Delta x_{t+1} \equiv x_{t+1} - x_t$, and $\bar{r}_{t+1} \equiv \ln \bar{R}_{t+1}$, the continuously compounded real return on wealth. Following the same steps as Campbell and Mankiw (1989) or Lettau and Ludvigson (2001), we obtain the following log-linearized expression (ignoring an unimportant constant term) :[7]

$$\ln C_t - \ln \bar{W}_t \simeq \rho_w \left(\ln C_{t+1} - \ln \bar{W}_{t+1} + \bar{r}_{t+1} - \Delta \ln C_{t+1} \right). \tag{2}$$

Equation (2) indicates that if today's consumption-to-wealth ratio is high, then either (a) tomorrow's consumption-to-wealth ratio will be high, or (b) the return on wealth between today and tomorrow $\bar{r}_{t+1}$ will be high, or (c) aggregate consumption growth $\Delta \ln C_{t+1}$ will be low. Since $\rho_w < 1$, Equation (2) can be iterated forward to obtain, under the usual transversality condition ($\lim_{j\to\infty} \mathbb{E}_t \rho_w^j cw_{t+j} = 0$), the following ex-ante present value relation:

$$\ln C_t - \ln \bar{W}_t \simeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(\bar{r}_{t+s} - \Delta \ln C_{t+s} \right). \tag{3}$$

[3]Private wealth includes financial assets, including private holdings of government assets, as well as non-financial assets, but excludes human wealth. See the appendix for a detailed data description.

[4]We focus on total private wealth and consumption, and not total national wealth or consumption, which includes government's net wealth and consumption. This is the appropriate focus under the -reasonable- assumption that Ricardian equivalance fails.

[5]For instance, if consumption decisions are taken by an infinitely lived representative household maximizing welfare defined as the expected present value of a logarithmic period utility $u(C) = \ln C$, then the consumption-to-wealth ratio is constant and equal to the discount rate of the representative agent.

[6]In steady state, $C/\bar{W}$ satisfies the following relation: $\Gamma/\bar{R} = 1 - C/\bar{W} \equiv \rho_w$, where Γ denotes the steady state growth rate of total private wealth and $\bar{R}$ the steady state gross return on wealth.

[7]See the appendix for a full derivation.

To understand equation (3), suppose that the consumption-to-wealth ratio (the left hand side of the equation) is currently higher than its unconditional mean. Since $C/\bar{W}$ is stationary, this ratio must be expected to decline in the future. Equation (3) states that this decline can occur in one of two ways. First, expected future return on total private wealth $\bar{r}_{t+s}$ could be high. This would increase future wealth, i.e. the denominator of the $C/\bar{W}$ ratio. Alternatively expected future aggregate consumption growth could be low, which would decrease the numerator of $C/\bar{W}$.

At this stage, it is important to emphasize that the assumptions needed to derive equation (3) are very minimal: we start from the law of motion of total private wealth, equation (1), which is an accounting identity. We then perform a log-linearization under mild stationarity conditions, and impose a transversality condition that rules out paths where wealth grows without bounds in relation to consumption. Equation (3)'s main economic message is that today's average propensity to consume out of wealth encodes relevant information about future consumption growth and/or future returns to wealth.

Before we can exploit this expression empirically, we need to make two important adjustments. First, as mentioned above, *total* private wealth is the sum of private wealth W_t and human wealth H_t. Because human wealth is not easily observable, Lettau and Ludvigson (2001) approximate the non-stationary component of human wealth with aggregate labor income and estimate a co-integration relation between consumption, financial wealth and labor income to construct a proxy for the left hand side of equation (3).[8] We follow a different route. Specifically, denote ω the aggregate share of private wealth in total wealth: $\omega = W/\bar{W}$. If ω is stationary, we can approximate (log) total wealth as $\ln \bar{W}_t = \omega \ln W_t + (1-\omega)\ln H_t$, and the log return on total wealth as $\bar{r}_t = \omega r_t^w + (1-\omega) r_t^h$ where r_t^w (resp. r_t^h) denotes the log return on private wealth (resp. human wealth).[9] Substituting these expressions into equation (3) and re-arranging we obtain:

$$\omega \left(\ln C_t - \ln W_t - \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^w - \Delta \ln C_{t+s} \right) \right) + (1-\omega) \left(\ln C_t - \ln H_t - \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^h - \Delta \ln C_{t+s} \right) \right) \simeq 0. \tag{4}$$

This equation makes clear that if the present value relation holds for private wealth, then it

[8] Lettau and Ludvigson (2001) also use a co-integration approach since the ratio C/W does not appear stationary in their shorter sample.

[9] The gross return on human wealth may be defined as $R_{t+1}^h = exp(r_{t+1}^h) = (H_{t+1} + WL_{t+1})/H_t$ where WL_{t+1} denotes the aggregate compensation of capital in period $t+1$. See Campbell (1996).

holds for human wealth, and vice versa. More generally, we can write:

$$\ln C_t - \ln W_t \backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^w - \Delta \ln C_{t+s} \right) + \varepsilon_t. \tag{5}$$

where ε_t represents the error term induced by ignoring human wealth.[10]

The second step is to realize that the return on private wealth r_t^w can always be decomposed into the sum of a real risk free rate r_t^f and a risk premium rp_t^w according to: $r_t^w = r_t^f + rp_t^w$. While we can construct reasonably accurate estimates of the real risk free rate r_t^f, we do not observe the risk premium on private wealth rp_t^w (or equivalently, the return to private wealth). This is so since private wealth includes a variety of financial assets such as portfolio holdings whose return could reasonably be approximated, but includes also non-financial assets such as housing, agricultural land and equipments whose returns are more difficult to measure. Our approach consists in proxying the return on private wealth by assuming that :

$$r_t^w = r_t^f + \nu\, \tilde{rp}_t, \tag{6}$$

where $\tilde{rp}_t$ is the observed excess global equity return, and ν is an adjustment parameter that will be estimated in order to maximize the empirical fit of equation (5).[11]

[10] From equation (4) and with some simple manipulations, we can write :

$$\varepsilon_t = (1-\omega) \left(\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^h - r_{t+s}^w \right) - (\ln W_t - \ln H_t) \right)$$

This error term is small when expected returns on human and private wealth are similar, and when the ratio of private to human wealth is stationary. Accordingly, the consumption-to-private wealth ratio may be low when expected future returns to human wealth are low relative to the returns on private wealth, or when human wealth is low relative to private financial wealth. The recent evidence on the decline in the labor share (see e.g. Karabarbounis and Neiman (2014)) and on the increase in income ineqality (see e.g. Piketty and Saez (2003)) could invalidate these assumptions. However, our focus on long run data should mitigate these concerns. For instance, as documented by Piketty and Saez (2003), the dynamics of income inequality over the last century is characterized by large and persistent fluctuations, but no historical trend: income inequality in the U.S. is today close to what it was at the beginning of the XXth century. Over shorter horizons, as in Lettau and Ludvigson (2001), the consumption-to-wealth ratio may appear non-stationary.

[11] Specifically we estimate ν via OLS so as to minimize the residuals in equation (5). This opens up the possibility that the risk premium component may be contaminated by the human capital component. Specifically, the OLS estimate of ν satisfies $\hat{\nu} = \nu + cov\,(\varepsilon, \tilde{cw}^{rp})/\text{var}(\tilde{cw}^{rp})$ where $\tilde{cw}^{rp} = \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \tilde{rp}_{t+s}$ is the estimated present value of future global equity risk premia. The possible bias on $\hat{\nu}$ attributes to the risk premium component the part of the variation in $\ln C - \ln W$ that should be attributed to fluctuations in human wealth and co-moves with the equity risk premium.

Substituting (6) into the present value relation (5), we obtain our fundamental representation:

$$\ln C_t - \ln W_t \backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s r_{t+s}^f + \nu \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s rp_{t+s} - \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \Delta \ln C_{t+s} + \varepsilon_t. \tag{7}$$
$$\equiv cw_t^f + cw_t^{rp} + cw_t^c + \varepsilon_t.$$

This equation states that the ratio of aggregate consumption to private wealth should contain information either about (a) future safe rates r_{t+s}^f, (b) future excess returns rp_{t+s}, or (c) future aggregate consumption growth $\Delta \ln C_{t+s}$. The terms cw_t^f, cw_t^{rp} and cw_t^c summarize the relative contributions of the risk free rate, the risk premia and consumption growth. Inspecting (7), it is well-known that aggregate consumption is close to a random walk, and that the risk premium is volatile and difficult to predict. Therefore, if anything we expect equation (7) to connect the aggregate consumption-to-wealth ratio and the expected path of future real risk free returns r_{t+s}^f.

2.2 Interpretation

Before we lay out our empirical strategy, we discuss how different fundamental shocks can affect returns, consumption and the consumption/wealth ratio.

2.2.1 Productivity shocks and consumption smoothing

As discussed, equation (7) does not provide a *causal* decomposition: the risk free, risk premium and consumption growth components are both endogenous and interdependent. To see this more clearly, let's consider a closed endowment economy with no government, so consumption C is equal to output Y. Equation (7) takes the form:

$$\ln C_t - \ln W_t \backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^w - \Delta \ln Y_{t+s} \right). \tag{8}$$

Suppose that total output growth is expected to decline. According to equation (8), for a given expected path of future returns, this should increase the consumption wealth ratio. However, expected future returns will not remain constant. Faced with a future slowdown in output growth, today's households may want to save more. In equilibrium, this will depress expected future returns, up to the point where consumption growth equals output growth. In turn, the decline in returns will exert downward pressure on the consumption-to-wealth ratio. To see this mechanism more explicitly, assume that the representative household has additively separable preferences over consumption, with a constant intertemporal elasticity of substitution (IES) $1/\sigma$ and a discount rate ρ. The usual log-linearized Euler equation takes the following form (up to the second order):

$$\sigma \mathbb{E}_t \Delta \ln C_{t+1} = \mathbb{E}_t r^w_{t+1} - \rho + \frac{1}{2}\sigma^2_{z,t},$$

where $\sigma^2_{z,t}$ denotes the conditional variance of $z_{t+1} = r^w_{t+1} - \sigma \Delta \ln C_{t+1}$ at time t.

Denote $g_t = \Delta \ln Y_t = \Delta \ln C_t$ the (exogenous) aggregate growth rate of output, and $\sigma^2_{g,t}$ its conditional variance. In equilibrium, the Euler equation expresses the expected return on wealth as :

$$\mathbb{E}_t r^w_{t+1} = \rho + \sigma \mathbb{E}_t g_{t+1} - \frac{1}{2}\sigma^2_{z,t}. \tag{9}$$

This expression encodes precisely the extent to which the return on wealth needs to respond to changes in expected output growth so as to clear the goods market: if output growth is expected to increase by 1%, the expected return on private wealth must increase by σ%. Substituting the Euler equation (9) into equation (8) one obtains:

$$\ln C_t - \ln W_t \backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left((\sigma - 1) g_{t+s} + \rho - \frac{1}{2}\sigma^2_{z,t+s} \right). \tag{10}$$

It is immediate from equation (10) that whether the consumption-to-wealth ratio increases or decreases with output growth depends on the sign of $\sigma - 1$, i.e. on the relative strength of the substitution and income effects. If $\sigma > 1$, the IES is low and interest rates need to decline a lot in order to stimulate consumption growth when productivity growth declines. The impact of productivity changes on interest rates dominates and C/W comoves positively with expected future productivity growth. If instead $\sigma < 1$, the IES is high and a modest decline in real interest rates is sufficient to push consumption growth down. The direct impact of productivity growth dominates and C/W comoves negatively with expected future productivity growth.[12]

Following similar steps, one can compute the various components cw^i_t as (up to some unimpor-

[12]In the special case where $\sigma = 1$, the consumption-wealth ratio is constant independent from $\mathbb{E}_t g_{t+s}$.

tant constants):

$$cw_t^f = \sigma \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(g_{t+s} - \frac{\sigma}{2} \sigma_{g,t+s}^2 \right)$$

$$cw_t^{rp} = \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(\sigma cov_t(r_{t+s}^w, g_{t+s}) - \frac{1}{2} \sigma_{r,t+s}^2 \right)$$

$$cw_t^c = -\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s g_{t+s},$$

where $\sigma_{r,t}^2$ is the conditional variance of the return on private wealth.

These expressions make clear that expected changes in productivity growth have direct opposite effects on the risk free and consumption components, scaled by the IES. This is most evident if there is no time-variation in second moments, in which case the risk premium component is constant while the risk free and consumption components are perfectly negatively correlated.[13]

2.2.2 Demographics

Consider now the effect of demographic forces on the consumption-to-wealth ratio. To do so, decompose total consumption growth $\Delta \ln C_{t+1}$ into per capita consumption growth $\Delta \ln c_{t+1}$, and population growth n_{t+1}: $\Delta \ln C_{t+1} = \Delta \ln c_{t+1} + n_{t+1}$. Substituting into (5) we obtain:

$$\ln c_t - \ln w_t \backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(r_{t+s}^w - \Delta \ln c_{t+s} - n_{t+s} \right),$$

where w_t denotes real private wealth per capita. It is obvious from this expression that an expected decline in population growth ($\mathbb{E}_t n_{t+s} < 0$) has a direct and positive effect on $c - w$, given a path of returns and consumption per capita. The effect of a decline in population growth on equilibrium interest rates, and therefore the indirect effect on the consumption-to-weath ratio, is more complex. As population growth slows down, this reduces the marginal product of capital, pushing down r^w. Aging, by increasing life expectancy, also leads to increased saving and therefore a decline in interest rates. On the other hand, a lower population growth increases the dependency ratio (i.e. the ratio of retirees to working age population). This tends to reduce aggregate savings, pushing interest

[13] Of course, risk premia may not be constant. For most models of interest, however, the correlation between returns on wealth and consumption growth is relatively small, indicating a small role for the macroeconomic risk premium that we measure here.

rates up (see Carvalho et al. (2016) for a discussion of the various channels). The empirical evidence as well as calibrated overlapping generation models generally indicate that slowdowns in population growth are associated with increased savings.[14] This should push down the consumption-to-wealth ratio with the strength of that effect, again, controlled by the intertemporal elasticity of substitution $1/\sigma$. In this case, as in the case of productivity shocks, the impact of demographic shocks will have opposite effects on the risk free and consumption growth components: $corr(cw_t^f, cw_t^c) < 0$. Importantly, we can measure the direct effect of demographic shocks on the consumption-to-wealth ratio by constructing $cw_t^n = -\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s n_{t+s}$. If demographic forces are an important driver of wealth and consumption movements, we expect that cw^n and cw^f be negatively correlated.

2.2.3 Deleveraging shock

Consider next what happens if there is a sudden shift in individuals' desire to save. At an abstract level, one can model this shift as a decrease in ρ, the discount rate of households. Such deleveraging shocks have been studied by Eggertsson and Krugman (2012), as well as Guerrieri and Lorenzoni (2011). To understand how these shocks may affect the consumption-to-wealth ratio, we need to consider two cases, depending on whether the economy is above or at the effective lower bound (ELB).

Consider first the case where the economy is above the ELB. For simplicity, assume that output is constant. With consumption equal to output in equilibrium, the Euler equation becomes

$$\mathbb{E}_t r_{t+1}^w = \rho_{t+1} - \frac{1}{2}\sigma_{r,t}^2, \tag{11}$$

where ρ_{t+1} is the now time-varying discount rate of the representative household between periods t and $t+1$, known at time t. A decline in ρ_{t+1} pushes down the equilibrium real return on assets and, under the assumption that the economy remains permanently above the ELB, the present-value equation (10) becomes:

$$\ln C_t - \ln W_t = \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (\rho_{t+s} - \frac{1}{2}\sigma_{r,t+s}^2).$$

[14] With open economies, the same phenomenon manifests itself in the form of current account surpluses for countries, such as Japan, Germany and China, with rapid slowdown in population growth and aging.

We can express the different components as:

$$
\begin{aligned}
cw_t^f &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \rho_{t+s} \\
cw_t^{rp} &= -\frac{1}{2}\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \sigma_{r,t+s}^2 \\
cw_t^c &= 0.
\end{aligned}
$$

An expected deleveraging shock, i.e. a decline in $\mathbb{E}_t \rho_{t+s}$, has a direct negative effect on the consumption-to-wealth ratio because it lowers the real risk free rate one for one, but no effect on the consumption or risk premia components.

Consider now what happens at the ELB. If prices are nominally rigid and nominal interest rates cannot decrease further to satisfy (11), the economy will experience a recession, as in Eggertsson and Krugman (2012) or Caballero and Farhi (2015). For simplicity, suppose that the effective lower bound is zero and that prices are permanently fixed so that $r^f = 0$ while the economy remains at the ELB. The Euler equation requires that:

$$
\sigma \mathbb{E}_t \Delta \ln C_{t+1} = -\rho_{t+1} + \frac{\sigma^2}{2} var_t \left(\sigma \Delta \ln C_{t+1} \right).
$$

Consumption is expected to increase at a rate that reflects the (positive) gap between the real interest rate and the natural real interest rate. Since potential output is constant this expression makes clear that the economy must experience a recession today (i.e. output and consumption need to be below potential). The expected return on wealth (equal to the expected excess return) now satisfies:

$$
\mathbb{E}_t r_{t+1}^w = cov_t(r_{t+1}^w, \sigma \Delta \ln C_{t+1}) - \frac{1}{2}\sigma_{r,t}^2,
$$

and may change as the economy hits the ELB, as emphasized by Caballero et al. (2016). If the economy is expected to remain permanently at the ELB, the different components of the consumption-to-wealth ratio can be expressed as:

$$
\begin{aligned}
cw_t^f &= 0 \\
cw_t^{rp} &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(cov_t(r_{t+s}^w, \sigma \ln C_{t+s}) - \frac{1}{2}\sigma_{r,t+s}^2 \right) \\
cw_t^c &= \frac{1}{\sigma}\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(\rho_{t+1} - \frac{\sigma^2}{2} var_t \left(\Delta \ln C_{t+1}\right) \right).
\end{aligned}
$$

This expression makes clear that at the ELB, the adjustment in the consumption-to-wealth ratio occurs through the consumption component. In the general case where the economy does not remain stuck at the ELB permanently, the adjustment will occur both via a decline in the real risk free rate -when the economy is expected to leave the ELB– and via an increase in consumption growth –while the economy is at the ELB. Both terms depress the consumption-to-wealth ratio, so cw^f and cw^c will be positively correlated.

2.2.4 Demand for safe asset.

A deleveraging shock increases the demand for savings and therefore depresses the returns on all assets. Let's now consider a shock to risk appetite, i.e. shift in the demand for safe versus risky assets. An easy way to capture such a shift would be an increase in the risk aversion coefficient σ. However, as is well known, σ also plays the role of the inverse of the intertemporal elasticity of substitution. In order to isolate the effect of a shift in risk appetite, assume that the representative household has the following Epstein-Zin recursive preferences :

$$
U_t = \left\{ (1-\beta)C_t^{1-\sigma} + \beta \left(\mathbb{E}_t U_{t+1}^{1-\gamma_t} \right)^{\frac{1-\sigma}{1-\gamma_t}} \right\}^{\frac{1}{1-\sigma}},
$$

where γ_t is the now time-varying coefficient of relative risk aversion and $\ln \beta = -\rho$. We maintain a constant IES of $1/\sigma$. Given these preferences, the risk-free rate satisfies:

$$
r_{t+1}^f = \rho + \sigma \mathbb{E}_t \Delta \ln C_{t+1} + \frac{\theta_t - 1}{2}\sigma_{r,t}^2 - \frac{\theta_t \sigma^2}{2}\sigma_{g,t}^2
$$

where $\theta_t \equiv (1-\gamma_t)/(1-\sigma)$. When $\theta_t = 1$, this formula collapses to the CRRA case. By contrast, when $\theta \neq 1$, the risk free rate depends on the variance of the market return. Standard derivations provide the following expression for the expected risk premium:

$$\mathbb{E}_t r^w_{t+1} - r^f_{t+1} = \theta_t \sigma cov_t(r^w_{t+1}, g_{t+1}) + (1-\theta_t)\sigma^2_{r,t}.$$

To highlight the role of fluctuations in risk appetite, consider an environment where output is constant, so $\sigma^2_{g,t} = 0$. It follows that the consumption-to-wealth ratio can be expressed as (up to some constant):

$$\ln C_t - \ln \bar{W}_t = \frac{1}{2}\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(1-\theta_{t+s}\right) \sigma^2_{r,t+s}.$$

A persistent increase in risk aversion $\mathbb{E}_t\gamma_{t+s}$ lowers $\mathbb{E}_t\theta_{t+s}$ and leads to an increase in the consumption-to-wealth ratio. This is intuitive: while consumption is unchanged (and equal to output), the decline in risk appetite lowers the current value of wealth. The decomposition (7) yields :

$$\begin{aligned} cw^f_t &= -\frac{1}{2}\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \left(1-\theta_{t+s}\right) \sigma^2_{r,t+s} \\ cw^{rp}_t &= \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s (1-\theta_{t+s}) \sigma^2_{r,t+s} \\ cw^c_t &= 0. \end{aligned}$$

An expected increase in future risk aversion increases the risk premium component cw^{rp} and decreases the risk free rate component cw^f. The consumption component remains unchanged. This is also intuitive: the decline in risk appetite requires an increase in risk premia. This increase in risk premia is achieved via an increase in the expected return on risky assets and a decline in the risk free rate. Overall, the increase in risk premia dominates (it is twice as large as the risk free component).

2.3 Summary.

The preceding discussion highlights that, while the decomposition (7) does not provide a causal interpretation of the different components, the co-movements of the different components offers a natural signature about the various economic forces at play: If the consumption and risk free rate components are negatively correlated, we would conclude that productivity and/or demographic shocks play an important role. If instead the consumption and risk free rate components are either poorly correlated or positively correlated, then we would conclude that deleveraging shocks are likely to be more relevant. Finally, if we find that the risk free and risk premium components are negatively correlated, we would infer that shocks to the demand for safe assets are an important part of the story.

3 Estimating the Present Value Decomposition

We implement our empirical strategy in three steps. First, we construct estimates of the consumption-to-wealth ratio over long periods of time. Next, we evaluate the empirical validity of equation (7) by constructing the empirical counterparts on the right hand side of that equation and testing whether they capture movements in the consumption-to-wealth ratio. Lastly, we investigate the role of various drivers of the consumption-to-wealth ratio.

3.1 Data description

We use historical data on private wealth, population and private consumption for the period 1871-2011 for the United States, the United Kingdom, Germany and France from Piketty and Zucman (2014a) and Jordà et al. (2016) to construct measures of real per capita consumption and (beginning of period) private wealth, expressed in constant 2010 US dollars. The top panel of Figure 3 reports real per capita private wealth and consumption for the United States between 1871 and 2011. As expected, historical time series on consumption and financial wealth show a long term positive trend. U.S. real per capita consumption increased from \$2,830 in 2010 dollars in 1871 to \$33,793 in 2011, while real per capita private wealth increased over the same period from \$12,584 to \$169,668. The middle panel reports the consumption-to-wealth ratio. It appears relatively stable over this long period of time, with a mean of 20.94 percent, slightly decreasing from 22.41 percent in 1871 to 19.91 percent in 2011. While the consumption-to-wealth ratio is strongly autocorrelated, formal tests of a unit root do not find conclusive evidence of a unit root.[15] We observe two periods during which the consumption-to- wealth ratio was significantly depressed: the first one spans the 1930s, starting shortly before the Great Depression and ending at the beginning of the 1940s. Interestingly, in 1939

[15] For instance, the ADF test with intercept and trend reports a value of -3.088, with a p-value of 0.113, while a similar Phillips-Perron test reports a value of -3.23 with a p-value of 0.0824.

Professor Alvin Hansen writes his celebrated piece about 'secular stagnation' (Hansen (1939)). The second episode of low consumption-to-wealth ratio starts around 1995 with a pronounced downward peak in 2008. The consumption-to-wealth ratio rebounds after 2008 – largely as a result in the decline in private wealth– back to its long run average. Perhaps not coincidentally, in the Fall 2013 at a conference at the International Monetary Fund, Larry Summers, professor at Harvard resuscitates the idea of secular stagnation, an idea which is still haunting us in 2017 (Summers (2015)).

The top panel of Figure 4 reports real consumption and wealth per capita for an aggregate of the U.S., the U.K., Germany and France since 1920. We label this aggregate the 'G-4'. Over the period considered, these four countries represent a sizable share of the world's financial wealth and consumption. London, New-York, and to a lesser extent Frankfurt, represent major financial centers. As for the U.S., real consumption and wealth per capita for the G-4 show a long term positive trend with a few major declines during the two World Wars and the Great Depression.[16] Real per capita consumption increased from \$3,742 in 1920 to \$26,967 in 2010 constant dollars while real per capita private wealth increased from \$19,952 to \$150,680 over the same period. The consumption-to-wealth ratio exhibits the same pattern as that of the U.S., with a mean of 20.71 percent.[17]

The bottom panel of figures 3 and 4 report the real risk free rate for the U.S. and for the G4 aggregate. The real risk free rate is constructed as the interest rate on 3-month Treasuries from Jordà et al. (2016), deflated by the realized CPI inflation rate.[18] Both figures document the decline in global real interest rates since the early 1980s, from 9.6 percent in 1981 to -1.8 percent in 2011. They also illustrate that the early 1980s may have been a relatively exceptional period: US real

[16] The effect of the wars on the financial wealth and consumption of Germany and France is the most dramatic. The U.S. consumption-to-wealth ratio is somewhat insulated from the shock of the wars and it does not show swings of the amplitude of the U.K., French and German consumption wealth ratios at the time of the first world war. This, and concerns about data quality prior to 1920 are the two reasons we only begin the 'G4' aggregate after 1920. In particular, as discussed in the appendix, wealth data is not available annually before 1954 for France, 1950 for Germany, 1920 for the U.K. and 1916 for the U.S. and is imputed based on savings data and estimates of the rate of capital gains on wealth for each country.

[17] We are unable to reject the null of a unit root in $\ln C - \ln W$ for the G4 aggregate at conventional levels of significance: the ADF test with trend and intercept reports a value of -2.43 with a p-value of 0.358 while the Phillips and Perron test reports a value of -19.3 with a p-value of 0.63. Nevertheless, we consider that this reflects the low power of unit root tests rather than definite evidence that the consumption-to-wealth ratio is non-stationary.

[18] For the G4 aggregate, we use the average of the U.S. and U.K. real interest rates, weighted by relative wealth. Appendix A provides the details of the aggregation procedure. We do not include the real rate for Germany and France, since episodes of monetary instability in the 1920s and during WWII in both countries generate very volatile measures of the ex-post real interest rate.

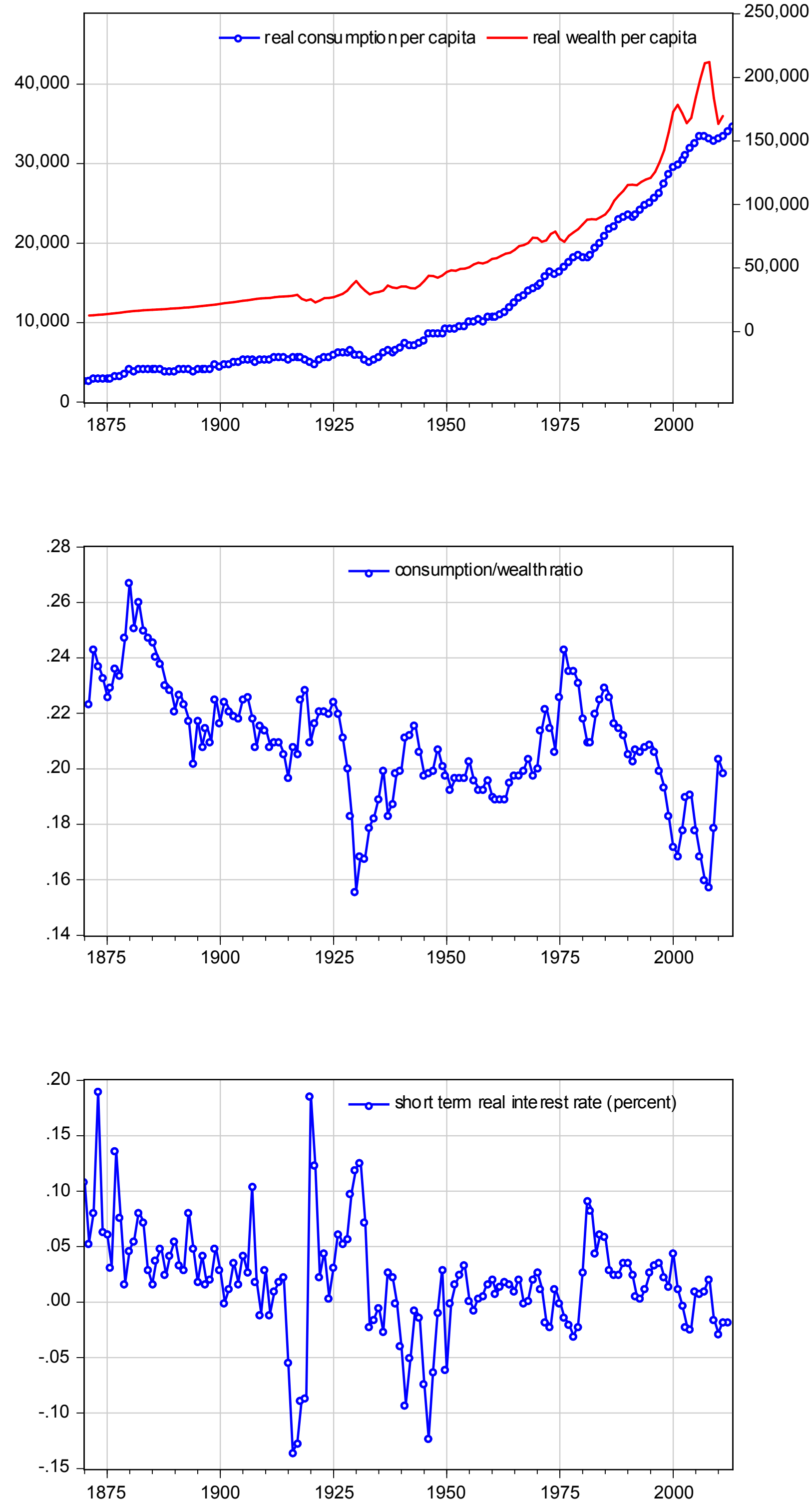

Figure 3: **Real Consumption and Wealth per capita (2010 USD), Consumption/Wealth Ratio and Short Term Real Interest Rate, United States, 1871-2011.**

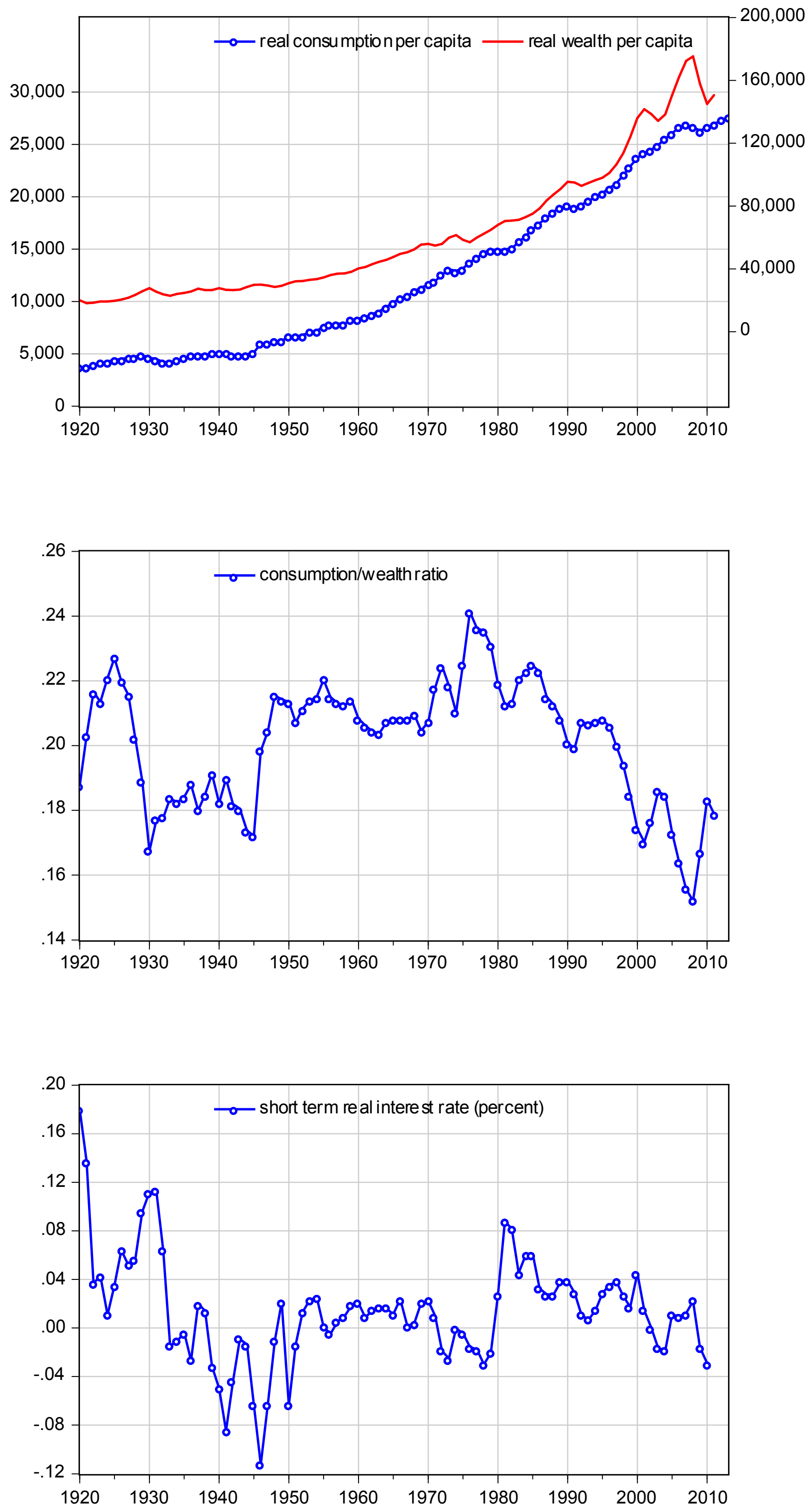

Figure 4: **Real Consumption and Wealth per capita (2010 USD), Consumption Wealth Ratio and Short Term Real Interest Rate, U.S., U.K., Germany and France (G4), 1920-2011.**

interest rates average 2.21 percent over the entire period, and only 0.1 percent per year between 1930 and 1980. For substantial periods of time over the last century, we find that the global real rate was significantly negative. During both World Wars as well as in the 1970s, this is a consequence of the elevated rate of price inflation. In the 1930s and the aftermath of the great recession, the decline in real rates reflects decreases in ex-ante real rates, rather than surprise inflation. Lastly, we define the excess return as the total return on equities, deflated by the CPI, minus the real return on 3-month Treasuries. [19]

3.2 VAR Results

We construct an empirical estimate of the right hand side of equation (7) using a Vector AutoRegression, we form the vector $\mathbf{z}_t = \left(\ln C_t - \ln W_t, r_t^f, rp_t, \Delta \ln C_t\right)'$ and estimate a Vector Auto Regression (VAR) or order p. Using this VAR, we then construct the forecasts $\mathbb{E}_t \mathbf{z}_{t+k}$ and we use these forecasts to construct :[20]

$$\begin{aligned} \hat{cw}_t^f &\backsimeq \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s r_{t+s}^f \\ \hat{cw}_t^{rp} &\backsimeq \hat{\nu} \mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s rp_{t+s} \\ \hat{cw}_t^c &\backsimeq -\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \Delta \ln C_{t+s} \end{aligned}$$

each of these components has a natural interpretation as the contribution of the risk free rate, the risk premium and the consumption growth components to the consumption-to-wealth ratio. We assume an annual discount rate $\rho_w = 0.96$. Recall that according to our derivations $\rho_w = 1 - C/\bar{W}$. Assuming $\rho_w = 0.96$ implies an average propensity to consume out of *total* private wealth of 4 percent. Since the consumption to private wealth ratio C/W we observe in the data is around 20 percent, this implies that $W/\bar{W} = 0.04/0.2 = 0.2$ and $H/W = 4$. A ratio of human wealth to financial wealth of 4 may seem high, but it is consistent with an economy that pays out 20 percent of income as capital income.[21]

[19] As for the risk free rate, we use a wealth-weighted average of the equity excess returns for the global excess return.

[20] See the details of the empirical VAR methodology in the Appendix.

[21] If a fraction δ of output is paid out as capital income, and the rest is paid out as 'human' income, the ratio of human wealth to private wealth is $H/W = (1 - \delta)/\delta$. With $\delta = 0.2$, we obtain a ratio $H/W = 4$. One might consider that this is a relatively high estimate of the human wealth. For instance, if payments to labor represent 2/3 of output

Finally, our approach requires an estimate of the adjustment parameter $\hat{\nu}$. As indicated earlier, we estimate this parameter by regressing $\ln C_t - \ln W_t - \hat{cw}_t^f - \hat{cw}_t^c$ on $\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s rp_{t+s}$. Recall that we do not observe the return on private wealth, so this method gives the highest chance to the model to match the observed consumption-to-wealth ratio. This calls for two observations. First, as noted above, this method leaves cw^f and cw^c unchanged so the correlation between the consumption growth component and the risk free rate component is unaffected by $\hat{\nu}$. Second, as we noted, while this method is appropriate if there is measurement error in the return to private wealth, it may induce some spurious movements if the residual in Eq. (7) due to fluctuations in human wealth relative to private wealth, is correlated with the cumulated excess return on equities. In that case, $\hat{cw}_t^{rp}$ is best interpreted as capturing both the risk premium as well as the component of the excess return on human wealth that is correlated with it.

Figure 5 shows the consumption wealth ratio as well the components of the right hand side of equation (7) for the US. The results are striking. First, we note that the fit of the VAR is excellent.[22] The grey line reports the predicted consumption-to-wealth ratio, i.e. the sum of the three components $cw_t^f + cw_t^{rp} + cw_t^c$.[23] Our empirical model is able to reproduce quite accurately the annual fluctuations in the consumption-to-wealth ratio over more than a century of data. This is quite striking since the right hand side of equation (7) is constructed almost entirely from the reduced form forecasts implied by the VAR estimation. Second, most of the movements in the consumption-to-wealth ratio reflect expected movements in the future risk-free rate, i.e. the cw_t^f component. By contrast, the risk premia cw_t^{rp} and per capita consumption growth cw_t^c components are not economically significant. It follows that the consumption-to-wealth ratio contains significant information on current and future real short term rates, as encoded in equation (7). As discussed above, the two historical periods of low consumption-to-wealth ratios occurred during periods of rapid asset price and wealth increases followed each time by a severe financial crisis. Our empirical results indicate that in the aftermath of these crises real short term rates remain low (or negative) for an extended period of time.

Finally, we find a strong negative correlation between the risk free component and both the risk

and payments to capital the remaining 1/3, the ratio of human wealth to private wealth is $(2/3)/(1/3) = 2$. This would be consistent with $C/W = 0.2$ if ρ_w =0.934. Our results remain unchanged with this alternative value for ρ_w.

[22] The lags of the VAR are selected by standard criteria.

[23] The overall fit is excellent, with an $R^2 = 0.74$. However, this result is obtained with a significant attenuation of the risk premium component since $\hat{\nu} = 0.37$.

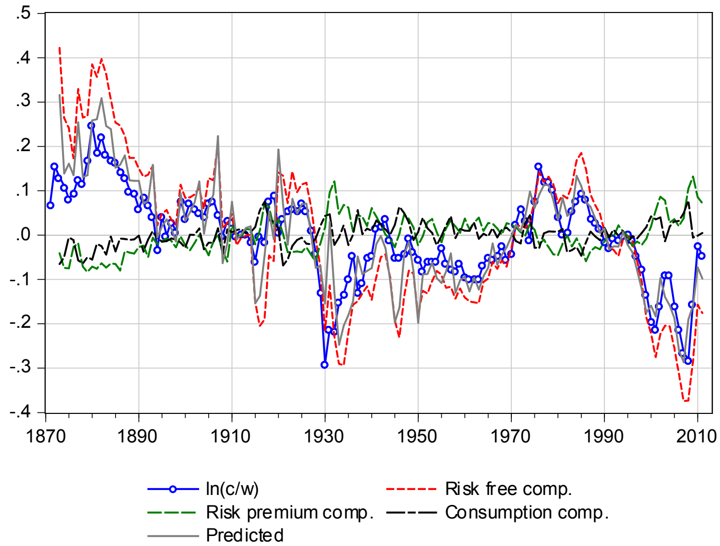

Figure 5: **Consumption Wealth, Risk-free, Equity Premium and Consumption Growth Components. United States, 1870-2011**. Note: The graph reports the (log, demeaned) private consumption-wealth ratio together with the riskfree, risk premium and consumption growth components. Estimates a VAR(3) with $\hat{\nu} = 0.37$. Source: Private wealth from Piketty and Zucman (2014a). Consumption and short term interest rates from Jordà et al. (2016). Equity return from Global Financial Database.

premium and consumption growth components.[24] While this could suggest some role for productivity shocks and demand for safe assets, we also observe that the magnitude of the consumption and risk premia components are generally too small. For instance, Table 1 decomposes the variance of $\ln C - \ln W$ into components reflecting news about future real risk-free rates, future risk premia, and future consumption growth. The model accounts for slightly more than 100 percent of the variance in the average propensity to consume, with the risk free rate representing 136 percent of the variation and the consumption growth component -32.9 percent. Overall, we infer from these results that deleveraging shocks must be a primary contributor that the observed dynamics, with productivity/demographic shocks playing a secondary role.

Figure 6 reports a similar decomposition for the 'G4' aggregate between 1920 and 2011. The results are very simular. First, the overall fit of the VAR remains excellent.[25] As before, we find that the risk free component explains most of the fluctuations in the consumption-to-wealth ratio. The adjusted risk premium and consumption growth components remain negligible. Next, the risk

[24]We estimate $corr\left(cw^f, cw^{rp}\right) = -0.77$ and $corr\left(cw^f, cw^c\right) = -0.66$.
[25]The $R^2 = 0.79$. The attenuation of the equity risk premium is stronger, however since we estimate $\hat{\nu} = 0.19$.

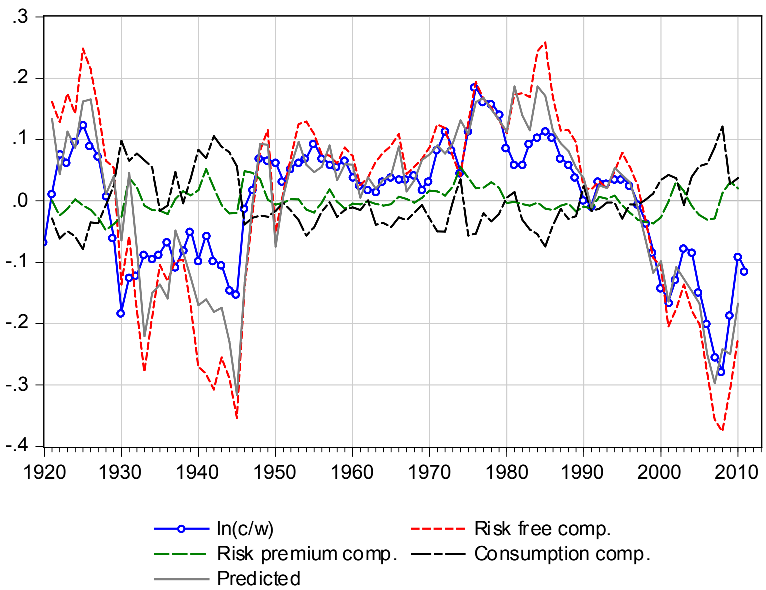

Figure 6: **Consumption Wealth: Risk-free, Equity Premium and Consumption Growth Components. United States, United Kingdom, Germany and France, 1920-2011**. Note: The graph reports the (log, demeaned) private consumption-wealth ratio for the U.S. U.K., Germany and France, together with the riskfree, risk premium and consumption growth components. Estimates a VAR(2) with $\hat{\nu} = 0.19$. Source: Private wealth from Piketty and Zucman (2014a). Consumption and short term interest rates from Jordà et al. (2016). Equity return from Global Financial Database.

free component remains strongly negatively correlated with the consumption growth component (the correlation is -0.82) but uncorrelated from the risk premium component (the correlation is -0.05). Finally, the variance decomposition, presented in Table 1 confirms again the importance of the risk free component. Overall, these results indicate confirm our interpretation that the main drivers of the fluctuations must be deleveraging shocks as well as productivity/demographic shocks.

To explore further the distinction between productivity and demongraphic shocks, Figure 7 reports an alternate decomposition where we separate total consumption growth into growth in consumption per capita and population growth: $\Delta \ln C = \Delta \ln c + n$. The results are largely unchanged. Table 1 provides the unconditional variance decomposition. This suggests that productivity shocks and demographic shocks play similar role in the dyamics of C/W. Both are negatively correlated with the risk free component (with correlation of -0.66 and -0.92 for the consumption per capita and population growth components respectively). Because of these components modest contribution to

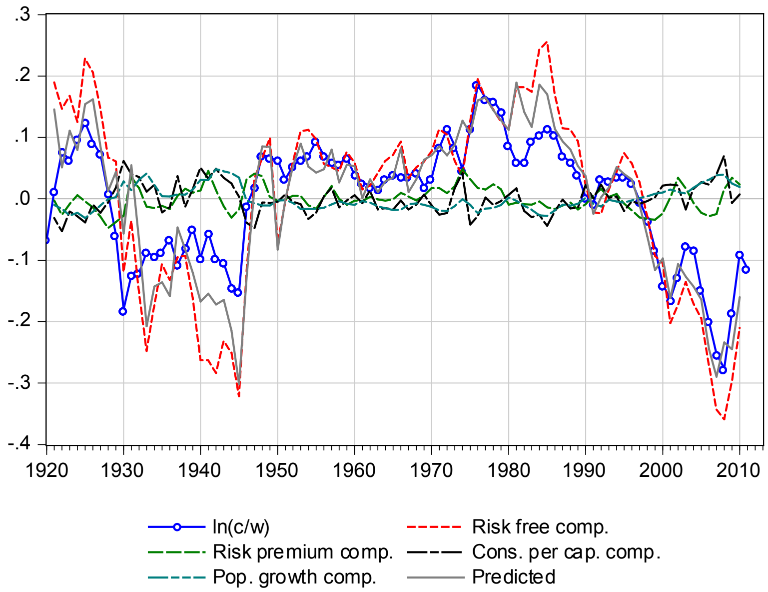

Figure 7: **Consumption Wealth: Risk-free, Equity Premium, Consumption per capita and Population Growth Components. United States, United Kingdom, Germany and France, 1920-2011**. Note: The graph reports the (log, demeaned) private consumption-wealth ratio together with the riskfree, risk premium, consumption per capita and population growth components. Estimates a VAR(2) with $\hat{\nu} = 0.19$. Source: Private wealth from Piketty and Zucman (2014a). Consumption, population and short rates from Jordà et al. (2016). Equity return from Global Financial Database.

the overall fluctuation in C/W, we conclude that deleveraging shocks remain a critical source of fluctuation.

The fact that equity risk premia account for almost none of the movements in C/W is perhaps surprising in light of Lettau and Ludvigson (2001)'s findings that a cointegration relation between aggregate consumption, wealth and labor income predicts reasonably well U.S. equity risk premia. A number of factors may account for this result. First and foremost, $\ln C/W$ appears reasonably stationary in our sample, hence we do not need to estimate a cointegrating vector with labor income. Second, we consider a longer sample period, going back to 1920. Thirdly, as argued above, our sample is dominated by two large financial crises and their aftermath. Lastly, we view our analysis as picking up low frequency determinants of real risk-free rates while Lettau and Ludvigson (2001) seem to capture business cycle frequencies.

4 Predictive regressions

The third and final step consists in directly evaluating the forecasting performance of the consumption-wealth variable for future risk-free interest rates, risk premia and aggregate consumption growth.

Our decomposition exercise indicates that the consumption-wealth ratio contains information on future risk-free rates. We can evaluate directly the predictive power of $\ln C_t/W_t$ by running regressions of the form:

$$y_{t+k} = \alpha + \beta \ln \left(C_t/W_t \right) + \epsilon_{t+k} \tag{12}$$

where y_{t+k} denotes the variable we are trying to forecast at horizon k . We consider the following candidates for y: the average real risk free rate between t and $t+k$; the average one-year excess return between t and $t+k$; the average annual real per capita consumption growth between t and $t+k$; the average annual population growth between t and $t+k$ and the average term premium between $t$and $t+k$.

Tables 2 presents the results for the US and the G4 aggregate. We find that the consumption-to-wealth ratio always contains substantial information about future short term risk free rates (panel A). The coefficients are increasing with the horizon and become strongly significant. They also have the correct sign, according to our decomposition: a low $\ln C/W$ strongly predicts a period of below average real risk-free rates up to 10 years out. By contrast, the consumption-to-wealth ratio has almost no predictive power for the equity risk premium and very limited predictive power for per capita consumption growth. The regressions indicate some predictive power for population growth: a low $\ln C/W$ predicts a low future population growth which suggests that the indirect effect (via changes in real risk-free rates) dominates the direct effect. Finally, there is significant predictive power for the term premium, i.e. the difference between the yield on 10-year Treasuries and short term rates. According to the estimates, a decrease in C/W is associated with a significant increase in term premia.

Figures 8-12 report our forecast of the risk free rate, equity premium, population growth, cumulated consumption growth per capita and term premium using the G-4 consumption-to-wealth ratio at 1, 2, 5 and 10 year horizon. For each year t, the graph reports $y^f_{t,k} = \frac{1}{k}\sum_{s=0}^{k-1} y^f_{t+s}$, the average of the variable z to forecast one-year real risk-free rate between t and $t+k$, where k is the forecasting horizon. The graph also reports the predicted value $\hat{y}^f_{t,k}$ based on predictive regression

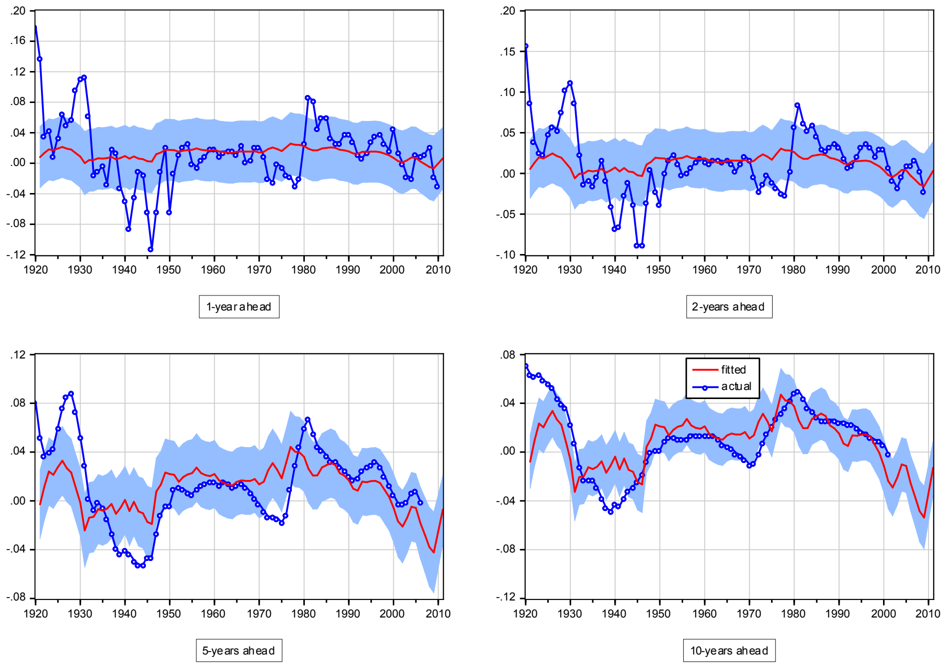

Figure 8: **Predictive Regressions: Risk Free Rate, 1920-2010.** Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global real risk free rate from a regression on past $\ln(C/W)$.

(12) together with a 2-standard error confidence band, computed using Newey-West robust standard errors. For two variables, the average future global short rate and the average future global term premium, the fit of the regression improves markedly with the horizon. The last forecasting point is 2011, indicating a forecast of -1.3 percent for the global short real interest rate until 2021 (bottom right graph) and of 1.22 percent for the term premium.

5 Conclusion.

Our results suggest that boom-bust financial cycles are a strong determinant of real short term interest rates. During the boom, private wealth increases rapidly, faster then consumption, bringing down the ratio of consumption to private wealth. This increase in wealth can occur over the course of a few years, fueled but increased leverage, financial exuberance, and increased risk appetite. Two such historical episodes for the global economy are the roaring 1920s and the 2000s. In the subsequent bust, asset prices collapse, collateral constraints bind, and households, firms and governments attempt to simultaneously de-leverage, as risk appetite wanes. The combined effect is

#	percent	U.S.	G4
1	β_{rf}	1.364	1.406
2	β_{rp}	0.005	0.025
3	β_c	-0.329	-0.336
	of which:		
3	β_{cp}	0.056	-0.168
4	β_n	-0.386	-0.168
5	Total (lines 1+2+3)	1.041	1.094

Table 1: **Unconditional Variance Decomposition of** $\ln C - \ln W$

Note: β_{rf} (resp.β_{rp}, and β_c) represents the share of the unconditional variance of $\ln C - \ln W$ explained by future risk free returns (resp. future risk premia and future total consumption growth); β_{cp} (β_n) represents the share of the unconditional variance of $\ln C - \ln W$ explained by per capita consumption growth (population growth). The sum of coefficients $\beta_{cp} + \beta_n$ is not exactly equal to β_c due to numerical rounding in the VAR estimation. Sample: U.S: 1871-2011; G4: 1920:2011

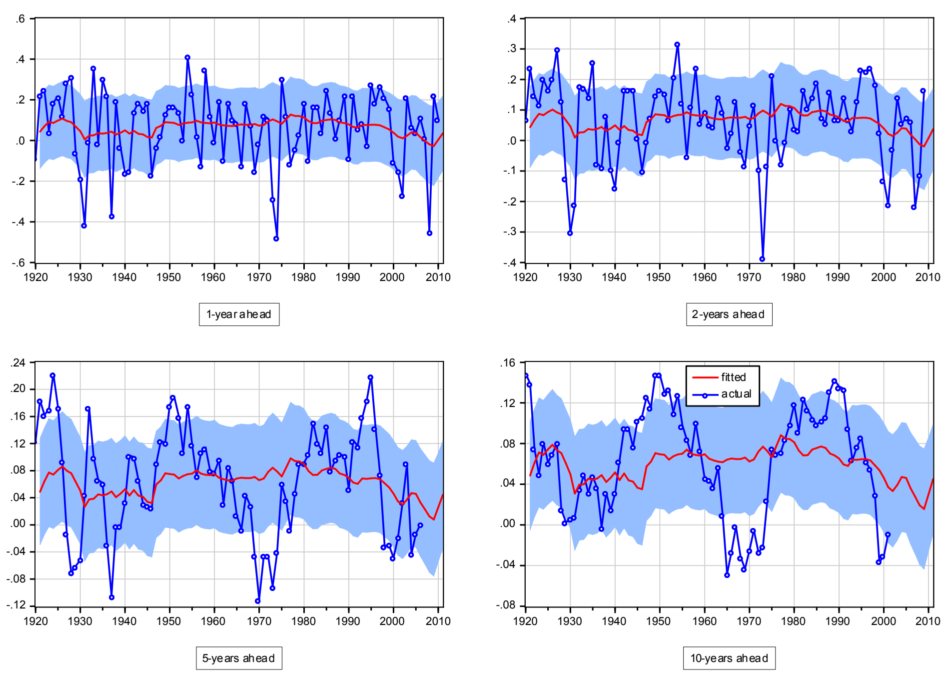

Figure 9: **Predictive Regressions: Equity Premium, 1920-2010.** Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized equity premium from a regression on past ln(C/W).

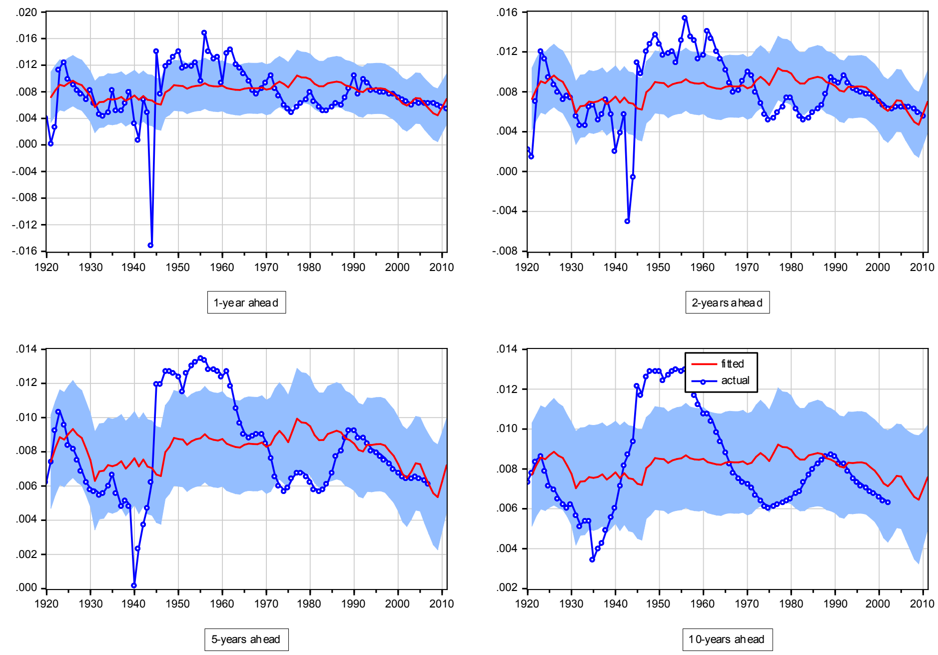

Figure 10: **Predictive Regressions: Population Growth, 1920-2010.** Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global population growth rate from a regression on past $\ln(C/W)$.

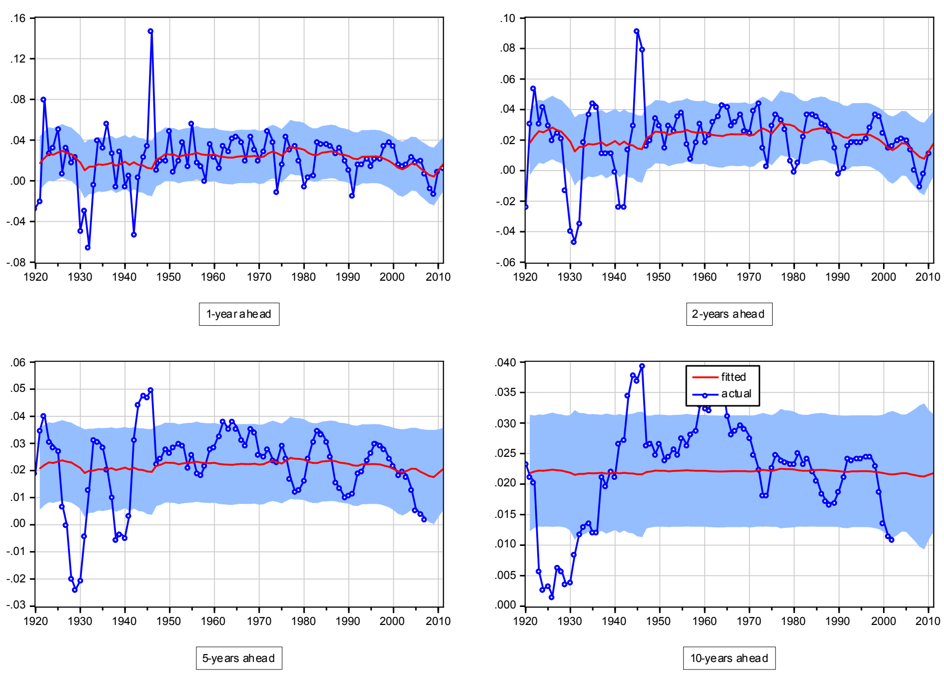

Figure 11: **Predictive Regressions: Consumption growth per capita, 1920-2010.** Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global per capita real consumption growth from a regression on past $\ln(C/W)$.

United States					U.S., U.K., France and Germany				
Forecast Horizon (Years)					Forecast Horizon (Years)				
	1	2	5	10		1	2	5	10
A. Short term interest rate					A. Short term interest rate				
$\ln C_t/W_t$	**.12**	**.14**	**.16**	**.18**	$\ln C_t/W_t$	**.07**	**.10**	**.19**	**.22**
	(.05)	(.05)	(.03)	(.03)		(.06)	(.06)	(.06)	(.04)
R^2	[.06]	[.10]	[.21]	[.34]	R^2	[.03]	[.07]	[.27]	[.43]
B. Consumption growth (per-capita)					B. Consumption growth (per-capita)				
$\ln C_t/W_t$	**.01**	**.01**	**-.02**	**-.04**	$\ln C_t/W_t$	**.06**	**.05**	**.02**	**.01**
	(.04)	(.04)	(.02)	(.02)		(.04)	(.04)	(.02)	(.02)
R^2	[0]	[0]	[.01]	[.12]	R^2	[.06]	[.06]	[.02]	[.00]
C. Equity Premium					C. Equity Premium				
$\ln C_t/W_t$	**.05**	**.02**	**-.08**	**-.07**	$\ln C_t/W_t$	**.27**	**.20**	**.01**	**-.06**
	(.22)	(.16)	(.08)	(.08)		(.25)	(.18)	(.11)	(.11)
R^2	[0]	[0]	[.01]	[.02]	R^2	[.02]	[.02]	[.00]	[.01]
D. Population Growth					D. Population Growth				
$\ln C_t/W_t$	**.03**	**.03**	**.03**	**.03**	$\ln C_t/W_t$	**.02**	**.02**	**.02**	**.02**
	(.01)	(.01)	(.01)	(.01)		(.01)	(.01)	(.01)	(.01)
R^2	[.35]	[.38]	[.42]	[.36]	R^2	[.07]	[.13]	[.18]	[.24]
E. Term Premium					E. Term Premium				
$\ln C_t/W_t$	**-.05**	**-.05**	**-.04**	**-.03**	$\ln C_t/W_t$	**-.05**	**-.06**	**-.06**	**-.03**
	(.01)	(.01)	(.01)	(.01)		(.02)	(.01)	(.01)	(.01)
R^2	[.09]	[.13]	[.19]	[.17]	R^2	[.14]	[.24]	[.40]	[.24]

Table 2: **Long Horizon Regressions**. Note: The table reports the point estimates, Newey-West corrected standard errors and the R^2 of the forecasting regression.

an increase in desired saving that depresses persistently safe real interest rates. An additional force may come from a weakened banking sector and financial re-regulation or repression that combine to further constrain lending activity to the real sector. Our estimates indicate that short term real risk free rates are expected to remain low or even negative for an extended period of time.

The central object of our analysis are risk free rates. In recent years, an abundant empirical literature has attempted to estimate the natural rate of interest, r^*, defined as the real interest rate that would obtain in an equivalent economy without nominal frictions. Many estimates indicate that this natural rate may well have become significantly negative. Our analysis speaks to this debate. Outside of the effective lower bound, monetary policy geared at stabilizing prices and economic activity will set the policy rate so that the real short term rate is as close as possible to the natural rate. Therefore, to the extent that the economy is outside the ELB, our estimate of future global real rates should coincide with estimates of r^*. At the ELB, this is not necessarily the case since global real rates must, by definition of the ELB, be higher than the natural rate.

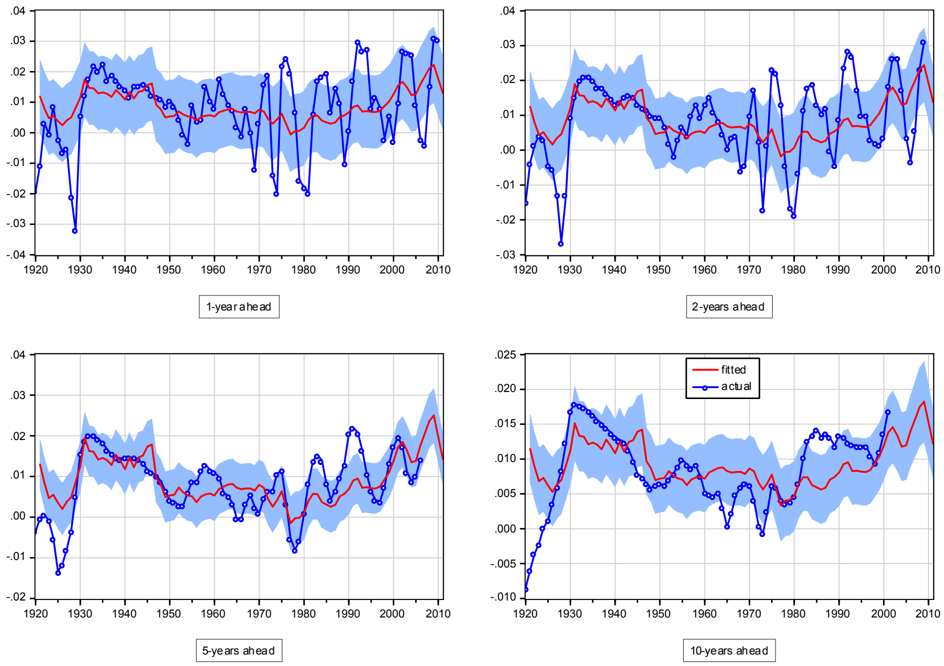

Figure 12: **Predictive Regressions: Term premium, 1920-2010.** Note: The graph reports forecasts at 1, 2, 5 and 10 years of the annualized global term premium from a regression on past $\ln(C/W)$.

Therefore, our estimates provide an upper bound on future expected natural rates. Given that our estimates are quite low (-1.3 percent on average between 2011 and 2021), we conclude that the likelihood of the ELB binding remains quite elevated.

Our empirical results do not provide strong evidence for the view that low real interest rates are the result of low expected future productivity. We don't find much explanatory power for future per capita consumption growth in the consumption-to-wealth ratio. Similarly, we find only limited support for demographic forces. Instead, our results points towards the importance of the global financial boom/bust cycle, both in the 1930s and in the 2000s. Under this interpretation, it is the increased desired savings, and the move away from risky asset, that drive real interest rate determination. Therefore, we view these empirical results very much in line with interpretations of recent events that emphasize the global financial cycle (Miranda-Agrippino and Rey (2015), Reinhart and Rogoff (2009), as well as the scarcity of safe assets (Caballero and Farhi (2015)).

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Appendix

A Data description

The data used in Section 3 were obtained from the following sources:

1. **Consumption:**
   Real per-capita consumption going back to 1870 and covering the two world wars was taken from Jordà et al. (2016) who in turn obtained the data from Barro and Ursúa (2010). As this consumption series is an index rather than a level, we convert it to a level using the consumption data from Piketty and Zucman (2014a). To convert to a level we could use any year we have level data for but chose to use the year 2006 (the year that the index of consumption was 100). In addition, the consumption data was adjusted so that instead of being based on a 2006 consumption basket, it was based on a 2010 consumption basket to match the wealth data.

2. **Wealth:**
   Real per capita wealth data was taken from Piketty and Zucman (2014b). The wealth concept used here is private wealth. As such it does not include government assets but includes private holdings of government issued liabilities as an asset. Where possible, wealth data is measured at market value. Human wealth is not included. Private wealth is computed from the following components: "Non-financial assets" (includes housing and other tangible assets such as software, equipment and agricultural land), and net financial assets (includes equity, pensions, value of life insurance and bonds). Prior to 1954 for France, 1950 for Germany, 1920 for the UK and 1916 for the USA, wealth data is not available every year (see Piketty-Zucman's appendix for details on when data is available for each country or refer to Table 6f in the data spreadsheets for each country). When it is available is is based on the market value of land, housing, other domestic capital assets and net foreign assets less net government assets. For the remaining years the wealth data is imputed based on savings rate data and assumptions of the rate of capital gains of wealth (see the Piketty-Zucman appendix for details of the precise assumptions on capital gains for each country. The computations can be found in Table 5a in each of the data spreadsheets for each country).

3. **Short term interest rates:**
   These were taken from Jordà et al. (2016) and are the interest rate on 3-month treasuries.

4. **Long term interest rates:**
   These were taken from Jordà et al. (2016) and are the interest rate on 10 year treasuries.

5. **Return on Equity:**
   This data is the total return on equity series taken from the Global Financial Database.

6. **CPI:**
   CPI data is used to convert all returns into real rates and is taken from Jordà et al. (2016).

7. **Population:**
   These were taken from Jordà et al. (2016).

Figure 4 reports consumption per capita, wealth per capita, the consumption/wealth ratio as well as the short term real risk free rate for our G4 aggregate between 1920 and 2011.

B Loglinearization of the budget constraints and aggregation

For a country i the budget constraint takes the form:

$$\bar{W}^i_{t+1} = R^i_{w,t+1}(\bar{W}^i_t - C^i_t) \tag{13}$$

where $\bar{W}^i_t$ denotes total private wealth at the beginning of period t, C^i_t is private consumption during period t and $\bar{R}^i_{w,t+1}$ is the gross return on total private wealth between periods $t$and $t+1$. All variables are in real terms measured. Lettau and Ludvigson (2001) propose a log-linear expansion around the steady state consumption-to-wealth ratio and steady state return. Define $cw^i_t = \ln C^i_t - \ln \bar{W}^i_t$. cw^i_t is stationary with mean cw^i. Dividing both side of (13) by $\bar{W}^i_t$ and taking logs, we obtain:

$$\begin{aligned}
\ln \bar{W}^i_{t+1}/\bar{W}^i_t &= \bar{r}^i_{w,t+1} + \ln(1 - C^i_t/W^i_t) \\
&= \bar{r}^i_{w,t+1} + \ln(1 - e^{cw^i}\exp(cw^i_t - cw^i)) \\
&\approx \bar{r}^i_{w,t+1} + \ln(1 - e^{cw^i} - e^{cw^i}(cw^i_t - cw^i)) \\
&\approx \bar{r}^i_{w,t+1} + \ln\left((1 - e^{cw^i})\left(1 - \frac{e^{cw^i}}{1 - e^{cw^i}}(cw^i_t - cw^i)\right)\right) \\
&\approx \bar{r}^i_{w,t+1} + \ln(1 - e^{cw^i}) - \frac{e^{cw^i}}{1 - e^{cw^i}}(cw^i_t - cw^i) \\
&\approx \bar{r}^i_{w,t+1} + k + \left(1 - \frac{1}{\rho_w}\right) cw^i_t
\end{aligned}$$

where $\rho_w = 1 - e^{cw^i} = (W - C)/W$ and k is an unimportant constant. The next step is to rewrite the left hand side as

$$\ln \bar{W}^i_{t+1}/\bar{W}^i_t = \ln(\bar{W}^i_{t+1}/C^i_{t+1}) - \ln(\bar{W}^i_t/C^i_t) + \Delta \ln C^i_{t+1} = -cw^i_{t+1} + cw^i_t + \Delta \ln C^i_{t+1}$$

to obtain (again, ignoring the constant):

$$cw^i_t = \rho_w \left(cw^i_{t+1} - \Delta \ln C^i_{t+1} + \bar{r}^i_{w,t+1}\right) \tag{14}$$

which can be iterated forward to obtain (under the usual transversality condition):

$$c^i_t - \bar{w}^i_t = \sum_{s=1}^{\infty} \rho^s_w \left(\bar{r}^i_{w,t+s} - \Delta \ln C^i_{t+s}\right)$$

C Aggregation

From Eq. (1) we can aggregate across countries:

$$\sum_i \frac{\bar{W}^i_{t+1}}{R^i_{w,t+1}} = \sum_i \bar{W}^i_t - C^i_t = \bar{W}_t - C_t$$

where $\bar{W}_t = \sum_i \bar{W}^i_t$ and $C_t = \sum_t C^i_t$. From this expression we can derive

$$\bar{W}_{t+1} = R_{w,t+1}(\bar{W}_t - C_t)$$

where

$$\frac{1}{R_{w,t+1}} = \sum_i \frac{\bar{W}_{t+1}^i}{\bar{W}_{t+1}} \frac{1}{R_{w,t+1}^i}$$

The global period return on private wealth is an harmonic weighted mean of the individual country returns.

D VAR methodology

Consider the present value relation in Eq. 4. We form $\mathbf{z}_t = (\ln C_t - \ln W_t, r_t, rp_t, \Delta \ln C_t,)'$ and estimate Vector AutoRegression of order p, VAR(p), which can be expressed in companion form as:

$$\bar{\mathbf{z}}_t = \bar{\mathbf{A}}\bar{\mathbf{z}}_{t-1} + \bar{\varepsilon}_t$$

where $\bar{\mathbf{z}}_t' = \left(\mathbf{z}_t', \mathbf{z}_{t-1}', ..., \mathbf{z}_{t-p}'\right)$. Using the estimated VAR matrix $\bar{\mathbf{A}}$, conditional forecasts of $\bar{\mathbf{z}}_t$ can be directly obtained as:

$$\mathbb{E}_t \bar{\mathbf{z}}_{t+k} = \bar{\mathbf{A}}^k \bar{\mathbf{z}}_t$$

from which we recover:

$$\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \bar{\mathbf{z}}_{t+s} = \sum_{s=1}^{\infty} \rho_w^s \bar{\mathbf{A}}^s \bar{\mathbf{z}}_t = \rho_w \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t.$$

Denote $\mathbf{e}_x$ the vector that 'extracts' variable x from $\bar{\mathbf{z}}$, in the sense that $\mathbf{e}_x'\bar{\mathbf{z}} = x$. It follows that

$$\mathbb{E}_t \sum_{s=1}^{\infty} \rho_w^s \mathrm{x}_{t+s} = \rho_w \mathbf{e}_x' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t$$

From this we can construct the various components as:

$$\begin{aligned}
cw_t^f &= \rho_w \mathbf{e}_r' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t \\
cw_t^c &= -\rho_w \mathbf{e}_{\Delta \ln C}' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t \\
cw_t^{rp} &= \nu \rho_w \mathbf{e}_{erp}' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t \\
cw_t^{\Delta \ln c} &= -\rho_w \mathbf{e}_{\Delta \ln c}' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t \\
cw_t^n &= -\rho_w \mathbf{e}_n' \bar{\mathbf{A}} \left(\mathbf{I} - \rho_w \bar{\mathbf{A}}\right)^{-1} \bar{\mathbf{z}}_t
\end{aligned}$$