Financial Cycles with Heterogeneous Intermediaries

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April 13, 2020

Abstract

This paper develops a dynamic macroeconomic model with heterogeneous financial intermediaries and endogenous entry. It features time-varying endogenous macroeconomic risk that arises from the risk-shifting behaviour of the cross-section of financial intermediaries. We show that when interest rates are high, a decrease in interest rates stimulates investment and increases financial stability. In contrast, when interest rates are low, further stimulus can increase aggregate risk while inducing a fall in the risk premium. In this case, there is a trade-off between stimulating the economy and financial stability.

JEL Codes: E32, E44, E52, G21.

Keywords: Macroeconomics, Financial Cycle, Risk-taking Channel of Monetary Policy, Leverage, Systemic Risk.

1 Introduction

The global financial crisis of 2007-2008 has called into question our modelling of the role of financial intermediaries in the economy. The financial sector, far from being a veil, plays a key role in the transmission and amplification of shocks and in driving fluctuations in aggregate risk. The precise mechanisms by which this happens are still debated. In particular, understanding the underlying forces driving endogenous systemic risk, the concentration of risk in some balance sheets and the interactions between monetary policy and financial stability are key issues. A long tradition of scholars such as Fisher (1933), Minsky (1977) and Kindleberger (1978) argued that financial sector expansions and contractions are important drivers of fluctuations in economic activity and financial stability. Kaminsky and Reinhart (1999), Reinhart and Rogoff (2009b) and Gourinchas and Obstfeld (2012) among others show that financial crises tend to be preceded by a rapid expansion of credit. Schularick and Taylor (2012) study the long run dynamics of money, credit and output over the period 1870-2008 and find that financial crises tend to be "booms gone bust".

Financial cycles have been analysed in the literature typically through the lenses of models featuring one representative financial intermediary subject to capital market frictions. In contrast, we emphasise the importance of heterogeneity in risk taking across financial intermediaries in driving aggregate outcomes. Changes in market shares due to increased risk taking by some intermediaries play a large role in the risk build-up phase of a crisis. For Sweden, Englund (2016) explains how between 1985 and 1990 the rate of increase of lending by financial institutions jumped to 16% due in part to deregulation with rapid shifts in market shares. There was a significant correlation between the rate of credit expansion of specific institutions and their subsequent credit losses in the crisis, leading to bailouts. For Spain, Santos (2017) emphasizes how between 2002 and 2009, the regional banks (cajas) leveraged a lot to invest in the real estate sector, their combined balance sheet reaching 40% of Spanish GDP in 2009. Some (Bancaja) more than tripled their balance sheet while more "conservative" ones (Catalunya Caixa) doubled it. They ended up all being nationalized in the crisis. In Germany, as described by Hellwig (2018), Landesbanken and local savings banks whose borrowing was guaranteed by German Lander and municipalities until 2005 took the

opportunity to gorge on cheap funds increasing their debt by around €250bn over the period 2001 to 2005. As the market for Asset Backed Commercial Paper (ABCP) doubled in size between 2000 and 2006 the share of Landesbanken in that market grew from 6% to 8%. The weaker was the stand-alone credit rating of the Landesbank, the larger the subsequent increase in risk taking. Deutsche Bank leveraged up to quadruple the size of its balance sheet from about €0.5 trillion in early 1990s to about €2 trillion in 2008 as a RoE of 25% was regularly targeted by the bank CEO. German taxpayers ended up paying about €70 billion to support their financial institutions. In the US, as noted by Korinek and Nowak (2017), large risk taking by some financial intermediaries also played a big role in the mortgage boom. Countrywide increased its size to capture more than 20% of the US market in 2006 and had to be rescued in 2008. Wilmarth (2013) mentions the high risk culture of the too-big-to-fail Citigroup as a possible explanation behind the massive expansion of its balance sheet during the boom years. Citigroup nearly doubled the share of its subprime mortgage business from 10% in 2005 to 19% in 2007. During the period 2007 to the spring of 2010, Citigroup recorded more than $130 billion in credit losses and write-downs. It received its first government bailout in October 2008 (it was bailed out 3 times in total).

Accounting for such heterogeneity in risk taking behaviour and its macroeconomic implications is important. A large literature has recognized the centrality of financial frictions such as costly state verification, collateral, net worth or Value-at-Risk (VaR) constraints for representative firms and intermediaries[1]. But that literature has not allowed for heterogeneity in risk taking; it has mostly focused on the transmission and the amplification of shocks rather than the endogenous risk build-up phase of the financial cycle and the concentration of risk in some balance sheets (see e.g. Kiyotaki and Moore (1997) where the interaction between endogenous credit limits and asset prices generates a powerful transmission mechanism for productivity shocks and Guerrieri and Lorenzoni (2017) for a Bewley type model with borrowing limits and credit crunch shocks). We build a novel framework with a continuum of financial intermediaries

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[1]See in particular, but not only, Bernanke and Gertler (1989), Kiyotaki and Moore (1997)), Lorenzoni (2008), Mendoza (2010), Gertler and Kiyotaki (2015), Gertler and Karadi (2011), Gertler, Kiyotaki and Queralto (2012), Farhi and Werning (2016), Aoki, Benigno and Kiyotaki (2016), Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013) , Danielsson, Shin and Zigrand (2010), Adrian and Shin (2010), Bruno and Shin (2015b), Coimbra (2016) and Acharya et al. (2017).

heterogeneous in their VaR constraints[2] and a moral hazard friction due to limited liability and government guarantees which generate risk-shifting. Heterogeneous VaRs may reflect different risk attitudes by the boards of financial intermediaries or different implementations of regulatory constraints across institutions and supervisors. In our model, the dynamics of the *distribution of leverage* across intermediaries is a key determinant of financial stability. When high risk-taking intermediaries are dominant, they increase the price of risky assets and concentrate most of the aggregate risk on their balance sheets. The leverage distribution across intermediaries is positively skewed and financial stability is lower, as a large fraction of assets are in the hands of intermediaries with a high probability of default. This tends to happen when financing costs are low, which may be due to deregulation, a savings glut, expansionary monetary policy or when volatility is low.

We link financial stability explicitly to the risk of intermediary default. In the model, default is costly and the cost of default is proportional to the balance sheet size of defaulting intermediaries (see Section 6). Credit booms generated by low costs of funds are associated with worsened financial stability and lower risk premia (these are "bad booms" in the terminology of Gorton and Ordoñez (2019)). This is consistent with the evidence reported in Krishnamurthy and Muir (2017) that spreads tend to be low before crises.[3] Risk-shifting and different VaR constraints across financial intermediaries jointly generate heterogeneous willingness to pay for risky assets and a link between aggregate risk taking and the distribution of leverage[4]. We provide therefore a different and complementary view of financial fragility from Gennaioli, Shleifer and Vishny (2012). For these authors, excess risk taking comes from neglecting some improbable risk. In our model, it is the presence of limited liability that leads bankers to optimally ignore downside risk within the default region, while government guarantees insure

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[2] See Adrian and Shin (2014) for microfoundations of VaR constraints.

[3] These authors note that standard models such as Gertler and Kiyotaki (2015), He and Krishnamurthy (2013) or Brunnermeier and Sannikov (2014) "will not match the pre-crisis spread evidence. In the(se) models, a prolonged period in which fragility and leverage rises will also be coupled with an increase in spreads and risk premia. That is, the logic of these models is that asset prices are forward looking and will reflect the increased risk of a crisis as fragility grows".

[4] Allen and Gale (2000) have shown that current and future credit expansion can increase risk-shifting and create bubbles in asset markets. Nuño and Thomas (2017) show that the presence of risk-shifting creates a link between asset prices and bank leverage.

depositors[5]. Our framework also generates booms driven by high expected productivity which increases investment, does not increase financial instability and has a very limited effect on the risk premium ("good booms").

In our model, there is an endogenous non-linearity in the trade-off between monetary policy (which affects the funding costs of intermediaries) and financial stability[6]. When the level of interest rates is high, a fall in interest rates leads to an increase in leverage (*intensive margin*) and to entry of less risk-taking intermediaries into the market for risky projects (*extensive margin*). The average intermediary is then less risky, so a fall in the cost of funds improves financial stability and expands the capital stock: there is no trade-off between stimulating the economy and financial stability. However, when interest rates are very low, a further decrease benefits the most leveraged risk-taking intermediaries and competition drives out the more prudent ones. Stimulating the economy shifts the distribution of assets towards the more risk-taking intermediaries, which have a higher default risk and increases aggregate risk-shifting. This non-monotonicity constitutes a substantial difference from the existing literature and is a robust mechanism coming from the interplay of the two margins and the general equilibrium feedback effect of asset returns. It provides a novel way to model the risk-taking channel of monetary policy analysed in Borio and Zhu (2012), Challe, Mojon and Ragot (2013)[7], Angeloni, Faia and Lo Duca (2015), Bruno and Shin (2015a) and Acharya and Plantin (2016).[8]

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[5]Baron and Xiong (2017) show that creditors of banks do not price the risk taken by bankers during credit expansions. Deposit guarantees have also the effect of ruling out bank runs in our framework. For models focusing on runs see Diamond and Dybvig (1983), Gertler and Kiyotaki (2015) and Angeloni and Faia (2013)). Kareken and Wallace (1978) point out that an important side effect of deposit insurance is excessive risk taking.

[6]Our model is about the behaviour of the real interest rate so the connection with monetary policy is only partial. Any change in regulation that affects funding costs would have similar implications. So would higher savings rates or large capital inflows. An extension of the model featuring nominal variables is left for future work.

[7]Challe, Mojon and Ragot (2013) describe a two-period model with heterogeneous intermediaries and limited liability which, like ours, features a link between interest rates and systemic risk. They focus on portfolio choice and heterogeneity in equity of intermediaries while we emphasize aggregate uncertainty and differences in risk taking. Unlike them, we embed the financial sector in a DSGE model. For a model of the deposit channel of monetary policy see Drechsler, Savov and Schnabl (2017).

[8]Recent empirical evidence on the risk-taking channel of monetary policy has been provided by Dell'Ariccia, Laeven and Suarez (2017) on US data, Jimenez et al. (2014) and Morais et al. (2019), exploiting registry data on millions of loans of the Spanish and Mexican Central Banks respectively. Using detailed Turkish data, Baskaya et al. (2017) highlight the importance of bank heterogeneity for credit creation and the transmission of global financing cost shocks. Coimbra and Rey (2018) show

A few papers have, like us, put their emphasis on the boom phase of the financial cycle. Lorenzoni (2008) shows that credit booms can be inefficient due to a pecuniary externality working through asset prices. In Martinez-Miera and Suarez (2014), bankers determine their exposure to systemic shocks by trading-off the risk-shifting gains due to limited liability with the value of preserving their capital after a crisis. Malherbe (2015) and Gersbach and Rochet (2017) present models with excessive credit during economic booms as increased lending by an individual bank exerts a negative externality on all other banks. Martinez-Miera and Repullo (2017) analyse "search for yield" in an environment with safe and risky entrepreneurs and banks (with no equity) facing a moral hazard friction a la Holmstrom and Tirole (1997). In their model, riskier entrepreneurs endogenously borrow from monitoring banks while safer entrepreneurs borrow from non-monitoring banks (shadow banks).[9] A small set of papers have analysed financial sectors with heterogeneous agents. Geanakoplos (2010) and Fostel and Geanakoplos (2012) study leverage cycles driven by wealth reallocations between optimists and pessimists. Kaplan, Mitman and Violante (2017) focus on household heterogeneity and beliefs to explain housing booms while Berger et al. (2018) shows the importance of heterogeneity across households to understand the transmission of changes in house prices to consumption. Kekre and Lenel (2020) emphasize the heterogeneity in household's marginal propensity to take risk to explain the stock market response to monetary policy shocks. Boissay, Collard and Smets (2016) features intermediaries that are heterogeneous in their intermediation skills. Incentives to divert funds in the interbank market lead to borrowing constraints that are a function of the pool of borrowers.[10] When returns on corporate loans fall, demand for funds is reduced and the interbank rate falls. More banks choose to give corporate loans rather than interbank ones. The selection in the pool of borrowers worsens, leading to a tightening of constraints which further reduces demand for interbank funds. This feedback may

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that in a cross section of countries, credit creation tends to be more elastic to decline in funding costs when the leverage distribution of the banking system is concentrated.

[9]Begenau and Landvoigt (2017) present a quantitative model of banks and shadow banks to analyse optimal banking regulation.

[10]But there is no heterogeneity due to asymmetric information, so all borrowing banks contract the same amount from interbank markets. Guerrieri and Uhlig (2016) present a model where, due to adverse selection, worse borrowers take loans when costs of funds are low, which may induce a crash of the credit market.

be strong enough to generate a freeze in the interbank market. In our paper, the heterogeneity of leverage elasticities with respect to changes in the cost of funds means that monetary policy has both a selection effect (how risky are levered intermediaries) and a composition effect (how are assets distributed across these intermediaries). This channel affects aggregate risk-taking in a setting with risk-shifting and default in equilibrium. Korinek and Nowak (2017) is closely related to our work but uses a very different modelling approach. Like us, the authors emphasize that heterogeneity in the financial sector drives the dynamics of aggregate risk. They use evolutionary dynamics to characterize the distribution of bankers' wealth: good shocks raise the fraction of wealth controlled by high risk takers in an incomplete market environment. They do not study the interaction between low rates and financial stability. On the empirical side, our work relates to recent work by Koijen and Yogo (2019) who test models where heterogeneity across institutional investors is an important driver of asset pricing.

Our model of financial intermediation has several advantages. First, it embeds an endogenous risk-taking channel in general equilibrium and allows to study the usual expansionary effect of monetary policy jointly with its effect on financial stability. Second, it is able to generate periods of low risk premium which coincide with periods of high endogenous macroeconomic risk (bad booms). The model also generates credit booms driven by high expected productivity, which do not increase financial fragility (good booms). Third, the model opens the door to a vast array of empirical tests based on microeconomic data on different classes of financial intermediaries. Indeed, heterogeneity in risk taking can in principle be matched to data on the leverage behavior of financial intermediaries or business lines within them.[11]

Section 2 provides stylized facts on the heterogeneous behaviour of leverage in the cross-section of intermediaries. Section 3 describes the model. Section 4 presents the main results in partial equilibrium to build intuition. Section 5 shows the general equilibrium results and responses to monetary policy and productivity shocks. The effect of large negative productivity shocks and the case of financial crises with costly intermediary default is analyzed in section 6. Section 7 concludes.

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[11]Our model attempts to perform in macro-finance something similar to what Melitz (2003) has done in international trade by relating aggregate outcomes to underlying microeconomic heterogeneity. We are not aware of any other paper in the macro-finance literature that pursues a similar aim.

2 Stylized facts on the cross-section of intermediary balance sheets

In this section we present some stylized facts on the cyclical properties of the cross-section of financial intermediary balance sheets. We use balance sheet data of financial intermediaries from Bankscope (see Appendix D) to compute leverage at the intermediary level. Leverage is defined as the ratio of assets over equity at book value, a definition that will be kept for the theoretical model of the following sections.
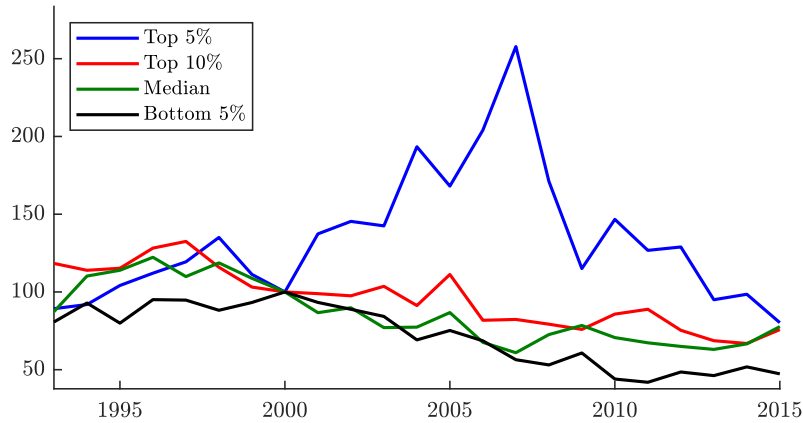
Figure 1: Evolution of leverage by quantiles (base year=2000)

In Figure 1 we show the time series of asset-weighted leverage for different quantiles of the leverage distribution, namely the top 5% (blue), top 10% (red), median (green) and bottom 5% (black). Values were rebased to 100 for the year 2000, to highlight the stark differences in dynamics and were weighted by assets to give a meaningful relevance to larger institutions. In the years that preceded the financial crisis there was a strong increase in leverage in the top 5% (leverage is multiplied by 2.5 between 2000 and 2008) but not in the other quantiles. If anything, these other quantiles had been trending downward until the crisis. There is a strong negative pre-crisis correlation between the top 5% and both the median (-0.77) and the bottom 5% (-0.84), while the correlation is positive between the median and the bottom 5% (0.74). The quantiles

are weighted by balance sheet size so the graph highlights that it was the large, highly levered intermediaries that were increasing leverage the most during the pre-crisis period. Figure 11 in the Appendix shows similar patterns for unweighted leverage. The only difference is that now both the Top 5 and 10% are increasing their leverage before the crisis, highlighting that some smaller institutions (though still in the top 10%) also increased leverage significantly before the crisis. The lower quantiles exhibit a similar behaviour as in Figure 1.
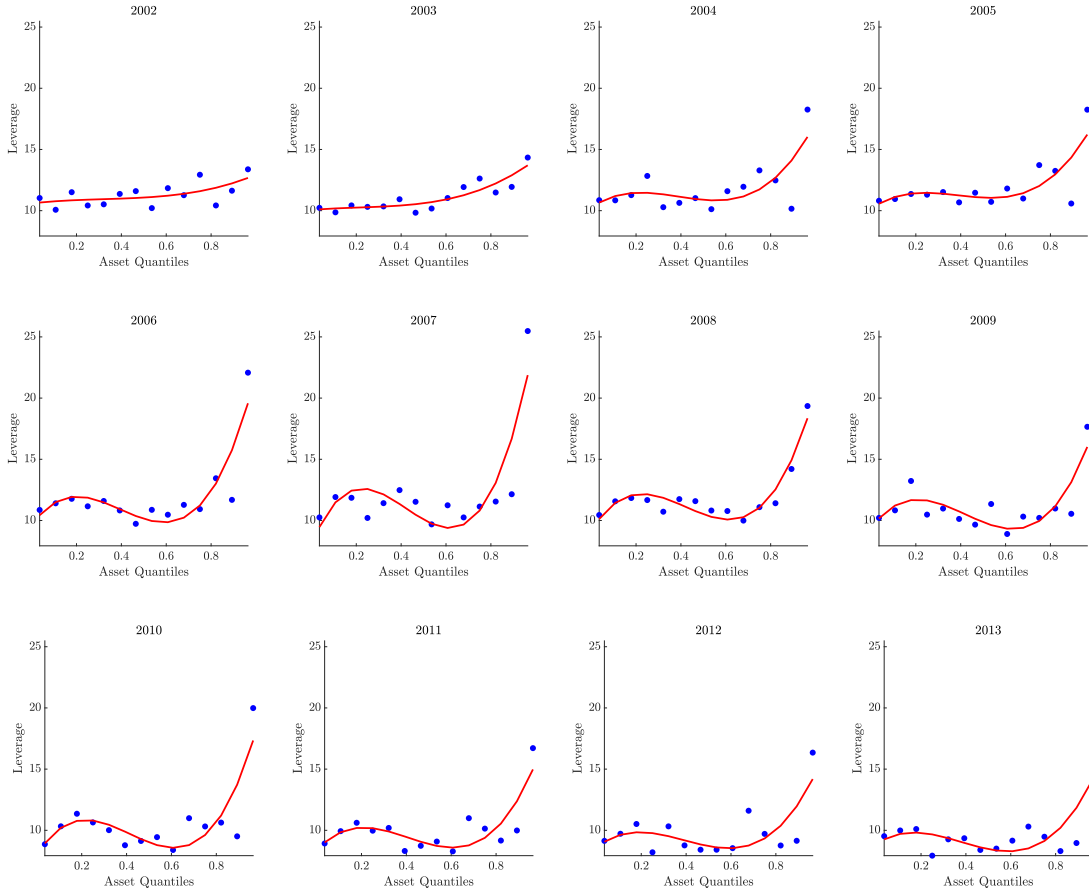

Figure 2: Binned scatter plot of leverage on asset quantiles (cubic fit)

In Figure 2, we show binned scatter plots of leverage as a function of asset quantiles for the years between 2002 and 2013. Each bin contains roughly 30 intermediaries and we plot the median leverage for each bin. We also plot in red a cubic fit to the points

plotted. Leverage is increasing in asset quantile, meaning that the larger intermediaries also tend to be the most levered. Moreover, as the cubic fit reveals, leverage is an increasing and convex function of balance sheet size and this pattern is present in every year of our data set as shown in Figure 17 in the Appendix. Strikingly, the convexity of leverage with respect to size increases significantly in the pre-crisis period to culminate around 2007. This period is characterized by low costs of funds due to abundant liquidity and light regulation. After the crisis and an increase in regulation (Basel 3), the convexity abates with a slight increase at the end of the sample. Hence leverage is not only largest for the bigger intermediaries, but it is also more reactive over the cycle.

Stylized fact 1: *Figures 1 and 2 show that leverage dynamics is heterogeneous across intermediaries. Leverage is a convex function of balance sheet size and the sensitivity of leverage to the cycle is higher for larger, more leveraged intermediaries.*

To illustrate the convexity of the leverage distribution over the cycle, we compute the share of assets of the top 5% most levered intermediaries in total intermediaries' assets. Figure 3, is a scatter-plot of that share (top 5%) on the vertical axis with the Effective Fed Funds Rate both in real terms (left panel) and in nominal terms (right panel) on the horizontal axis. Each point is a yearly observation. There is a strong negative correlation between the Effective Fed Funds Rate and the top 5% share. There are some outliers, in particular the three points which are above the regression lines (share of top 5% above 65% of total assets despite relatively high real or nominal rates). They correspond to the years 2006, 2007 and 2008, immediately before the crisis. One possible interpretation is that the real cost of funds in those years declined more than the Fed Funds rate proxy would suggest due to the substantial increase in the use of very short term repo markets (overnight repo).[12]

The evidence presented above indicates that here is a strong heterogeneity within the financial sector in terms of correlation of leverage and interest rate over time. Up to 2007, the correlation between the top 5% and the real effective Fed Funds rate is -0.34 but positive for the median (0.58) and bottom 5% (0.41).[13] To investigate this

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[12]We also computed the top 5% share using only US bank data, results were very similar.

[13]Results are qualitatively very similar if we use instead nominal rates. The correlation between the
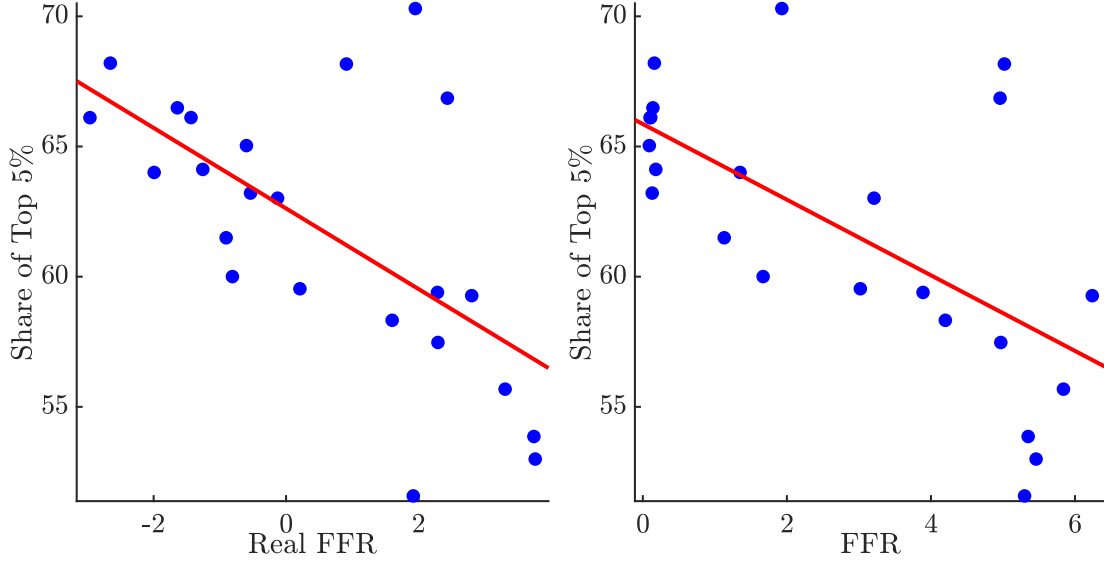
Figure 3: Share of assets of the top 5% most levered intermediaries in total intermediaries' assets and the real Effective Fed Funds Rate (left panel) and the nominal effective Fed Funds rate (right panel) in pp.

further, we look at the relationship between individual bank leverage and interest rates, allowing for different responses across the distribution. We run the following baseline panel regression:

$$Lev_{i,t} = \beta_0 + \beta_1 Lev_{i,t-1} + \beta_2 FF_t + \beta_3 Top5_{i,t} + \beta_4 FF_t \times Top5_{i,t} + \alpha_i + \varepsilon_{i,t} \quad (1)$$

$Top5$ is a dummy variable that takes the value 1 if the intermediary is in the top 5% by asset-weighted leverage. These are large, highly levered intermediaries. FF is the real Fed Funds Rate. β_1 measures persistence of leverage and β_2 the response of leverage to interest rates. β_3 picks up the average difference in leverage in the two groups. Our object of interest is β_4, which captures the heterogeneity of the response of leverage to interest rates for the top 5%. Results can be seen in Table 2. The first column is the baseline specification as in Equation (1). All regressions have financial intermediary fixed effects. As expected, there is significant persistence in leverage as β_1 is positive

top 5% and the nominal effective Fed Funds rate is -0.20 and again positive for the median (0.49) and bottom %5 (0.24).

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and highly significant. Also, leverage is significantly larger on average for the $Top5$ group. The coefficient of the real Fed Funds Rate is not significant when $Top5 = 0$ but is highly significant and negative when $Top5 = 1$. That is, leverage and interest rates are negatively correlated for large, highly leveraged intermediaries, but this is not the case for other intermediaries. The following columns show the robustness of this relation. In the second column we add the top 10% and the median; the negative interaction with the fed funds rate is only significant for the top 5%. In the third column we run the specification in first-differences and again we have a negative and signifiant interaction coefficient. Falls in interest rates strongly correlate with increases in leverage for the top 5% group but not for the other intermediaries. In the last three columns, we add time fixed effects to control for other possible macroeconomic confounding effects. The results remain robust, with coefficients of the interaction terms remaining highly significant.[14]

Stylized fact 2: *Figure 3 and Table 2 show that there is a negative correlation between the share of assets of the top 5% most levered intermediaries and the cost of funds as proxied by the real or the nominal effective Fed Funds rate. Concentration of leverage increases as costs of funds go down.*

These stylized facts highlight the presence of strong heterogeneity in the dynamics of the cross-section of leverage. In the next section we present a model that is able to rationalize these new facts. Heterogeneity is a first order determinant of the dynamics of aggregate risk and of the macroeconomy in the model.

3   The Model

The general equilibrium model is composed of a representative risk-averse household who faces an intertemporal consumption saving decision, a continuum of risk-neutral, heterogeneous financial intermediaries, and a stylized central bank and government. There is aggregate uncertainty, in the form of productivity and monetary policy shocks. Given the heterogeneity in bank balance sheets, this will lead to heterogeneity in default risk in the intermediation sector. Default is costly (Section 6).

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[14]The same results hold for the nominal effective Fed Funds rate.

3.1 Households and the production sector

The representative household has an infinite horizon and consumes a final good C_t^H. She finances her purchases using labour income W_t and returns from a savings portfolio. We assume that the household has a fixed labour supply and does not invest directly in the capital stock K_t.[15] She can either save using a one-to-one storage technology S_t^H and/or deposit D_t^H with financial intermediaries at interest rate r_t^D. The return on deposits $R_t^D \equiv 1 + r_t^D$ is risk-free and guaranteed by the government. Intermediaries use deposits, along with inside equity ω_{it}, to invest in capital and storage. In Section 5 we will introduce monetary policy as a source of wholesale funding. Monetary policy will therefore affect the weighted average cost of funds for intermediaries.

The production function combines labour and capital in a typical Cobb-Douglas function. Since labour supply is fixed, we normalize it to 1. Output Y_t is produced according to the following technology:

$$Y_t = Z_t K_{t-1}^\theta L_t^{1-\theta} \tag{2}$$

where Z_t represents total factor productivity and θ the capital share of output. Given $L_t = 1$, in equilibrium firm maximization implies that wages $W_t = (1 - \theta)Z_t K_{t-1}^{\theta-1}$. We will introduce some idiosyncratic risk to financial intermediation, so the return on a unit of capital will be intermediary specific $R_{it}^K = \theta Z_{it} K_{t-1}^{\theta-1} + (1 - \delta)$ (more on this later).

The household program can be written as follows:

$$\max_{\{C_t, S_t^H, D_t^H\}_{t=0}^\infty} E_0 \sum_{t=0}^\infty \beta^t u(C_t^H) \qquad \text{s.t.} \tag{3}$$

$$C_t^H + D_t^H + S_t^H = R_t^D D_{t-1}^H + S_{t-1}^H + W_t - T_t \qquad \forall t \tag{4}$$

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[15]Given households are risk-averse and intermediaries are risk neutral (and engage in risk-shifting), relaxing the assumption households cannot invest directly would make no difference to their portfolio in equilibrium unless all intermediaries are constrained. There are also little hedging properties in the asset, since the correlation of the shock to returns with wage income is positive. In the numerical exercises it is never the case that all intermediaries are constrained as some choose not to leverage, so to simplify notation and clarify the household problem, we assume directly that only intermediaries can invest in the risky capital stock.

where β is the subjective discount factor and $u(.)$ the period utility function. T_t are lump sum taxes and S_t^H are savings invested in the one-to-one storage technology. Note that the return on deposits is risk-free despite the possibility of intermediary default. The reason is that deposits are guaranteed by the government, which may need to raise taxes T_t in the event intermediaries cannot cover their liabilities. Households understand that the higher the leverage of intermediaries, the more likely it is for them to be taxed in the future. However, they do not internalize this in their individual portfolio decisions since each household cannot by itself change aggregate deposits nor the expectation of future taxes.

The return on storage is also risk-free, which implies that households will be indifferent between deposits and storage if and only if $R_t^D = 1$. Therefore, they will not save in the form of deposits if $R_t^D < 1$ and will not invest in storage if $R_t^D > 1$. In equilibrium, the deposit rate will be bounded from below by the unity return on storage, implying that $R_t^D \geq 1$. In the case $R_t^D = 1$, the deposit quantity will be determined by financial intermediary demand, with the remaining household savings being allocated to storage.

3.2 Financial intermediaries

The financial sector is composed of two-period financial intermediaries which fund themselves through inside equity and household deposits[16]. They use these funds to invest in the aggregate risky capital stock and/or in the riskless one-to-one storage technology. Intermediaries are risk neutral agents who maximize expected second period consumption subject to a Value-at-Risk (VaR) constraint. They also benefit from limited liability. To capture the diversity of risk attitudes among financial intermediaries, we assume that they are heterogeneous in α^i, the maximal probability their return on equity is negative according to their VaR constraint. α^i is exogenously given and is the key parameter in the VaR constraint. This probability varies across intermediaries and is continuously distributed according to the measure $G(\alpha^i)$ with $\alpha^i \in [\underline{\alpha}, \overline{\alpha}]$.

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[16]We will extend the funding options to include wholesale funding, whose cost is influenced by monetary policy, in section 5. The economy in our benchmark case does not feature an interbank market or other funding possibilities. We relax this assumption and allow for interbank market in Appendix E. Qualitative results are unchanged.

The balance sheet of intermediary i at the end of period t is as follows:

Assets	Liabilities
k_{it}	ω_{it}
s_{it}	d_{it}

where k_{it} are the shares of the aggregate capital stock held by intermediary i, s_{it} the amount of storage held, d_{it} the deposit amount contracted at interest rate r_t^D, and ω_{it} the inside equity. At the beginning of the next period, aggregate and idiosyncratic shocks are revealed and the net cash flow $\pi_{i,t+1}$ is:

$$\pi_{i,t+1} = R_{i,t+1}^K k_{it} + s_{it} - R_t^D d_{it} \tag{5}$$

Intermediary return on capital $R_{i,t+1}^K$ is risky and depends on the ex-post productivity of the capital held by the intermediary. It features an idiosyncratic and an aggregate productivity component. With probability ζ, the intermediary is hit by a negative idiosyncratic shock and its capital fails to produce anything, although it still recovers undepreciated capital at $t+1$. With probability $(1 - \zeta)$ it is not hit by the negative idiosyncratic shock.[17] We can then describe idiosyncratic returns $R_{i,t+1}^K$ as follows:

$$R_{i,t+1}^K = \begin{cases} 1 - \delta & \text{with probability} \quad \zeta \\ \theta \tilde{Z}_{t+1} K_t^{\theta-1} + (1 - \delta) & \text{with probability } 1 - \zeta \end{cases} \tag{6}$$

where $\tilde{Z}_t$ is the aggregate component and can be interpreted as the productivity of capital conditional on no idiosyncratic shock. ζ is a measure of idiosyncratic risk. The aggregate component follows a simple AR(1) process in logs

$$\log \tilde{Z}_{t+1} = (1 - \rho^z)\mu_z + \rho^z \log \tilde{Z}_t + \varepsilon_{t+1}^z \tag{7}$$

$$\varepsilon_{t+1}^z \sim N(0, \sigma_z) \tag{8}$$

ε_t^z is the shock to the log of exogenous productivity (conditional on no idiosyncratic shock) with persistence ρ^z and standard deviation σ_z. μ_z is a scaling parameter such

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[17]We can think of ζ as an operational risk shock. It is mainly introduced for computational purposes in order to ensure that the lowest (positive) probabilities of default of leveraged intermediaries are never numerically indistinguishable from zero.

that $E(Z) = E(\tilde{Z}(1 - \zeta)) = 1$. Let $F(\epsilon_t^z)$ be the cumulative distribution function (cdf) of $\exp(\epsilon_t^z)$, a notation which will be convenient later. Expected return on capital will be equal across intermediaries and we define $E\left[R_{t+1}^K\right] \equiv E\left[R_{i,t+1}^K\right]$. Differences in willingness to pay for shares of the capital stock will however arise in the presence of heterogeneous default risk and limited liability, generating an intermediary-specific option value of default.

3.2.1 Value-at-Risk constraint

Financial intermediaries are assumed to be constrained by a *VaR* condition. This constraint imposes that intermediary i invests in such a way that the probability its return on equity is negative must be smaller than an exogenous intermediary-specific parameter α^i.[18] The *VaR* constraint for intermediary i can then be written as:

$$\Pr(\pi_{i,t+1} < \omega_{it}) \leq \alpha^i \tag{9}$$

The probability that the net cash flow is smaller than starting equity ω_{it} must be less or equal than α^i. This constraint follows the spirit of the Basel Agreements, which aim at limiting downside risk and preserving an equity cushion. Furthermore, Value-at-Risk techniques are used by banks and other financial intermediaries (for example asset managers) to manage risk internally. When binding, it also has the property of generating procyclical leverage, which can be observed in the data for some intermediaries as described in Geanakoplos (2011) and Adrian and Shin (2014) when equity is measured at book value. Using a panel of European and US commercial and investment banks Kalemli-Özcan, Sorensen and Yesiltas (2012) also provide evidence of procyclical leverage while emphasizing cross-sectional variations across types of intermediaries.

Heterogeneity in the parameter of the *VaR* constraint can be rationalized in different ways. It could be understood as reflecting different risk management practices or differentiated implementation of regulatory requirements by different supervisors. For

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[18]Alternatively we could posit that the threshold is at a calibrated non-zero return on equity. There is a mapping between the distribution $G(\alpha^i)$ and such a threshold, so for any value we could find a $\tilde{G}(\alpha^i)$ that would make the two specifications equivalent given expected returns. We decide to use the current one as it reduces the parameter space.

example, the Basel Committee undertook a review of the consistency of risk weights used when calculating how much capital a sample of banks put aside for precisely defined portfolio. When given a diversified test portfolio the banks surveyed produced a wide range of results in terms of modelled *VaR* and gave answers ranging from 13 million to 33 million euros in terms of capital requirement with a median of about 18 million (see Basel Committee on Banking Supervision (2013) p.52). Some of the differences are due to different models used, some to different discretionary requirements by supervisors and some to different risk appetites, as "Basel standards deliberately allow banks and supervisors some flexibility in measuring risks in order to accommodate for differences in risk appetite and local practices" (p.7). In the data, leverage is highly heterogeneous in the cross-section of financial intermediaries as can be seen in the descriptive statistics in Table 3 of appendix D.

3.2.2 Intermediary investment problem

We assume that the risk neutral intermediaries live for two periods, receive a constant endowment of equity $\omega_{it} = \omega$ in the first and consume their net worth in the second. This assumption of constant equity is a simplifying assumption but we find that book value equity is indeed very sticky in the data. We show in Figure 13 the almost one-for-one correlation between changes in the size of debt and assets at book value, for a very broad sample of banks using Bankscope data. Figure 13 also shows the stickiness of book value equity relative to assets and debt. Balance sheet expansions and contractions tend to be done through changes in debt and not through movements in equity. Krishnamurthy and Vissing-Jorgensen (2015) present remarkable evidence on the time series of bank long-term assets, short-term debt and equity as a percentage of GDP for the US. We replicate their findings and show in Figure 14 a strong correlation between long-term assets and short term debt (0.994) and a far smaller one between equity and assets (0.283). In addition, if we detrend the series, the correlations are, respectively, 0.972 and -0.02174 so still very high for assets and debt but virtually zero between equity and assets. Furthermore the magnitudes of long term assets and short term debt are comparable throughout, highlighting the central role of leverage in funding investment in the economy. The macro-finance literature often focuses on the dynamics of net worth, assuming a representative agent (see e.g. Gertler and

Kiyotaki (2015)), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2014), Jermann and Quadrini (2012)) and abstracting from the cross sectional differences in intermediaries. We take a complementary approach. To highlight the novel nature of our mechanism, we instead assume constant equity, thus abstracting from the net worth channel and putting a sharp focus on the effects generated by the heterogeneous dynamics of leverage in the cross-section.

When the net cash flow $\pi_{i,t}$ is positive, it is consumed by financial intermediary i and we denote its consumption by c_{it}.[19] When the net cash flow is negative, $c_{it} = 0$ and the intermediary defaults. Government steps in to repay depositors as it upholds deposit insurance. This is a pure transfer, funded by a lump sum tax on households. Hence, in our model, households are forward-looking and do intertemporal optimization while most of the action in the intermediation sector comes from heterogenous leverage and risk taking in the cross-section. This two-period modeling choice is made for simplicity and allows us to focus on the role of different leverage responses across financial intermediaries[20].

Each intermediary has to decide whether it participates or not in the market for risky assets or invests in the storage technology (*participating intermediary* versus *non-participating intermediary*) and, conditionally on participating, whether it uses deposits to lever up (*risky intermediary*) or just invests its own equity (*safe intermediary*). Note that this label of *risky* or *safe* is based on the possibility (or not) of defaulting on lenders, not in terms of the volatility of their return on assets or equity. These will only be risk free for *non-participating* ones, which invest only in storage. In Appendix E, we show that an alternative model where intermediaries can choose to lend to each other as an outside option has very similar implications.[21]

Intermediaries are assumed to be (constrained) risk-neutral price takers, operating in a competitive environment. Each maximizes consumption over the next period by picking k_{it} (investment in risky assets) and s_{it} (investment in the storage technology), under its *VaR* constraint, while taking interest rates on deposits R_t^D and asset return

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[19]When intermediary j is inactive, then $c_{jt} = \omega$ as the return of the storage technology is one.

[20]Other papers in the literature have used related assumptions, for example exogenous death of intermediaries in Gertler and Kiyotaki (2015).

[21]In Appendix E, we consider a standard centralized market for intermediary borrowing. For a model of financial stability issues arising from banking networks see Aldasoro, Gatti and Faia (2017).

distributions $R_{t+1}^K(\varepsilon)$ as given. The program of each intermediary i is given by:

$$V_{it} = \max \; \mathbb{E}_t(c_{i,t+1}) \tag{10}$$

$$\text{s.t.} \;\; \Pr(\pi_{i,t+1} < \omega_{it}) \leq \alpha^i \tag{11}$$

$$k_{it} + s_{it} = \omega_{it} + d_{it} \tag{12}$$

$$c_{i,t+1} = \max\left(0, \pi_{i,t+1}\right) \tag{13}$$

$$\pi_{i,t+1} = R_{i,t+1}^K k_{it} + s_{it} - R_t^D d_{it}$$

where α^i is the *VaR* threshold (the maximum probability of not being able to repay stakeholders fully) and $\pi_{i,t+1}$ the net cash flow.

Intermediaries can choose not to lever up ($d_{it} = 0$) or even stay out of capital markets and not participate ($k_{it} = 0$). In this case, they have the outside option of investing all their equity in the storage technology and collect it at the beginning of the next period. The value function of a non-participating intermediary investing in the outside option is:

$$V_{it}^O = V^O = \omega \tag{14}$$

3.2.3 Limited liability

The presence of limited liability truncates the profit function at zero, generating an option value of default that intermediaries can exploit. For a given expected value of returns, a higher variance increases the option value of default as intermediaries benefit from the upside but do not suffer from the downside. For a given choice of k_{it} and d_{it} we have that:

$$\mathbb{E}_t\left[\max(0, \pi_{i,t+1})\right] \geq \mathbb{E}_t\left[\pi_{i,t+1}\right] \tag{15}$$

with the inequality being strict whenever the probability of default is strictly positive. Deposit insurance transfers t_t^i happen when the net cash flow is negative and are given by:

$$t_{t+1}^i = \max\left(0, -\pi_{t+1}\right) \tag{16}$$

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The max operator selects the appropriate case depending on whether intermediary i can repay its liabilities or not. If it can, then deposits repayments are lower than return on assets and deposit insurance transfers are zero. Total intermediary consumption C_t^I and aggregate transfers/taxes T_t are given by integrating over the mass of intermediaries:

$$C_t^I = \int c_{it} \, dG(\alpha^i) \qquad (17)$$

$$T_t = \int t_t^i \, dG(\alpha^i) \qquad (18)$$

For now we assume default is costless in the sense that there is no deadweight loss when the government is required to pay deposit insurance. In section 6, we will drop the assumption of costless default by having a more general setup that allows for a lower return on assets held by defaulting intermediaries.

3.3   Investment strategies and financial market equilibrium

Financial intermediaries are price takers, therefore the decision of each one depends only on the expected return on assets (taking into account limited liability) and the cost of liabilities. Since the mass of each intermediary is zero, individual balance sheet size does not affect returns on the aggregate capital stock. Intermediary i will be a *participating intermediary* in the market for risky assets whenever $V_{it} \geq V^O$. This condition determines entry and exit into the market for risky capital endogenously.

There is however another important endogenous decision. Intermediaries which participate in the market for risky assets have to choose whether to lever up and, if they do, by how much. We will refer to the decision to lever up or not, i.e. to enter the market for deposits as the *extensive margin*. We will refer to the decision regarding how much to lever up as the *intensive margin*. Financial intermediaries which lever up are *risky* intermediaries. Financial intermediaries which participate in the market for risky capital but do not lever up are *safe* intermediaries.

Proposition 3.1 *When $\mathbb{E}[R_{t+1}^K] \geq 1$, participating intermediary i will either lever up to its VaR constraint or not raise deposits at all.*

Proof: See Appendix B.

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Proposition 3.1 states that if the return to risky capital is higher in expectation than the return on the storage technology then whenever an intermediary decides to lever up, it will do so up to its *VaR* constraint and will not invest in storage. Hence all *risky* intermediaries will be operating at their constraint.

When expected return on risky capital is smaller than return on storage: $\mathbb{E}[R_{t+1}^K] < 1$, it might still be the case that capital is preferred to storage in equilibrium by some intermediaries due to limited liability. We would then have equilibria in which some intermediaries invest in storage and possibly some of the most risk-taking ones leverage up a lot taking advantage of the option value of default. In what follows we focus on cases where $\mathbb{E}[R_{t+1}^K] \geq 1$ which is always the case in our simulations.

3.3.1 Intensive margin and endogenous leverage

Let $Z_{t+1}^e \equiv \mathbb{E}_t(\tilde{Z}_{t+1})$, an expectation known at t. For a participating intermediary i deciding to lever up, the *VaR* condition will bind (see Proposition 3.1):

$$\Pr\left[\pi_{t+1}^i \leq \omega\right] = \alpha^i \tag{19}$$

Hence, after some straightforward algebra, we obtain the following:

$$\zeta + (1-\zeta)\Pr\left[e^{\varepsilon_{t+1}^z} \leq \frac{r_t^D + \delta - \frac{\omega}{k_{it}}r_t^D}{\theta Z_{t+1}^e K_t^{\theta-1}}\right] = \alpha^i \tag{20}$$

The leverage λ_{it} of an active intermediary is given by:

$$\lambda_{it} \equiv \frac{k_{it}}{\omega} = \frac{r_t^D}{r_t^D - \theta Z_{t+1}^e K_t^{\theta-1} F^{-1}\left(\frac{\alpha^i - \zeta}{1-\zeta}\right) + \delta} \tag{21}$$

where we defined leverage as assets over equity and F^{-1} as the inverse cdf of the technology shock $e^{\varepsilon_{t+1}^z}$ evaluated at probability $\frac{\alpha^i - \zeta}{1-\zeta}$. Note that intermediaries with $\alpha^i < \zeta$ will never participate.

Let $r_t^{\alpha^i} \equiv \theta Z_{t+1}^e K_t^{\theta-1} F^{-1}\left(\frac{\alpha^i - \zeta}{1-\zeta}\right) - \delta$ be the ex-post return on capital for which the return on equity of *risky* intermediary α^i is zero. The expression above can then be

simply written as:

$$\lambda_{it} = \frac{r_t^D}{r_t^D - r_t^{\alpha^i}} \tag{22}$$

This expression for leverage is only true when the constraint is binding for *risky* intermediary α^i. In equilibrium, decreasing marginal returns to K ensure that the denominator is always positive. Otherwise the constraint would not be binding and *risky* intermediaries would increase K, which in turn would reduce $r_t^{\alpha^i}$.

Proposition 3.2 *For a participating intermediary i, the leverage λ_{it} has the following properties: it is increasing in α^i, decreasing in the cost of funds r_t^D and increasing in expected marginal productivity of capital $\theta Z_{t+1}^e K_t^{\theta-1}$. Furthermore, $\frac{\partial^2 \lambda_{it}}{\partial (r_t^D)^2} > 0$ and $\frac{\partial^2 \lambda_{it}}{\partial r_t^D \partial \alpha^i} < 0$.*

Proof: Immediate from Equation (21) and given the monotonicity of the cdf and the shape of $F^{-1}()$.

Proposition 3.2 implies that, from the perspective of a participating individual intermediary (i.e. absent general equilibrium effects on K_t), leverage will be decreasing in the cost of funds r_t^D. For a given balance sheet size, decreasing the cost of liabilities increases expected *net* cash flows and thus decreases the probability of distress. From 3.1, intermediaries would then choose to increase leverage until their probability of distress hits the VaR constraint. Furthermore, when interest rates are low the probability of default is lower *ceteris paribus*. In that region, the pdf is flatter therefore increases in leverage translate into small increases in probability of distress. This means that intermediaries can increase leverage by sizable amounts until they hit the *VaR* constraint. So the lower r_t^D, the stronger the intensive margin effect. Similarly for high α (looser constraints) leverage can be increased a lot before the constraint is hit. Therefore the leverage of the most risk-taking intermediaries will react more to interest rate changes. This heterogeneity of the *intensive margin* to changes in the cost of funds means that as interest rates fall, the more skewed will be the distribution of leverage in the cross-section. This generates a composition effect, where the proportion of assets being held by the more risk-taking intermediary rises. Since the intensive margin effect is larger the lower are interest rates to begin with, it follows that this composition effect is particularly strong at low levels of interest rates.

Generally, intermediary leverage will also be decreasing in the volatility of the productivity shocks σ_z. This will be true whenever $F^{-1}\left(\frac{\alpha^i - \zeta}{1 - \zeta}\right)$ is increasing in σ_z, implying realistically that the probability of a negative return on equity is (ceteris paribus) increasing in the volatility of returns.

3.3.2 Extensive margin and endogenous leverage

We now focus on the *extensive margin*, that is to say whether intermediaries who participate in risky capital markets choose to lever up using deposits or not.[22]

Let V^L denote the value function of *risky* intermediaries who decide to lever up using deposits and V^N the value function of the *safe* ones who only invest at most their equity in the risky capital stock.

$$V_{it}^L = \mathbb{E}_t \left[\max\left(0, R_{i,t+1}^K k_{it} - R_t^D d_{it}\right)\right] \tag{23}$$

$$V_{it}^N = \mathbb{E}_t[R_{i,t+1}^K] k_{it}^N + \omega - k_{it}^N \tag{24}$$

with $k_{it}^N \in \{0, \omega\}$ and the max operator in V^L being the effect of limited liability. Since there is no risk of defaulting on deposits if you have none, there is no option value of default for non-levered intermediaries. This N group could in principle also include intermediaries who invest only a fraction of their equity in the capital stock. Given our choice of *VaR* constraint, *safe* intermediaries will either invest ω in the capital stock or not at all.[23]

We can then use the condition $V_{it}^L = V_{it}^N$ to find the cut-off value $\alpha_t^L = \alpha_t^j$ for which intermediary j is indifferent between leveraging up or not. Above α_t^L (looser *VaR* constraints), all intermediaries will be levered up to their respective constraints and do not invest in storage as shown in Proposition 3.1. For any levered intermediary i, we

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[22]Intermediaries can also decide not to invest in risky capital markets and instead to use the storage technology. If they do so, then their value function is $V^O = \omega$ given the unit return to storage and linear utility.

[23]Note that the *VaR* condition of a *safe* intermediary can be written as $\Pr\left(e^{\varepsilon_{t+1}} < \frac{\delta K^{1-\alpha}}{\theta Z_{t+1}^e}\right) \leq \frac{\alpha^i - \zeta}{1 - \zeta}$. Since this is not a function of k_{it}, the inequality will either be true and the intermediary will invest up to ω, or it won't and he cannot invest any amount in the capital stock without violating it. Note also that the inequality is always false for $\alpha^i < \zeta$.

have:

$$\mathbb{E}\left[\max\left(0, k_{it}R_{i,t+1}^K - R_t^D d_{it}\right)\right] \geq \omega\, \mathbb{E}_t\left[R_{t+1}^K\right] \tag{25}$$

where the left hand side is the expected payoff on the assets of intermediary i and the right hand side is the expected payoff when it invests only its equity ω in capital markets. Using the balance sheet equation $k_{it} = d_{it} + \omega$, we can substitute for deposits, which leads to the following condition:

$$\mathbb{E}_t\left[\max\left(0, k_{it}\left(R_{i,t+1}^K - R_t^D\right) + R_t^D\omega\right)\right] \geq \omega\, \mathbb{E}_t\left[R_{t+1}^K\right] \tag{26}$$

For the marginal intermediary j, equation (26) holds with equality:

$$\mathbb{E}_t\left[\max\left(0, k_{jt}\left(R_{j,t+1}^K - R_t^D\right) + R_t^D\omega\right)\right] = \omega\, \mathbb{E}_t\left[R_{t+1}^K\right] \tag{27}$$

Since all *risky* intermediaries will be at the constraint, we can combine equation (27) with equation (21) evaluated at the marginal intermediary (whose *VaR* parameter is α_t^L). Moreover, $\mathbb{E}_t\left[R_{t+1}^K\right]$ is a function of Z_{t+1}^e and K_t but is independent of i. Therefore equation (27) and equation (21) jointly define an implicit function of the threshold *VaR* parameter $\alpha_t^L \, (= \alpha^j)$ with variables (r_t^D, Z_{t+1}^e, K_t).

Hence we have the following result:

Proposition 3.3 *There exists a cut-off value α_t^L in the distribution of VaR parameters such that all intermediaries with VaR constraints looser than the cut-off will borrow to leverage up to their constraint. All intermediaries with VaR constraints tighter than the cut-off will choose to not leverage. Equations (27) and (21) define an implicit function of the threshold $\alpha_t^L = A(r_t^D, Z_{t+1}^e, K_t)$.*

3.3.3 Financial market equilibrium and deposit demand curve

To close the financial market equilibrium, we need to use the market clearing condition. The aggregate capital stock of the economy is equal to the total investment

in risky projects by all intermediaries.

$$K_t = \int_{\underline{\alpha}}^{\overline{\alpha}} k_{it}\, dG(\alpha^i) \tag{28}$$

This integral can be divided into capital held by *risky* levered intermediaries (above α_t^L) and capital held by *safe* intermediaries who do not lever up but invest all their equity in the capital stock (between α_t^N and α_t^L). Below α_t^N all intermediaries invest all their equity in storage.

For *safe* intermediaries who invest all their equity in capital shares, the VaR constraint is given by $\zeta + (1-\zeta)F\left(\frac{\delta K_t^{1-\theta}}{\theta Z_{t+1}^e}\right) \leq \alpha^i$. We can pin down α_t^N by looking at the marginal *safe* intermediary for whom the constraint binds exactly.

$$\alpha_t^N = \zeta + (1-\zeta)F\left(\frac{\delta K_t^{1-\theta}}{\theta Z_{t+1}^e}\right) \tag{29}$$

In equilibrium, the market clearing condition for K can then be written as:

$$K_t = \int_{\alpha_t^L}^{\overline{\alpha}} k_{it}\, dG(\alpha^i) + \left[G(\alpha_t^L) - G(\alpha_t^N)\right]\omega \tag{30}$$

Where k_{it} is given by the asset purchases of *risky* intermediaries described in equation (21). Along with the expression for α_t^N in equation (29), the market clearing equation (30) defines an implicit function of $(\alpha_t^L, r_t^D, Z_{t+1}^e, K_t)$. Since Z_{t+1}^e is determined at t by state variables and intermediaries are price takers, the financial market clearing function together with the implicit function $\alpha_t^L = A(r_t^D, Z_{t+1}^e, K_t)$ pin down the aggregate capital stock K_t and the marginal levered intermediary α_t^L, for a given deposit rate r_t^D and expected productivity Z_{t+1}^e.

Together they determine the *aggregate demand curve for deposits* as a function of deposit rates and expected productivity. By pinning down (α_L, K), they also determine the entire distribution of leverage in the financial sector for a given (r_t^D, Z_{t+1}^e). In general equilibrium, the deposit rate r_t^D will be determined in conjunction with the *aggregate deposit supply curve* coming from the recursive household problem described in section 5.

3.4 Measuring Financial Stability

The model establishes an important relation between funding costs and the cross-sectional distribution of risk taking by financial intermediaries. Financial stability is a multidimensional object depending on time-varying distributions of leverage and risk taking which are functions of present and future states. For expositional purposes, we summarize this object into a few simple but relevant measures of financial instability in order to track its evolution.

Our baseline measure M^1 is *the probability that in the next period all leveraged intermediaries will be in distress*, defined as the inability to repay in full their stakeholders (deposits and equity). This has a very direct link with the *VaR* constraint, as for each levered intermediary the probability of distress will be simply the parameter α^i. Given aggregate shocks by definition affect all intermediaries, then $M_t^1 = \alpha_t^L$. If the least risk-taking leveraged intermediary is in distress, so must all the intermediaries with higher leverage.[24] In the model, a rise in α_t^L (meaning that the marginal entrant has a looser *VaR* constraint) is then a fall in financial stability according to M^1. The baseline measure has the advantage of not only describing the risk of the whole sector but also of tracking the marginal investor in financial markets an important concept in leverage cycles, as highlighted by Geanakoplos (2011).

The model features significant risk-shifting behavior, as levered financial intermediaries take advantage of limited liability and the option value of default. Moreover, the riskier the intermediary, the larger will be their option value of default. To have a sense of aggregate distortions to investment caused by risk-shifting, we calculate a *Weighted Option Value of Default* by weighing each intermediary's option value of default by their total assets. This measure M^2 can therefore be interpreted as the average option value of default per unit of capital in the economy.

In the following sections we will use measures M^1 and M^2 to track the dynamics of financial stability in response to monetary and productivity shocks.[25]

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[24]More precisely, in the presence of idiosyncratic shocks M^1 would be an affine transformation of α_t^L, with $M_t^1 = \zeta + (1 - \zeta)\alpha_t^L$. Given this transformation is time-invariant, for simplicity we set $M^1 = \alpha_t^L$ even in the case with idiosyncratic shocks.

[25]Given that we can describe the whole cross-sectional distribution of leverage and intermediary risk we can also use a range of potential alternative measures. We highlight this point by providing 3 other measures in Appendix C.

4 Partial equilibrium results

To provide a better illustration of the financial sector mechanics in the model, we first show a set of partial equilibrium results taking as given the deposit rate, before moving on to general equilibrium in section 5 where the household problem will close the model. From now on we study the properties of the model using numerical simulations.[26]

We begin by analysing the distribution of intermediary leverage conditional on the deposit rates r_t^D and on expected productivity Z_{t+1}^e. In Figure 4, we show an example of the cross-sectional distribution of leverage for three different values of the deposit rate. The calibration of the model is discussed in more detail in section 5.

In the three cases, the area below each line[27] is proportional to the aggregate capital stock $K_t = \int k_{it} \, dG(\alpha^i)$. The vertical line showing a drop in leverage marks the cut-off and identifies the marginal levered intermediary α_t^L. To the left of the cut-off α_t^L, intermediaries are not levered, which corresponds to the more conservative VaR constraints. They are the *safe* intermediaries. To the right of the cut-off, leverage and balance sheet size k_{it} increase with α_t^i. That is, the more risk-taking is the intermediary, the larger will be its balance sheet for a given r_t^D and Z_{t+1}^e. Those are *risky* intermediaries.

The graph illustrates how the *intensive* and *extensive* margins affect leverage and the aggregate capital stock as the deposit interest rate changes. For the three cases displayed, as deposit rates fall, the *intensive* margin for the most risky intermediaries is always increasing. That is, for each such intermediary that is levered up, the balance sheet grows when the cost of funds falls. For a given balance sheet size, a lower rate would reduce the probability of default as it reduces the amount that needs to be repaid next period. This relaxes the *VaR* constraint, so intermediaries at the top of the distribution expand their balance sheet up to the new limit and grow in size.

Perhaps less intuitively, the effect for intermediaries in the middle of the distribution and on the *extensive* margin is ambiguous. One would expect that a fall in interest rate would lead to higher leverage by all intermediaries and entry of more risk averse

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[26]We performed many different calibrations but only report a few. Results (available upon request) are qualitatively robust across simulations.

[27]Assuming a uniform distribution for $G(\alpha^i)$ as in the baseline calibration. The details of the numerical method to solve the model are given in Appendix A.
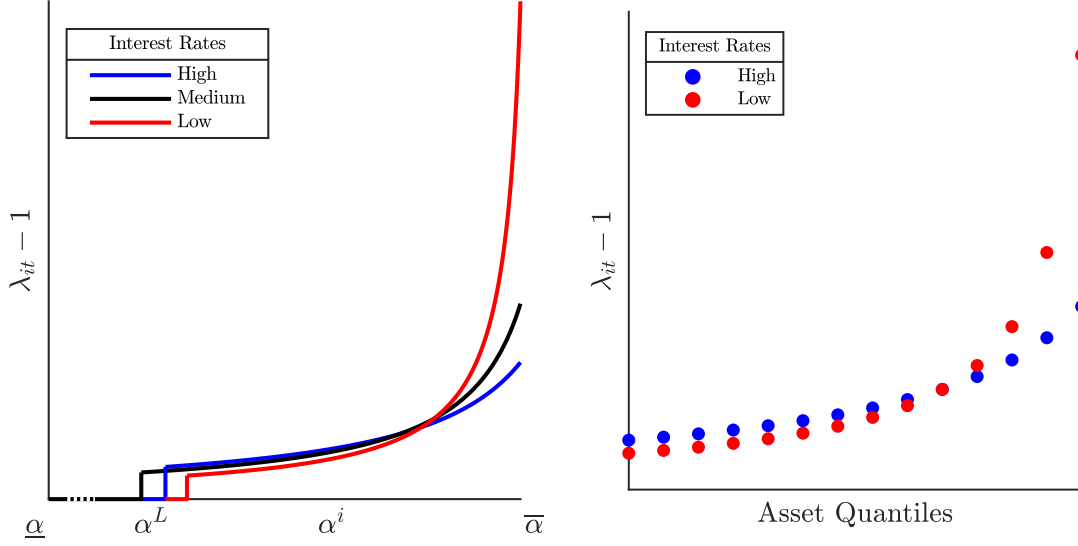
Figure 4: *Cross-sectional distribution of leverage λ_{it} as a function of the VaR parameter α^i (left panel) and quantiles of asset size among leveraged institutions (right panel). $\lambda_{it} - 1$ is used so that unlevered participants have a value of 0.*

intermediaries. Entry does occur when one goes from a high level of interest rate to a medium level of interest rate (the cut-off moves to the left). But this is no longer the case when one moves from a medium level of interest rate to a low level of interest rate: the cut-off moves to the right. Depending on the level of interest rates, a fall in interest rates can lead to more or fewer intermediaries choosing to lever up. The intensive margin effect at the middle and tail of the distribution is also not positive, in fact it decreases for many intermediaries. This is due to the fall in expected returns as K increases, driven by the large balance sheet expansion of the riskier intermediaries. We explain below this strong non-linearity of the effect of interest rates on financial stability and the leverage of intermediaries in the middle of the distribution.

On the right panel, we show the model implied leverage per asset quantile of levered institutions for high and low rates. The graph is strikingly similar to Figure 12 obtained from the data. Leverage among lower quantiles does not vary much across the two cases, while those at the top exhibit significantly larger leverage whenever interest rates are low. Those patterns are consistent with the stylized facts described in Section 2.

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4.1 Non-linear trade-off between increased output and financial stability

Following a fall in deposit rates, the riskier intermediaries expand their asset holdings raising the aggregate capital stock. This lowers the return on risky asset holdings due to decreasing returns to (aggregate) capital. As seen in the graph above, there are very interesting asymmetries depending on the *level* of the interest rate.

When the interest rate level is high, the lower cost of liabilities reduces the probability of default for a given balance sheet size. Hence all intermediaries with a risky business model are able to lever more (*intensive* margin). In this case, there are also positive returns for the (previously) marginal intermediary due to the now lower cost of leverage. More intermediaries can lever up and enter the market for deposits (*extensive* margin), reducing the cut-off α^L. The financial system then becomes less risky since newly entered intermediaries have a stricter *VaR* constraint. According to measure $M^1 = \alpha^L$ there is no trade-off in this case between using lower interest rates to stimulate investment and financial stability.

When the interest rate level is low, the *intensive* margin effect of a decrease in the interest rate is strong (see Proposition 3.2), leverage and investment are high and the curvature of the production function leads to a decrease in expected asset returns which is large enough to price out of the market the most risk averse of the previously levered intermediaries. The sign of the effect on α^L depends on whether the fall in asset returns is stronger than the fall in the cost of liabilities. In the case of initially low interest rates, a further fall (in those rates) leads to fewer intermediaries choosing to lever up. The intermediaries remaining are larger and more risk taking on average. There is therefore a clear trade-off between an expansionary monetary policy (that lowers funding costs) and financial stability.

In order to gain some intuition, we can look at two polar cases. In the first, aggregate capital is infinitely elastic and return distributions $R^K_{t+1}(\varepsilon)$ are fixed. In this case, a decrease in the cost of funding can only lead to entry as the (previously) marginal intermediary will now make positive profits. The cut-off falls and there is no trade-off. In the second example, aggregate capital is fixed and returns adjust to clear the

market[28]. If a fall in the cost of funding allows more leverage from the more risk-taking intermediaries, then it must be that the (previously) marginal intermediary no longer holds capital and returns fall enough to price him out. In this case, there will always be a trade-off (housing markets where the supply is relatively inelastic are a possible example of that case). In intermediate cases, the strength of the intensive margin effect is important as it determines the extent to which returns fall due to decreasing returns in the aggregate capital stock. The stronger is this effect (i.e. the more leverage increases following a fall in interest rates or the more *interest-elastic* the intermediaries are), the more likely a trade-off will be present. As stated in Proposition 3.2, leverage increases faster as the interest rate falls (conditional on being levered). This means the *intensive* margin effect is particularly strong when interest rates are low. Proposition 3.2 also states that leverage reacts more, the more risk-taking is the intermediary implying additionally that the most risk-taking intermediaries grow faster. This leads to additional skewness in the cross-sectional distribution of leverage.

Hence, as shown in Figure 4, when interest rates fall from high to medium to low, balance sheets become more heterogeneous in size and the difference between the most leveraged and the least leveraged intermediary rises. We highlight the following properties of our model:

1) Skewness of leverage, aggregate investment, funding costs and fundamental volatility

In Figure 5, the left panel plots the cut-off α_t^L as a function of deposit rates r_t^D for three different productivity levels, while the middle panel does the same for the aggregate capital stock K_t. K_t is monotonically decreasing with r_t^D. As expected, the lower is the interest rate, the higher will be aggregate investment and we have a standard deposit demand curve. However, the change in *financial structure* underlying the smooth response in the capital stock is non-monotonic. As we can see from the left panel, the cut-off α_t^L first decreases when we go from high interest rates to lower ones and then goes up sharply as we approach zero. There is a change in the composition of

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[28]In this case, the price of capital will adjust as it is no longer pinned down by the investment technology. For recent macroeconomic models in which extensive and intensive margin have interesting interactions (albeit in very different contexts) see Martin and Ventura (2015) and Bergin and Corsetti (2015).

intermediaries. Less risk-taking intermediaries reduce their exposure and decrease asset holdings as they are priced out by more risk-taking institutions due to decreasing returns to capital. The latter use low interest rates to increase their leverage significantly.
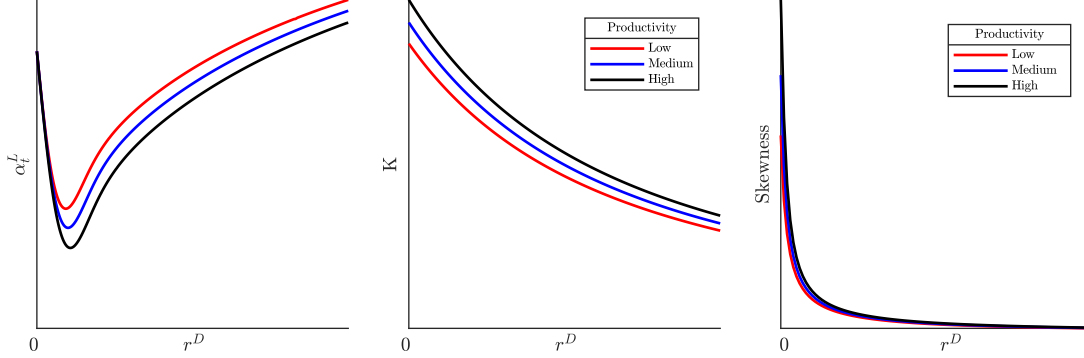

Figure 5: *Cut-off level α_t^L and aggregate capital stock as a function of deposit rates r_t^D*

The lower is the interest rate, the more heterogeneous is leverage across intermediaries. Since the intensive margin of high α^i intermediaries responds more than low α^i, when interest rates are low there is an increased concentration of assets in the most risk-taking intermediaries. Also, a fall in the extensive margin is more likely at low rates, which amplifies this effect. In the right panel of Figure 5 we show the cross-sectional skewness of leverage is a decreasing function of the interest rate. The concentration of assets in riskier intermediaries generates more risk-shifting in aggregate. Hence, similar aggregate investment outcomes can be supported by different underlying financial structures with very different implications for financial stability. Similarly, as shown in Figure 25, a lower level of volatility of the productivity shocks increases concentration in the banking sector for a given level of funding cost as more risk taking financial intermediaries can capture a higher market share. Therefore low fundamental volatility periods may be associated *ceteris paribus* with high concentration in the banking sector and more systemic fragility.

2) Trade-off between financial stability and economic activity

When interest rates are high, a fall in interest rates leads to entry by less risk-taking intermediaries (a fall in the cut-off α_t^L) into levered markets. But when interest rates

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are low, a fall in interest rates leads to a rise in the cut-off α_t^L, which means the least risk-taking intermediaries reduce their exposure to the risky asset through deleveraging, while the more risk-taking intermediaries increase their balance sheet size and leverage.

We illustrate this point in our partial equilibrium setting by doing a 100 basis points monetary expansion for different target rates. As we will see in section 4, these results carry over to the general equilibrium setting. For this experiment, we assume a very simple monetary policy rule:

$$R_t = R_{t-1}^\nu \bar{R}^{1-\nu} \varepsilon_t^R \tag{31}$$

where $R_t = 1 + r_t^D$ is the return on deposit or the cost of leverage for intermediaries. ε_t^R is a monetary policy shock and ν is the persistence of the shock, calibrated[29] to 0.24. $\bar{R}$ is the long-run level of interest rates therefore each of the lines above is calibrated to a different $\bar{R}$. For simplicity, in this simple partial equilibrium exercise, we assume that the monetary authority can directly affect the deposit rate. We relax this assumption in section 5 and show how it can be mapped into this exercise.
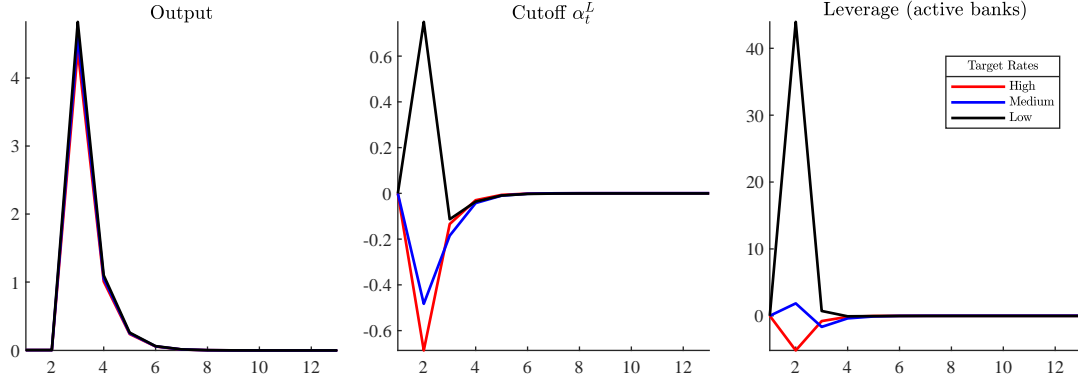

Figure 6: *Partial equilibrium IRF to a 100 basis points fall in deposit rates. Scale in percentage point deviations from the baseline*

Results can be seen in Figure 6, plotted as percentage changes from their respective values at target rates $\bar{R}$.[30] The time period corresponds to one year and the state of the

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[29] Annualized value as estimated by Curdia et al. (2015)

[30] Note that there is no truly dynamic aspect in the partial equilibrium model and it can be seen

economy when the shock hits is the one corresponding to the target rates. In the left graph we see that the rise in output is relatively insensitive to the level of the target interest rate. The behaviour of the cut-off α_t^L is, however, very differentiated. When the target rates are high, there is a negative effect of a monetary expansion on the cut-off. That means that less risk-taking intermediaries enter risky markets and the average probability of intermediary default falls. In this case, there is no trade-off between financial stability and monetary expansion. This is definitely not the case when target interest rates are low. In that case, average leverage of active banks increases massively by 43% and the cut-off also rises. The large increase in leverage by very risk-taking intermediaries then prices out the less risk-taking ones at the margin, raising the average probability of default among levered intermediaries. This large effect on leverage is a combination of both the intensive margin effect, and a composition effect due to exit of the most risk averse intermediaries. For intermediate levels, we see that this effect is muted, with leverage increasing only slightly and financial stability improving (cutoff going down). Hence, according to our baseline measure $M^1 = \alpha^L$ of financial stability, there is a trade-off between financial stability and monetary policy when interest rates are low, but not when they are high.

The level of the interest rate matters since it affects the sensitivity of the intensive margin to changes in the cost of funds. The fact that risk-taking intermediaries are able to lever more when the cost of funds is low increases the capital stock and can price out of the market less risk-taking intermediaries. The financial sector becomes less stable, as risky assets concentrate in very large, more risk-taking financial institutions. There is also potentially a large mispricing of risk since the riskier intermediaries are those who engage the most in risk-shifting (measured in the aggregate by M^2).[31]

We note that all the effects described above regarding the dispersion and the cyclicality of leverage, financial stability and aggregate risk-shifting can occur even in the absence of monetary policy shocks. Similar effects could be due to the deregulation of the financial sector or to low volatility environments. The cyclicality of the savings

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as a sequence of static problems. The general equilibrium model of section 5 will feature a fully dynamic household problem which affects the banking problem, since the household inter-temporal maximization will determine the deposit supply curve and the equilibrium level of deposit rates.

[31]Other measures of financial stability presented in Appendix C also highlight the presence of an important trade-off which occurs only at low levels of the interest rate.

behaviour or of capital flows and their effect on equilibrium deposit rates will also lead to cyclical movements in leverage and investment. To understand this more fully, we now close the general equilibrium model by adding the intertemporally optimizing household sector to determine the deposit rate endogenously.

5 General Equilibrium

In this section, we solve the model in general equilibrium by joining the household and intermediary problems. We show that the financial sector equilibrium can be easily integrated in a standard dynamic stochastic general equilibrium framework, with monetary policy and productivity shocks. We introduce costly default in section 6.

5.1 Monetary policy as a change in the cost of external funds

In this section we allow intermediaries to fund themselves through wholesale funding l_{it}. We assume that the monetary authority can control the rate of wholesale funding relative to deposits, by providing funds at a spread γ_t from deposits.[32] Wholesale funding is remunerated at rate $R_t^L = 1 + r_t^L$ and we denote the deposit rate r_t^D as before. We assume that:

$$R_t^L = R_t^D(1 - \gamma_t) \tag{32}$$

Monetary policy is exogenous, akin to a funding subsidy γ_t which follows a simple AR(1) process in logs.

$$\log \gamma_t = (1 - \rho_\gamma)\mu_\gamma + \rho_\gamma \log \gamma_{t-1} + \varepsilon_t^\gamma \tag{33}$$

$$\varepsilon_t^\gamma \sim N(0, \sigma_\gamma) \tag{34}$$

where μ_γ is the central bank target subsidy, ρ_γ the subsidy's persistence and ε^γ are monetary policy shocks with σ_γ standard deviation.

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[32]The monetary authority is assumed to be a deep-pocketed institution which can always fund wholesale funding. Like deposits, wholesale funds are always repayed (by bailout if necessary). To avoid dealing with the monetary authority's internal asset management, we assume that the cost of fund is a deadweight loss (or gain).

If the central bank were to provide unlimited funds to intermediaries at this rate, they would leverage using only wholesale funding. We assume that wholesale funding is given in a fixed proportion χ of other liabilites, which in this case are simply deposits. Total wholesale funding for intermediary i is then:

$$l_{it} = \chi d_{it} \tag{35}$$

The balance sheet of an intermediary i is then:

Assets	Liabilities
k_{it}	ω
s_{it}	d_{it}
	l_{it}

Given our assumptions, we can then define R_t^F as the total cost of a unit of funding and f_{it} as total external funds of bank i.

$$R_t^F = \frac{1 + \chi(1 - \gamma_t)}{1 + \chi} R_t^D \tag{36}$$

$$f_{it} = (1 + \chi)d_{it} \tag{37}$$

The balance sheet can be rewritten as follows:

Assets	Liabilities
k_{it}	ω
s_{it}	f_{it}

With external funds being remunerated at rate R_t^F. We obtain the same banking problem as before, replacing deposits by total funds f_{it} and the deposit rate by the unit cost of funds R_t^F. We can solve as before by mapping f_{it} and R_t^F easily into deposits d_{it} and their rate R_t^D. By moving γ_t the central bank will be able to change R_t^F as long as changes in equilibrium R_t^D do not offset perfectly the changes in the spread on the total cost of funding.

5.2 Solving the dynamic model

The financial sector equilibrium determines investment given funding costs R_t^F and expected productivity Z_{t+1}^e. We can then solve for the aggregate capital stock K and cut-off α_t^L as a function of R_t^F and expected productivity Z_{t+1}^e.

$$K = K^*(R^F, Z^e) \tag{38}$$

$$\alpha^L = \alpha^{L,*}(R^F, Z^e) \tag{39}$$

By integrating balance sheet equations, we obtain an expression for total funds F_t and deposit supply D_t:

$$F_t = \int_{\alpha_t^L}^{\overline{\alpha}} (k_{it} - \omega) \; dG(\alpha^i) \tag{40}$$

$$D_t = \int_{\alpha_t^L}^{\overline{\alpha}} d_{it} \; dG(\alpha^i) = \frac{F_t}{1 + \chi} \tag{41}$$

where $F_t = \int f_{it} \; dG(\alpha^i)$ are total liabilities held by *leveraged* intermediaries and D_t is the aggregate deposit demand. Market clearing in the deposit market requires supply and demand to be equal.

$$D_t^H = D_t \tag{42}$$

Goods market clearing requires that output is used in consumption of intermediaries and households, investment and the accumulation of storage. The investment good is the consumption good and there are no capital or investment adjustment costs[33]. Aggregate investment I_t is given by the law of motion of the capital stock $K_t = (1 - \delta)K_{t-1} + I_t$. The resource constraint of the economy is as follows:

$$S_{t-1}^H + S_{t-1}^I + Y_t = C_t^H + C_t^I + S_t^H + S_t^I + I_t + T_t^L \tag{43}$$

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[33]We also do not constrain investment to be necessarily positive.

where $C_t^I = \int c_{it} \, dG(\alpha^i)$ and $T_t^L = \int l_{it} \, dG(\alpha^i) - R_{t-1}^L \int l_{i,t-1} \, dG(\alpha^i)$ is the net whole-sale funding. S_t^H are the holdings of storage held by households and $S_t^I = \int s_{it} \, dG(\alpha^i)$ are aggregate storage holdings held by financial intermediaries at t.

Definition 2: Equilibrium.

Let $\mathcal{S} = \{D_{t-1}, S_{t-1}^H, S_{t-1}^I, K_{t-1}, Z_{t-1}, \gamma_{t-1}, \varepsilon_t^z, \varepsilon_t^\gamma\}_{t=0}^\infty$ be the vector of state variables and shocks. Given a sequence of rates $\{r_t^D\}_{t=0}^\infty$, monetary policy rule and financial market rules $K(\mathcal{S}), \alpha^L(\mathcal{S}), S(\mathcal{S})$, let us define the optimal decisions of the representative household as $C^H(\mathcal{S}), D^H(\mathcal{S}), S^H(\mathcal{S})$.

An equilibrium is a sequence of rates $\{r_t^D\}_{t=0}^\infty$, and policy rules $C^H(\mathcal{S})$, $D^H(\mathcal{S})$, $S^H(\mathcal{S})$, $S^I(\mathcal{S})$, $K(\mathcal{S})$, $\alpha^L(\mathcal{S})$, such that:

- $C(\mathcal{S}), D^H(\mathcal{S}), S^H(\mathcal{S}), S^I(\mathcal{S}), K(\mathcal{S}), \alpha_t^L(\mathcal{S})$ are optimal given $\{r_t^D\}_{t=0}^\infty$

- Asset and goods markets clear at every period t

In equilibrium, we need to find a deposit rate which, conditional on exogenous variables and the financial sector equilibrium, is consistent with the household problem. We proceed by iterating on r_t^D, imposing the financial market equilibrium results. For a given deposit rate r_t^D, we find the law of motion for household wealth and consumption, use the Euler equation errors to update the deposit rate and repeat until convergence. A more detailed explanation of the algorithm used for our global solution method can be seen in Appendix A.

5.3 Calibration

To solve the model numerically, we need to specify the period utility function, the shape of the distribution of the VaR probabilities and calibrate the remaining parameters. Given the interaction between extensive and intensive margin effects, the mass of intermediaries in a given section of the distribution could have an important role in determining which of the two effects dominates. To highlight that the results described are not a consequence of this distribution, we assume that $G(\alpha^i)$ is uniform

between $[0, \overline{\alpha}]$. For the utility function, we assume a standard CRRA representation.

$$u(C) = \frac{C^{1-\psi} - 1}{1 - \psi} \tag{44}$$

Table 1: Calibration of selected parameters

Parameter	Value	Description
ψ	4	Risk aversion parameter
β	0.96	Subjective discount factor
ρ^z	0.9	AR(1) parameter for TFP
σ_z	0.03	Standard deviation of TFP shock
μ_γ	0.023	Target spread over deposit rates
ρ_γ	0.816	Spread persistence
σ_γ	0.0128	Standard deviation of spread
$\frac{\chi}{1+\chi}$	0.41	Wholesale funding percentage
θ	0.35	Capital share of output
δ	0.1	Depreciation rate
ω	0.697	Equity of intermediaries
$\overline{\alpha}$	0.4961	Upper bound of distribution $G(\alpha^i)$
ζ	0.01	Idiosyncratic unproductive capital probability

The calibration can be seen in Table 1. For the utility function parameters, risk aversion ψ, the subjective discount factor β, the TFP parameters ρ^z and σ_z we use standard values from the literature. Similarly for θ, the capital share of output, and for δ the depreciation rate of the capital stock. To calibrate the monetary policy parameters, we calculate the subsidy as the difference between the Effective Fed funds Rate and $1/\beta$, the long-run deposit rate. We then fit an AR(1) process to get the parameters used.

The wholesale funding percentage used to calibrate χ was calculated from the time series mean of the cross-sectional asset-weighted average in Bankscope data[34] for the period 1993-2015. For the purpose of this calibration, wholesale funding was assumed to be all non-deposit liabilities of each financial intermediary.

We calibrate $\overline{\alpha}$ to match the probability of default of the median *risky* intermediary

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[34]Bankscope contains a large panel of financial intermediaries' balance sheet data. See Appendix D.

when deposit rates are at steady-state. Using FDIC data on failed banks, we find that the median age of failed banks in the US was around 20.5 years. The full sample distribution of ages at failure can be seen in Figure 20. We then calibrate $\overline{\alpha}$ to match a default probability 5% for the median intermediary when $R_t^D = 1/\beta$. This also implies a default probability for the riskiest intermediary $\overline{\alpha}$ of 22%. Turan G. Bali, Stephen J. Brown and Mustafa O. Caglayan (2014) report that the median lifespan of a hedge fund is slightly less than 5 years, a value close to what our calibration implies for $\overline{\alpha}$.

ω is chosen to fit leverage at steady-state. Some of the intermediaries are leveraged and others are not, so we cannot use only Bankscope data (which contains mostly leveraged banks) to calibrate leverage. According to the "broad measure" of Other Financial Institutions (OFIs) in the Global Shadow Banking Report (Financial Stability Board (2015)), non-levered intermediaries hold about 137 trillions of assets while banking assets are around 135 trillion. We use these figures to calculate an asset-weighted average of leverage of 7.3, which is reached by combining the Bankscope asset-weighted average leverage of 13.5 for 2015 and assuming a leverage of 1 for the OFIs. We target our calibration of ω so that the median *risky* intermediary matches this value.

The size of the equity endowment ω and the volatility of aggregate shocks σ_z will also contribute to determine the financial sector reaction to changes in deposit rates. For that reason, we also conducted some comparative statics on both σ_z and ω to see how the model changes with those parameter calibrations. There is very little effect on the first moments of real variables such as output and consumption but there are important changes on equilibrium leverage and financial stability when we vary ω and/or σ_z. In general, the easier it is for riskier intermediaries to absorb the market, then less stable will financial markets be. Increases in ω and decreases in σ_z both *worsen* financial stability. Low volatility of the fundamental shocks σ_z will lead to lower financial stability since riskier intermediaries will find it easier to capture the market. More details can be found in Appendix F.

The value of $\overline{\alpha}$ and the shape of its distribution will also matter for financial stability. Increasing $\overline{\alpha}$ leads to a less financially stable financial sector. We leave for future work to perform a (technically challenging) estimation of the model where the distributions of α or ω could potentially be backed out from the data and focus here on understanding the qualitative implications of the model.

5.4 Monetary policy and leverage

We now look at the impact of a positive subsidy shock, which we will refer to as an expansionary monetary policy shock or a decrease in the cost of funds. In Figure 7 we see the impact of a 100 basis points to the subsidy[35] in three different scenarios to illustrate the non-linear effects of monetary policy on financial stability. Impulse response functions are expressed as deviations from the respective scenario in the absence of the shock. This monetary policy loosening decreases the funding rate of the banks by 8 bp as can be seen in the left panel of Figure 8. Scenario 1 (blue line) features a low initial capital stock (corresponding to high equilibrium levels of the interest rate). Scenario 2 (red line) is for a larger capital stock (corresponding to a low level of equilibrium interest rate). Scenario 3 (black line) is at the risky steady-state[36]. As in Coeurdacier, Rey and Winant (2011) we define the risky steady-state as the steady-state in which there are no shocks but economic agents take into account the full stochastic structure of the model when they optimize (unlike in the deterministic steady-state where they expect no shocks).
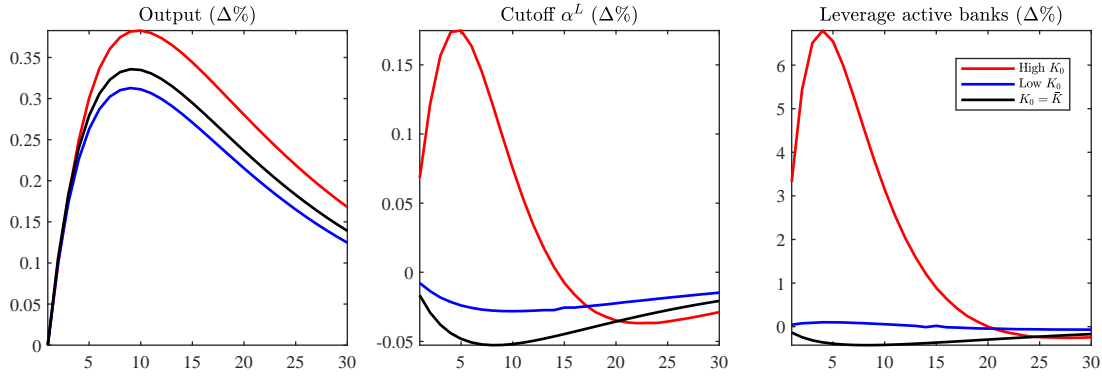

Figure 7: *Monetary policy shock of 100 basis points to γ_t*

We can easily relate the general equilibrium results to the partial equilibrium

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[35]Note that this translates into a lower reduction in the total cost of funds (see Figure 8). This is due to the fact that the cost of funds is a composite of deposits and wholesale funds, but also due to endogenous movements in the deposit rate.

[36]These three scenarios were chosen to illustrate the parallel with the partial equilibrium setting, since the solution of the model is such that there is, ceteris paribus, a negative correlation between the initial capital stock and the funding rate.

intuitions developed above. In the case of a low initial capital stock (associated with a high equilibrium funding rate), a positive monetary policy shock expands output, increases aggregate leverage and at the same time it reduces the cut-off α^L, due to the entry of less risk-taking intermediaries in deposit markets. We are in the "no trade-off zone of monetary policy" where a decrease in the interest rate increases investment and financial stability. In the case of a high initial capital stock (associated to a low funding cost for intermediaries), an expansionary shock has a larger positive effect on output and leverage but this time intermediaries at the margin choose not to lever up. In contrast, the most risk-taking intermediaries leverage significantly and financial stability is affected negatively.

This is a very different trade-off from the traditional Phillips curve which has been the benchmark model driving monetary policy analysis for many years. Aggregate economic variables such as consumption, wealth or capital behave smoothly as evidence in Figure 15, but the underlying change in financial structure supporting these macroeconomic outcomes can become less stable depending on the level of the interest rate.
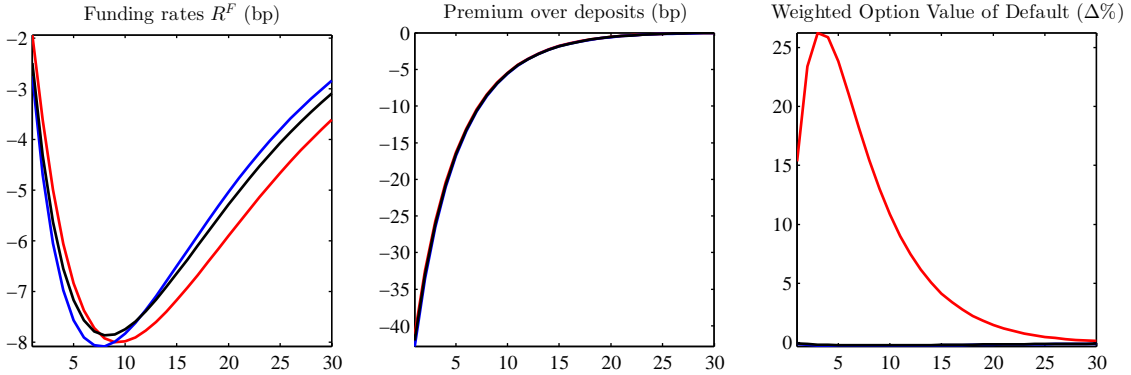

Figure 8: *Monetary policy shock of 100 basis points to γ_t: Financial variables*

As seen in Figure 8, the Weighted Option Value of Default also increases drastically with a monetary policy loosening when interest rates are low. The option value of default is defined as the difference between expected profits under limited liability and the (untruncated) cash flow. The larger this difference, the bigger the distortions coming from the presence of the limited liability and the worse for financial stability. Since the option value of default is intermediary-specific due to the heterogeneity of balance

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sheets, we construct an asset-weighted mean to illustrate the aggregate effect. When the interest rate is lower, the decrease in the cost of funds generates a very large increase due to the exit of safer intermediaries but also to the increase in leverage skewness in the cross-section. The impulse response functions for the alternative risk measures of Figure 19 in Appendix C also illustrate the presence of a strong trade-off when interest rates are low. Finally, the premium over deposits goes down as monetary policy expands since the demand for deposits goes up and the expected return to risky capital goes down due to decreasing returns and the mispricing of aggregate risk. Scenario 2 has the characteristics of a "bad boom" in the terminology of Gorton and Ordoñez (2019).

5.5 Productivity driven leverage

Cycles in leverage can be driven by movements in the cost of funds, but also by changes in expected productivity. When leverage is driven by an increase in productivity then the ensuing leverage growth does not come at the cost of financial stability. There is a fundamental difference between a credit boom driven by a shift in supply (i.e. cheaper access to funds) and a boom driven by demand for credit (i.e. better investment opportunities). Productivity shocks in our framework are an example of the latter as they forecast larger productivity in the future. In general equilibrium supply and demand of credit are interdependent so this distinction is simply to clarify the intuition and relate it to the original shock leading to credit growth.

We now look at a shock to productivity. In Figure 9 we see the impact of a one standard deviation positive productivity shock in the same 3 scenarios as the previous section.

The effects are similar irrespective of the position in the state space and the level of interest rates at the time of the shock. Total leverage goes up due to increased investment opportunities and is hump-shaped, as can be seen in the first panel. The hump-shape is due to the initial pressure of credit demand which requires higher deposit rates to clear the market. After impact household wealth accumulates and deposit rates start to fall, leading to a hump-shaped response of credit and investment as the positive productivity shock fades out.

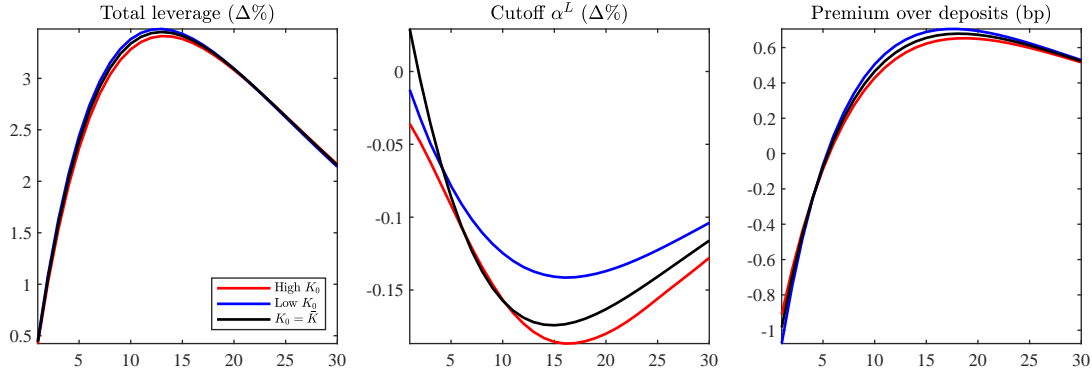This effect can be also seen on the premium over deposits (right panel). On impact,

Figure 9: *Shock to exogenous productivity*

there is a larger rise of deposit rates than expected returns, despite the better investment opportunities coming from higher expected productivity. The effect on the premium is however very small (1bp decrease on impact), only a small fraction of the effect seen after a monetary policy shock (40bp). In the middle panel, we also see that financial stability overall slightly improves in all scenarios, apart from a short-lived marginal uptick on impact in the middle scenario. Again these are small effects, indicating that productivity driven leverage booms are not a concern for financial stability in the same way that credit supply driven ones are. As Krishnamurthy and Muir (2017) show, credit booms accompanied by the tightening of spreads can predict financial crises, while those without such a tightening do not. We are able to rationalize this fact through the cross-sectional composition of the financial sector and the difference between productivity driven and credit supply driven leverage. The positive productivity shock leads to a "good boom".

6 Costly intermediary default

We now consider the case of costly intermediary default. As in the previous section, leveraged intermediaries in risky financial markets will default on depositors if the realisation of the productivity shock is low enough. This requires intervention by the government to pay for deposit insurance, which is now less benign than previously

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assumed as there is a deadweight loss[37]. We assume that capital held by defaulting intermediaries suffers a proportional productivity loss $\overline{\Delta}$ relative to the productivity of capital held by non-defaulting intermediaries. This loss can arise from (real) bankruptcy costs or some degree of inalienability in investment projects. The main assumption is that these costs are proportional to the output of the respective capital shares. Let μ_t^d be the share of capital held by defaulting intermediaries. We define an aggregate productivity loss $\Delta_t = \mu_t^d \overline{\Delta}$; this is an increasing function in the share of capital held by defaulting intermediaries. Note that the productivity of capital held by healthy intermediaries is unaffected at t, so the impact on aggregate productivity is coming only from cross-sectional differences between defaulting and non-defaulting intermediaries. The extent of the "loss given default" is therefore endogenous in our model and increasing with the severity of the banking crisis.

We also consider the possibility that this disruption spreads to the entire financial market in the following periods by affecting productivity of *all* intermediaries in future periods by Δ_t, which reflects the endogenous severity of the financial crisis. The loss of aggregative productivity is then intermediary-specific during default, but it can affect the whole economy moving forward (the allocative process of the whole economy is impaired). When it happens we call this the *crisis* state. We model the persistence of the crisis state through a Poisson process, with a constant probability p of exiting the crisis at each period. Depending on the process, variable ξ_t takes the value of one if the crisis carries on to the next period or zero if it does not. This reflects for example the (unmodelled) capacity of the state to intervene and solve the banking crisis quickly or not[38]. Our specification nests both the case of costless default ($\overline{\Delta} = 0$) and the case where there is no disruption of financial markets in subsequent periods ($p = 1$). We have:

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[37]As before, deposit guarantees will be financed by lump sum taxation of households. The welfare analysis of our setup is left for future work.

[38]States have shown a very uneven ability, for example, to clean financial sectors balance sheets and dispose of non performing loans. Politics and economics are closely intertwined in that process. Our assumptions on real default costs are similar to the sovereign default literature, i.e. Arellano (2008) who calibrates output costs due to default and the probability of re-entry in capital markets (like we do with the Poisson process) from the data.

$$\mu_t^d = \frac{\int k_{it} \; \mathbb{1}_{(\pi^i < 0)} \; dG(\alpha^i)}{K_t}$$

$$\Delta_t = \xi_{t-1} \max(\mu_{t-1}^d \; \overline{\Delta}, \Delta_{t-1})$$

where the indicator function takes the value of 1 if intermediaries of type i default or 0 if not. If there are also defaults during a crisis state, then the max operator ensures that the largest penalty applies going forward. Whenever the economy is in crisis, productivity for all financial intermediaries is scaled down by a factor μ_t^d proportional to the percentage of total capital held by defaulting intermediaries. ξ_{t-1} is known to agents when they make their investment decisions at period $t - 1$, so the uncertainty on the returns on their capital investment is only on the realization of the exogenous productivity process[39]. This timing assumption allows us to keep tractability as the main difference in the financial sector block is that now $Z_{t+1}^e = (1 - \Delta_t)Z_t^{\rho^Z}$. Since both Δ_t and Z_t are state variables, we can still solve for the financial sector equilibrium as before.

This set up is tractable and allows us to parameterize crises of different severity and length. Reinhart and Rogoff (2009a) present a classic description of the characteristics of crises across history, and evidence that crises associated with banking crises are more severe. Borio et al. (2016) and Laeven and Valencia (2012) present empirical evidence showing that there can be substantial and long lasting productivity drops after financial crises. To calibrate these parameters we refer to the database of Laeven and Valencia (2012), setting $p = 0.5$ to target an average crisis length of 2 years as in the data, and $\overline{\Delta} = 0.115$ implying a maximal efficiency loss of 11.5% per year[40].

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[39] There is still uncertainty on asset returns if the intermediary defaults but this is not considered in the intermediary problem due to limited liability truncating the profit functions at zero in those states.

[40] In the database of Laeven and Valencia (2012), the average cumulative output loss is 23% over the length of the crisis, which is on average two years.

6.1 Productivity shocks and financial crises

In this section we study the impact of a financial crisis with costly default on the path of the economy, following a productivity shock. Figure 10 shows the impact of a large productivity shock in 3 possible scenarios[41].
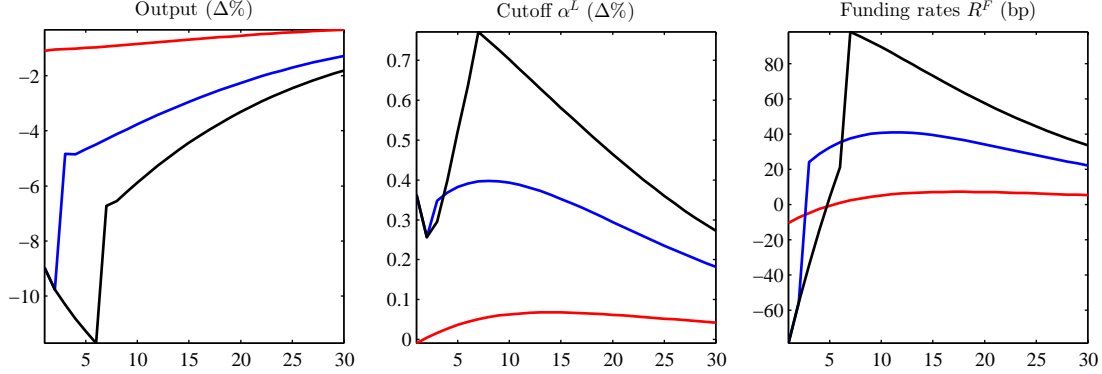

| Output ($\Delta\%$) | Cutoff α^L ($\Delta\%$) | Funding rates R^F (bp) |

Figure 10: *Large shock to exogenous productivity*

In scenario 1 (red line) the economy at the risky steady-state is hit at period t by the largest possible shock that does not trigger any defaults. In scenarios 2 (blue line) and 3 (black line) the economy is hit with the smallest shock such that all *levered* intermediaries default. The difference between scenarios 2 and 3 is in the length of the crisis. Scenario 2 is the short crisis scenario, where the crisis only carries on to the next period, $\xi_1 = 1$. Scenario 3 is the "unlucky" scenario, where the crisis carries on for an additional 5 periods, possibly because of (unmodelled) policy mistakes: $\xi_s = 1$ for $t = 1$ to $t = 6$. The length of the crisis is unknown beforehand to the agents in the economy, although as mentioned before they observe the value of ξ_t when they make their investment decisions at t. Not surprisingly, when the crisis hits there is a large decline in output. As expected productivity is low, only the intermediaries with the looser *VaR* constraints can operate. There is a strong fall in deposit demand due to the low expected productivity, which severely tightens the constraint.

In equilibrium the fall in deposit demand generates a fall in funding costs due to

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[41]Impulse response functions expressed in basis points deviations for rates or otherwise in percent deviations from the risky-steady state

decreased deposit rates. For the cases with defaults, one can observe a small initial decrease in the cut-off after an initial jump. This is because on impact the economy jumps to the trade-off region. As interest rates start rising from that point onward, the economy travels through the U-shape with the cut-off falling initially and then increasing as interest rates rise.

The length of the crisis also has very interesting dynamic effects on wealth. Given that households expect to exit the crisis state with probability p, when exit fails to materialize in Scenario 3 they are running down their wealth and their consumption dips down (see Figure 16). As wealth falls, deposit rates and funding costs (see Figure 10) grow as it becomes more costly for the household to save and fund bank leverage. When eventually the economy exits the crisis state, household wealth is low and demand for leverage jumps, leading to a jump in funding rates to compensate households for decreased consumption today. This leads also to a higher risk premium as expected return to capital jumps up. Total leverage and investment, which had seen severe contractions start to go up again (see Figure 16). This effect is also present with a short crisis, but is particularly stark for the longer crisis.

7   Conclusion

This paper develops a novel framework for modeling a financial sector with heterogeneous financial intermediaries and aggregate risk. The heterogeneity in the VaR constraints coupled with limited liability generates endogenous time variation in leverage, risk-shifting and financial stability. The interaction between the intensive and the extensive margins of investment creates a rich set of non-linear dynamics where the level of interest rates plays a key role. When interest rates are high, a monetary expansion increases both the intensive margin and the extensive margin. The monetary authority is able to stimulate the economy, while at the same time increasing financial stability. When interest rates are already low, a further reduction can lead to large increases in leverage by the most risk-taking institutions, pricing out previously active intermediaries, due to decreasing aggregate returns to capital. Importantly, the intermediaries which decrease their balance sheet size have lower probabilities of default than those that remain levered, leading to an increase in systemic risk. Our

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model, unlike the existing literature, generates a trade-off between economic activity and financial stability depending on the level of the interest rate. During booms driven by low funding costs and increased credit supply, risk premia are low as there is a lot of risk-shifting by the most risk-taking intermediaries in the economy. Booms driven by positive productivity shocks do not lead to an increase in financial instability nor to such low levels of risk premia.

Because our framework has heterogeneity at its heart, it allows us to make use of cross-sectional data on intermediary balance sheets. We derive novel implications linking the times series of the concentration of leverage and monetary policy which are strikingly borne out in the data. We believe we are the first paper to link changes in the cross-sectional distribution of leverage, macroeconomic developments and fluctuations in financial stability. We show that similar macroeconomic outcomes can be supported by very different underlying financial structures. This has important implications for the transmission of monetary policy, the effect of fundamental volatility, of financial deregulation or of capital flows and the sensitivity of the economy to interest rate movements.

A major advantage of our framework is that our financial block is easy to embed in a standard dynamic stochastic general equilibrium framework. We plan to extend our model to environments with sticky prices and a more complex portfolio choice on the bank side as well as to study boom and bust cycles in emerging markets. We also plan to apply it to explain the dynamics of the real estate market. The model could also be calibrated to fit a distribution of financial intermediaries characteristics. One could in principle back out the distribution of α^i from leverage data and allow for a distribution of intermediary-specific equity ω^i. That said, allowing for time variation in equity would require the introduction of an additional state-variable in the financial sector problem which would make the solution more computationally intensive.[42] We leave these issues and the welfare implications of our model for future research.

References

Acharya, Viral, and Guillaume Plantin. 2016. "Monetary Easing and Financial Insta-

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[42]And having together time-varying and intermediary-specific equity would require an infinitely dimensional state-space without additional assumptions.

bility." *mimeo NYU Stern and Sciences Po.*

Acharya, Viral V., Lasse H. Pedersen, Thomas Philippon, and Matthew Richardson. 2017. "Measuring Systemic Risk." *The Review of Financial Studies*, 30(1): 2–47.

Adrian, Tobias, and Hyun Song Shin. 2014. "Procyclical Leverage and Value-at-Risk." *Review of Financial Studies*, 27(2): 373–403.

Adrian, Tobias, and Hyun S. Shin. 2010. "Financial Intermediaries and Monetary Economics." In *Handbook of Monetary Economics*. Vol. 3 of *Handbook of Monetary Economics*, , ed. Benjamin M. Friedman and Michael Woodford, Chapter 12, 601–650. Elsevier.

Aldasoro, Inaki, Domenico Delli Gatti, and Ester Faia. 2017. "Bank networks: Contagion, systemic risk and prudential policy." *Journal of Economic Behavior & Organization*, 142: 164 – 188.

Allen, Franklin, and Douglas Gale. 2000. "Bubbles and Crises." *The Economic Journal*, 110(460): 236–255.

Angeloni, Ignazio, and Ester Faia. 2013. "Capital regulation and monetary policy with fragile banks." *Journal of Monetary Economics*, 60(3): 311–324.

Angeloni, Ignazio, Ester Faia, and Marco Lo Duca. 2015. "Monetary policy and risk taking." *Journal of Economic Dynamics and Control*, 52(C): 285–307.

Aoki, Kosuke, Gianluca Benigno, and Nobuhiro Kiyotaki. 2016. "Monetary and Financial Policies in Emerging Markets ." *mimeo,Tokyo University, London School of Economics, and Princeton University.*

Arellano, Cristina. 2008. "Default Risk and Income Fluctuations in Emerging Economies." *American Economic Review*, 98(3): 690–712.

Bali, Turan G., Stephen J. Brown, and Mustafa O. Caglayan. 2014. "Macroeconomic risk and hedge fund returns." *Journal of Financial Economics*, 114(1): 1–19.

Baron, Matthew, and Wei Xiong. 2017. "Credit Expansion and Neglected Crash Risk." *The Quarterly Journal of Economics*, 132(2): 713–764.

Basel Committee on Banking Supervision. 2013. "Analysis of risk-weighted assets for market risk." Bank for International Settlements Regulatory Consistency Assessment Programmme (RCAP).

Baskaya, Yusuf Soner, Julian di Giovanni, Sebnem Kalemli-Özcan, and Mehmet Fatih Ulu. 2017. "International Spillovers and Local Credit Cycles." *NBER Working Paper*, 23149.

Begenau, Juliane, and Tim Landvoigt. 2017. "Financial Regulation in a Quantitative Model of the Modern Banking System." *mimeo Stanford GSB and Austin.*

Berger, David, Veronica Guerrieri, Guido Lorenzoni, and Joseph Vavra. 2018. "House Prices and Consumer Spending." *Review of Economic Studies*, 10(2): 250–74.

Bergin, Paul R, and Giancarlo Corsetti. 2015. "Beyond Competitive Devaluations: The Monetary Dimensions of Comparative Advantage." *CEPR Discussion Papers*, 10718.

Bernanke, Ben, and Mark Gertler. 1989. "Agency Costs, Net Worth, and Business Fluctuations." *American Economic Review*, 79(1): 14–31.

Boissay, Frederic, Fabrice Collard, and Frank Smets. 2016. "Booms and Systemic Banking Crises." *Journal of Political Economy*, 124(2).

Borio, Claudio, and Haibin Zhu. 2012. "Capital regulation, risk-taking and monetary policy: A missing link in the transmission mechanism?" *Journal of Financial Stability*, 8(4): 236–251.

Borio, Claudio, Enisse Kharroubi, Christian Upper, and Fabrizio Zampolli. 2016. "Labour reallocation and productivity dynamics: financial causes, real consequences." Bank

for International Settlements BIS Working Papers 534.

Brunnermeier, Markus K., and Yuliy Sannikov. 2014. "A Macroeconomic Model with a Financial Sector." *American Economic Review*, 104(2): 379–421.

Bruno, Valentina, and Hyun Song Shin. 2015*a*. "Capital flows and the risk-taking channel of monetary policy." *Journal of Monetary Economics*, 71(C): 119–132.

Bruno, Valentina, and Hyun Song Shin. 2015*b*. "Cross-Border Banking and Global Liquidity." *Review of Economic Studies*, 82(2): 535–564.

Challe, Edouard, Benoit Mojon, and Xavier Ragot. 2013. "Equilibrium risk shifting and interest rate in an opaque financial system." *European Economic Review*, 63(C): 117–133.

Coeurdacier, Nicolas, Hélène Rey, and Pablo Winant. 2011. "The Risky Steady State." *American Economic Review*, 101(3): 398–401.

Coimbra, Nuno. 2016. "Sovereigns at Risk: a dynamic model of sovereign debt and banking leverage." *mimeo Paris School of Economics*.

Coimbra, Nuno, and Helene Rey. 2018. "Financial Cycles and Credit Growth across Countries." *AEA Papers and Proceedings*, 108: 509–12.

Curdia, Vasco, Andrea Ferrero, Ging Cee Ng, and Andrea Tambalotti. 2015. "Has U.S. monetary policy tracked the efficient interest rate?" *Journal of Monetary Economics*, 70: 72 – 83.

Danielsson, Jon, Hyun Song Shin, and Jean-Pierre Zigrand. 2010. "Risk Appetite and Endogenous Risk." Financial Markets Group FMG Discussion Papers dp647.

Dell'Ariccia, Giovanni, Luc Laeven, and Gustavo Suarez. 2017. "Bank Leverage and Monetary Policy's Risk-Taking Channel: Evidence from the United States." *The Journal of Finance*, 72(2): 613–654.

Diamond, Douglas W, and Philip H Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy*, 91(3): 401–19.

Drechsler, Itamar, Alexi Savov, and Philipp Schnabl. 2017. "The Deposits Channel of Monetary Policy." *The Quarterly Journal of Economics*, 132(4): 1819–1876.

Englund, Peter. 2016. "The Swedish 1990s Banking Crisis." *mimeo Stockholm School of Economics*.

Farhi, Emmanuel, and Ivan Werning. 2016. "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities." Harvard University OpenScholar Working Paper 95131.

Financial Stability Board. 2015. "Global Shadow Banking Report 2015." Financial Stability Board.

Fisher, Irving. 1933. "The Debt-Deflation Theory of Great Depressions." *Econometrica*, 1(4): 337–57.

Fostel, Ana, and John Geanakoplos. 2012. "Why does bad news increase volatility and decrease leverage?" *Journal of Economic Theory*, 147(2): 501–525.

Geanakoplos, John. 2010. "The Leverage Cycle." *NBER Macroeconomics Annual*, 24(1): 1–66.

Geanakoplos, John. 2011. "What's Missing from Macroeconomics: Endogenous Leverage and Default." In *Approaches to Monetary Policy Revisited – Lesson from the Crisis, Sixth ECB Central Banking Conference*. 220–238.

Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny. 2012. "Neglected risks, financial innovation, and financial fragility." *Journal of Financial Economics*, 104(3): 452 – 468.

Gersbach, Hans, and Jean-Charles Rochet. 2017. "Capital Regulation and Credit

Fluctuations." *Journal of Monetary Economics*, 90: 113–124.

Gertler, Mark, and Nobuhiro Kiyotaki. 2015. "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy." *American Economic Review*, 105(7): 2011–43.

Gertler, Mark, and Peter Karadi. 2011. "A model of unconventional monetary policy." *Journal of Monetary Economics*, 58(1): 17–34.

Gertler, Mark, Nobuhiro Kiyotaki, and Albert Queralto. 2012. "Financial crises, bank risk exposure and government financial policy." *Journal of Monetary Economics*, 59(S): S17–S34.

Gorton, Gary, and Guillermo Ordoñez. 2019. "Good Booms, Bad Booms." *Journal of the European Economic Association*.

Gourinchas, Pierre-Olivier, and Maurice Obstfeld. 2012. "Stories of the Twentieth Century for the Twenty-First." *American Economic Journal: Macroeconomics*, 4(1): 226–65.

Guerrieri, Veronica, and Guido Lorenzoni. 2017. "Credit Crises, Precautionary Savings and the Liquidity Trap." *Quarterly Journal of Economics*, 132(3): 1427–1467.

Guerrieri, Veronica, and Harald Uhlig. 2016. "Housing and Credit Markets: Booms and Busts." *Handbook of Macroeconomics*, 2B: 1427–1496.

Hellwig, Martin. 2018. "Germany and the Financial Crises 2007-2017." *mimeo Max Planck Institute for Research on Collective Goods and Bonn University*.

He, Zhiguo, and Arvind Krishnamurthy. 2013. "Intermediary Asset Pricing." *American Economic Review*, 103(2): 732–70.

He, Zhiguo, and Arvind Krishnamurthy. 2014. "A Macroeconomic Framework for Quantifying Systemic Risk." NBER Working Paper 19885.

Holmstrom, Bengt, and Jean Tirole. 1997. "Financial Intermediation, Loanable Funds, and The Real Sector." *The Quarterly Journal of Economics*, 112(3): 663–691.

Jermann, Urban, and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review*, 102(1): 238–71.

Jimenez, Gabriel, Steven Ongena, Jose-Luis Peydro, and Jesus Saurina. 2014. "Hazardous Times for Monetary Policy: What Do TwentyThree Million Bank Loans Say About the Effects of Monetary Policy on Credit RiskTaking?" *Econometrica*, 82(2): 463–505.

Kalemli-Özcan, Sebnem, Bent Sorensen, and Sevcan Yesiltas. 2012. "Leverage across firms, banks, and countries." *Journal of International Economics*, 88(2): 284–298.

Kaminsky, Graciela, and Carmen Reinhart. 1999. "The Twin Crises: The causes of banking and Balance of Payments Problems." *American Economic Review*, 89(3): 473–124–500.

Kaplan, Greg, Kurt Mitman, and Giovanni L Violante. 2017. "The Housing Boom and Bust: Model Meets Evidence." National Bureau of Economic Research Working Paper 23694.

Kareken, John H, and Neil Wallace. 1978. "Deposit Insurance and Bank Regulation: A Partial-Equilibrium Exposition." *The Journal of Business*, 51(3): 413–38.

Kekre, Rohan, and Moritz Lenel. 2020. "Monetary Policy, Redistribution and Risk Premia." Unversity of Chicago Booth and Princeton.

Kindleberger, Charles. 1978. *Manias, Panics, and Crashes: A History of Financial Crises.* New York: Basic Books.

Kiyotaki, Nobuhiro, and John Moore. 1997. "Credit Cycles." *Journal of Political Economy*, 105(2): 211–48.

Koijen, Ralph S.J., and Motohiro Yogo. 2019. "A Demand System Approach to Asset Pricing." *Journal of Political Economy*, forthcoming.

Korinek, Anton, and Martin Nowak. 2017. "Risk-taking dynamics and financial stability." *mimeo, Harvard and John Hopkins.*

Krishnamurthy, Arvind, and Annette Vissing-Jorgensen. 2015. "The impact of Treasury supply on financial sector lending and stability." *Journal of Financial Economics*, 118(3): 571–600.

Krishnamurthy, Arvind, and Tyler Muir. 2017. "How Credit Cycles across a Financial Crisis." National Bureau of Economic Research Working Paper 23850.

Laeven, Luc, and Fabian Valencia. 2012. "Systemic Banking Crises Database: An Update." *International Monetary Fund, Working Papers, 12/163.*

Lorenzoni, Guido. 2008. "Inefficient Credit Booms." *Review of Economic Studies*, 75(3): 809–833.

Malherbe, Frédéric. 2015. "Optimal capital requirements over the business and financial cycles." C.E.P.R. Discussion Papers CEPR Discussion Papers 10387.

Martin, Alberto, and Jaume Ventura. 2015. "Financial Reforms and Capital Flows: Insights from General Equilibrium." In . Vol. 21 of *Central Banking, Analysis, and Economic Policies Book Series*, Chapter 7, 109–137. Central Bank of Chile.

Martinez-Miera, David, and Javier Suarez. 2014. "Banks' Endogenous Systemic Risk-taking." *mimeo, CEMFI.*

Martinez-Miera, David, and Rafael Repullo. 2017. "Search for Yield." *Econometrica*, 85(2): 351–378.

Melitz, Marc J. 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica*, 71(6): 1695–1725.

Mendoza, Enrique G. 2010. "Sudden Stops, Financial Crises, and Leverage." *American Economic Review*, 100(5): 1941–66.

Minsky, Hyman. 1977. "The Financial Instability Hypothesis: an Interpretation of Keynes and Alternative to Standard Theory." *Challenge*, 1: 20–27.

Morais, Bernardo, José-Luis Peydró, Jessica Roldá-Peña, and Claudia Ruiz-Ortega. 2019. "The International Bank Lending Channel of Monetary Policy Rates and QE: Credit Supply, Reach-for-Yield, and Real Effects." *The Journal of Finance*, forthcoming.

Nuño, Galo, and Carlos Thomas. 2017. "Bank leverage cycles." *American Economic Journal: Macroeconomics*, 9(2).

Reinhart, Carmen, and Kenneth Rogoff. 2009a. *This Time Is Different: Eight Centuries of Financial Folly.* . 1 ed., Princeton University Press.

Reinhart, Carmen M., and Kenneth S. Rogoff. 2009b. *This Time Is Different: Eight Centuries of Financial Folly.* Princeton, New Jersey:Princeton University Press.

Santos, Tano. 2017. "El Diluvio: The Spanish Banking Crisis, 2008-2012." *mimeo Columbia University.*

Schularick, Moritz, and Alan M. Taylor. 2012. "Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870-2008." *American Economic Review*, 102(2): 1029–61.

Tauchen, George, and Robert Hussey. 1991. "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models." *Econometrica*, 59(2): 371–96.

Wilmarth, Arthur. 2013. "Citigroup: A Case Study in Managerial and Regulatory Failures." *Indiana Law Review*, 47: 69–137.
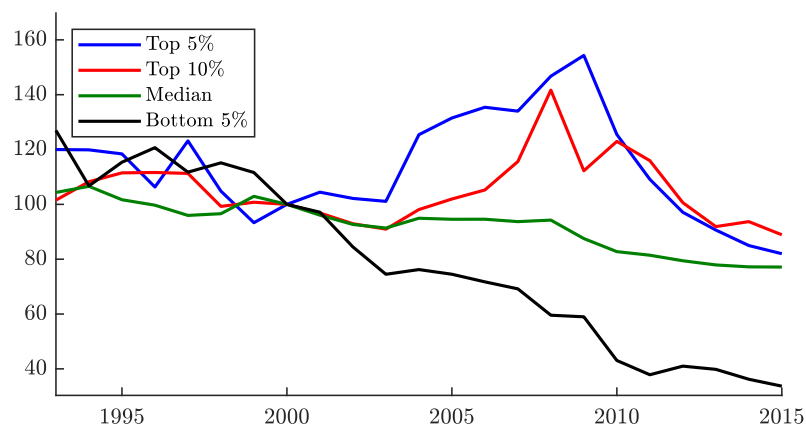
Figures and Tables

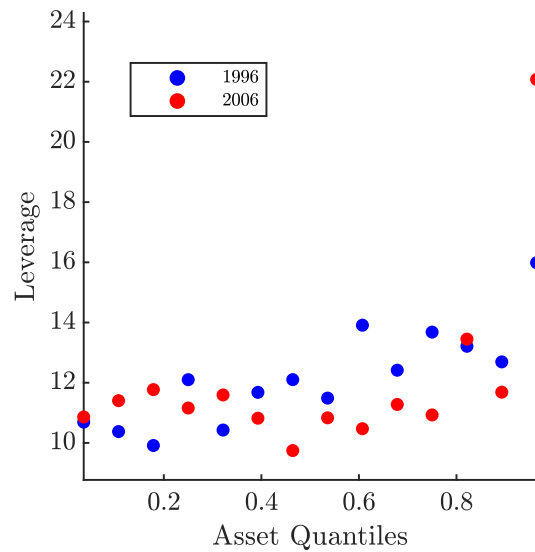Figure 11: Evolution of leverage by quantiles (unweighted, base year=2000)

Figure 12: Binned scatter plot of leverage on asset quantiles

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Table 2: Heterogeneity in leverage and the Fed Funds Rate

|	$Lev_{i,t}$	$Lev_{i,t}$	$\Delta Lev_{i,t}$	$Lev_{i,t}$	$Lev_{i,t}$	$\Delta Lev_{i,t}$
$Lev_{i,t-1}$	0.459***	0.433***		0.449***	0.432***	
	(0.013)	(0.013)		(0.013)	(0.013)	
FF_t	0.066	0.019				
	(0.063)	(0.085)				
$Top5_{i,t}$	26.53***	25.81***	15.04***	26.60***	25.91***	14.99***
	(1.369)	(1.441)	(1.503)	(1.371)	(1.442)	(1.505)
$Top5_{i,t} \times FF_t$	-1.870***	-2.334***		-1.873***	-2.349***	
	(0.287)	(0.403)		(0.286)	(0.402)	
$Top10_{i,t}$		6.488***		6.627***		
		(1.040)		(1.041)		
$Top10_{i,t} \times FF_t$		0.321		0.333		
		(0.309)		(0.309)		
$Median_{i,t}$		2.346***		2.482***		
		(0.562)		(0.566)		
$Median \times FF_t$		0.156		0.160		
		(0.130)		(0.130)		
ΔFF_t			-0.045			
			(0.078)			
$Top5_{i,t} \times \Delta FF_t$			-0.911***			-0.915***
			(0.347)			(0.347)
N	5325	5325	5325	5325	5325	5325
Intercept	Yes	Yes	Yes	Yes	Yes	Yes
Bank FE	Yes	Yes	Yes	Yes	Yes	Yes
Time FE	No	No	No	Yes	Yes	Yes
R^2	0.67	0.62	0.02	0.67	0.61	0.02

***$p < 0.01$, **$p < 0.05$, *$p < 0.1$
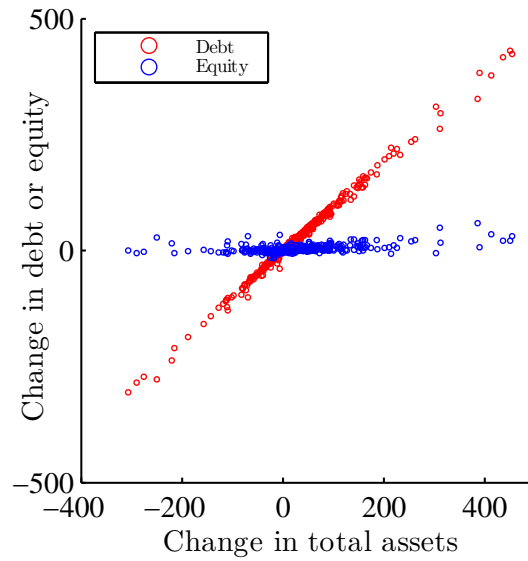
Figure 13: *Yearly changes in total asset against yearly changes in equity or debt from 1993 to 2015. Billions of USD. Source: Bankscope*
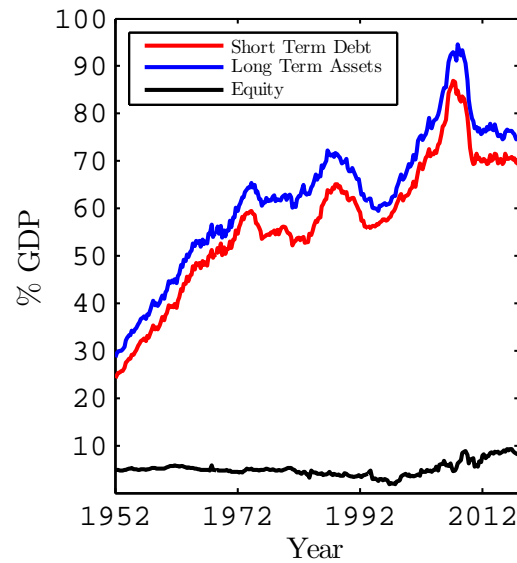

Figure 14: *Bank short-term debt, long-term assets and equity as a percentage of US GDP. Data constructed as in Krishnamurthy and Vissing-Jorgensen (2015).*
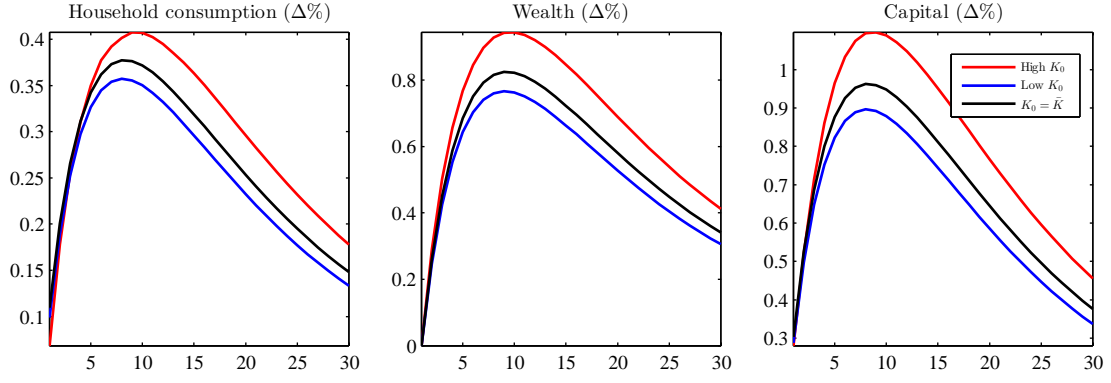
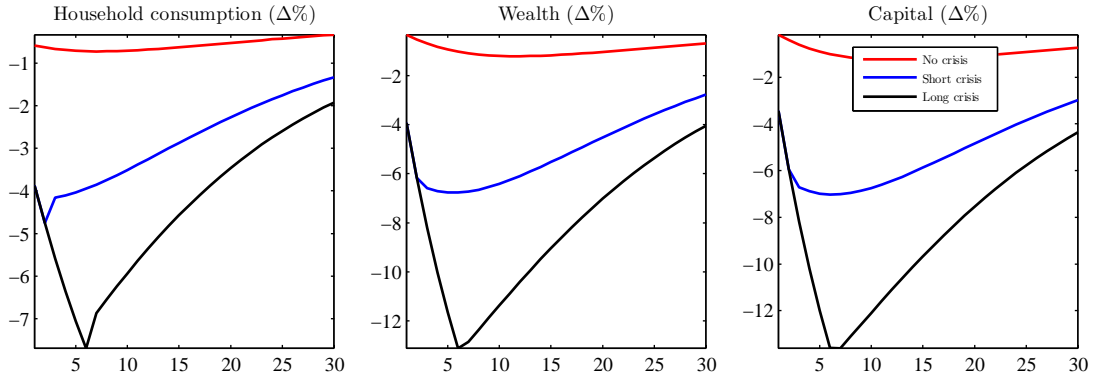Figure 15: *Monetary policy shock of 100 basis points to γ_t: Real variables*

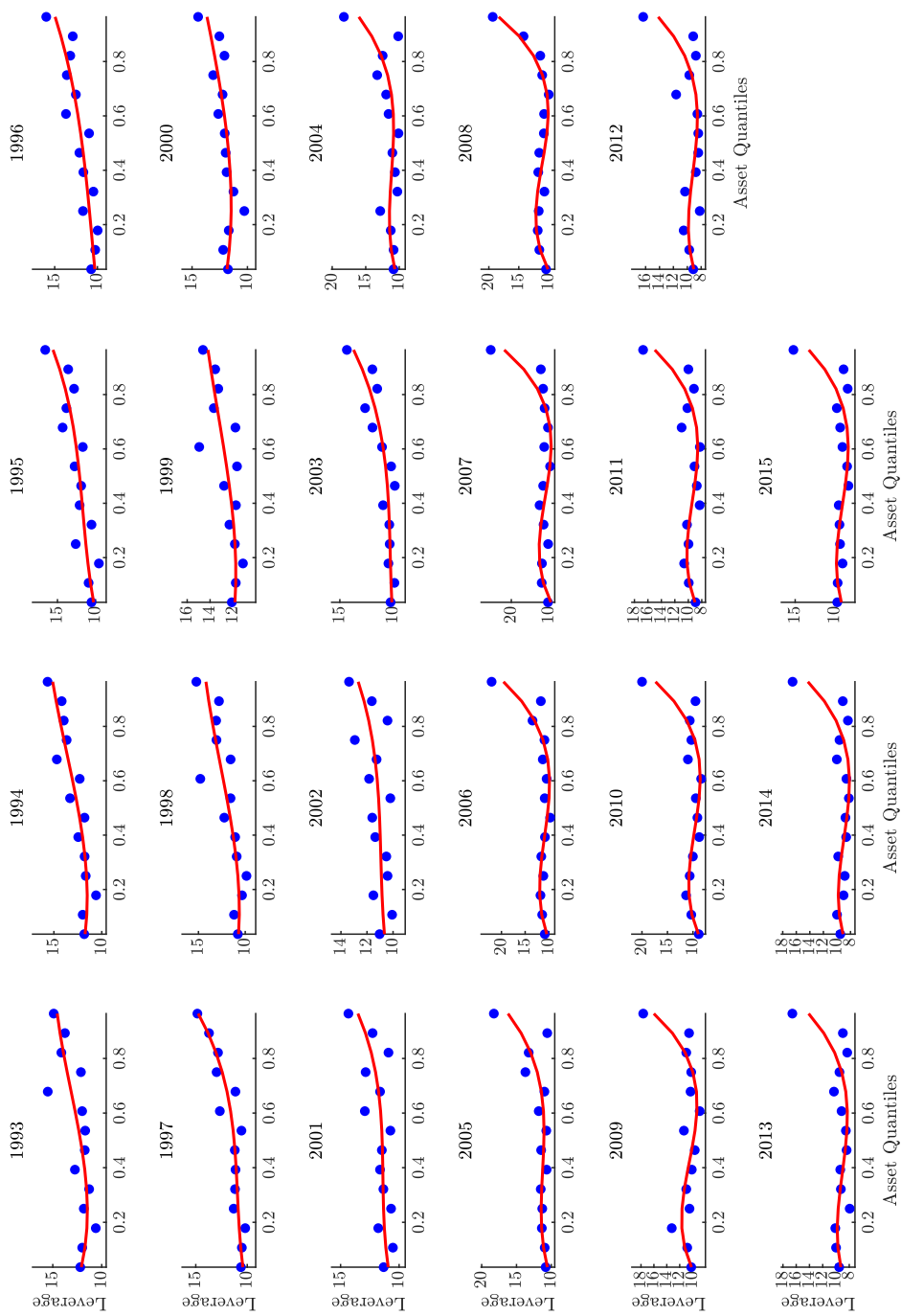Figure 16: *Large shock to exogenous productivity: Real variables*

Figure 17: Binned scatter plot of leverage on asset quantiles for all sample years. Red lines are fitted cubic polynomials.

Appendix A. Numerical solution method

The solution method is composed of two main blocks. The first block solves the partial equilibrium problem for a grid of points for variables r^F and Z^e. We discretize the state space using 100 nodes for Z^e and 200 for r^F. Given funding costs r^F and expected productivity Z^e we can solve jointly for equations (26) and (28), plugging in equation (21) in the latter. We also use the property that levered intermediaries never invest in storage. This gives us policy functions $K^*(r^F, Z^e)$ and $\alpha^{L,*}(r^F, Z^e)$.

The second block is the recursive one. First we define the household savings problem as a function of disposable wealth Ω_t, productivity $\tilde{Z}_t$, efficiency adjustment Δ_t and monetary policy γ_t.

$$\Omega_t = (1 - \theta)Y_t - T_t + D^H_{t-1} + S^H_{t-1}$$

The procedure entails the following steps

1. Discretize the state space $\mathcal{S}$ for the variables $(\Omega, Z, \Delta, \gamma)$. The process for Z and γ are approximated using a Tauchen and Hussey (1991) quadrature procedure with 11 and 7 nodes respectively. The state space for the variable Ω is discretized using 500 nodes and we use 10 for Δ.

2. Iterate on prices r^D and policy function $C^*(\mathcal{S})$ starting with an initial guess $r^D(\mathcal{S})$ for deposit prices and $C^*(\mathcal{S})$. For every point $\mathcal{S}_j \in \mathcal{S}$:

   (a) Using the state vector and r^D_j, calculate r^F_j and Z^e_j.

   (b) Solve for (K_j, α^L_j) using $K^*(r^F_j, Z^e_j)$ and $\alpha^{L,*}(r^F_j, Z^e_j)$. Back out deposit supply D_j from the balance sheet equations.

   (c) Plug D_j in the budget constraint of the agent. Together with $C_j = C^*(\mathcal{S}_j)$ this pins down S^H_j.

   (d) Calculate expectations of $(\mathcal{S}'|\mathcal{S})$ and update deposit prices and policy functions using the optimality conditions and numerical integration.

   (e) Check for convergence. If $||(r'_j - r_j)|| + ||(C^*_j)' - C^*_j||$ is smaller than a threshold value stop. Else, go back to (a) and repeat.

To numerically integrate intermediary variables, Gauss-Legendre quadrature using 51 points is used. To calculate expectations of future net disposable wealth, we also need to calculate taxes conditional on future shocks. For a given productivity draw $Z'|Z_j$ we identify the threshold intermediary for which no bailout is needed: $(R^K k_i - R^D d_i) = \omega$. We can then calculate the amount T_t of taxes required by numerical integration.

Appendix B. Proof of Proposition 3.1

When $\mathbb{E}[R^K_{i,t+1}] \geq 1$, participating intermediary i will either lever up to its Value-at-Risk constraint: $d_{it} = \overline{d}^i_t$, or not raise deposits at all : $d_{it} = 0$.

Given the option value of default and the condition $\mathbb{E}[R^K_{i,t+1}] \geq 1$, participating intermediaries will not invest in storage. The Value-at-Risk constraint bounds the maximum level of leverage of intermediary i, therefore $d_{it} \in [0, \overline{d}^i_t]$. The expected profits of intermediary i as a function of deposits are:

$$\pi^i_t(d_{it}) = (1 - \zeta) \int_{\varepsilon^i_t(d_{it})}^{\infty} \left[R^K_{t+1}(\omega + d^i_t) - R^D_t d^i_t \right] dF(\varepsilon) \tag{45}$$

where ε^i_t is the max of 0 (the lower bound of the support for ε) and the shock for which profits are zero).

$$\varepsilon^i_t(d^i_t) = \max \left(0, \frac{\frac{R^D_t d^i_t}{\omega + d^i_t} - 1 + \delta}{\theta Z^{\rho^Z} K^{\theta-1}_t} \right) \tag{46}$$

Taking derivatives:

$$\frac{\partial \pi^i_t}{\partial d^i_t} = (1 - \zeta) \int_{\varepsilon^i_t(d^i_t)}^{\infty} \left(R^K_{t+1}(\varepsilon) - R^D_t \right) dF(\varepsilon) - \pi^i_t(\varepsilon^i) \frac{\partial \varepsilon^i}{\partial d^i_t} \tag{47}$$

Lemma 1 *Given equations (45) and (46), then* $\pi^i_t(\varepsilon^i_t)\frac{\partial \varepsilon^i}{\partial d^i_t} = 0$

For any $d_{it} \geq \frac{\omega(1-\delta)}{R^D_t - 1 + \delta}$, then $\pi^i_t(\varepsilon^i_t) = 0$ by definition of ε^i_t. For $d^i_t < \frac{\omega(1-\delta)}{R^D_t - 1 + \delta}$, then $e^i = 0$ and $\frac{\partial \varepsilon^i}{\partial d^i_t} = 0$ due to the max operator.

We have as first and second derivatives:

$$\frac{\partial \pi_t^i}{\partial d_{it}} = (1 - \zeta) \int_{\varepsilon_t^i(d_t^i)}^{\infty} \left(R_{t+1}^K(\varepsilon) - R_t^D \right) dF(\varepsilon)$$

$$\frac{\partial^2 \pi_t^i}{\partial d_{i,t}^2} = -(1 - \zeta) \left[R_{t+1}^K(\varepsilon(d_{it})) - R_t^D \right] \frac{\partial \varepsilon_t^i}{\partial d_{it}} \tag{48}$$

Given the monotonicity of $R_{t+1}^K(\varepsilon)$, then $\forall \tilde{d}$ such that $\left. \frac{\partial \pi_t^i}{\partial d_{it}} \right|_{\tilde{d}} = 0$, it follows that $R_{t+1}^K(\varepsilon_t^i(\tilde{d})) - R_t^D < 0$ or all elements in the integral are non-negative and it cannot be zero. Since $\frac{\partial \varepsilon_t^i}{\partial d_{it}} > 0$, then $\left. \frac{\partial^2 \pi_t^i}{\partial d_{it}^2} \right|_{\tilde{d}} > 0$ by equation (48). If $\tilde{d}$ exists, it must be a minimum and we therefore conclude that the maximum must be at the bounds: $d_{it} = \arg\max \left(\pi_t(0), \pi_t^i(\overline{d}_t^i) \right)$.

Appendix C. Alternative Measures of Financial Stability

We present three alternative measures of financial stability. M^3 is the *asset-weighted mean of active* α^i. We have that $M_t^3 = \int_{\alpha_t^L}^{\overline{\alpha}} \alpha^i \frac{k_{it}}{K_t^L} dG(\alpha^i)$, where K_t^L is the total asset holdings of leveraged intermediaries. This measure has the advantage of not only capturing the extensive margin effect but also capturing the effect of skewness on aggregate financial stability. That is, a financial sector with the same cut-off α_t^L but with a more skewed distribution of leverage will on aggregate be more risky, as a larger share of the capital would be held by more risk-taking intermediaries.

We also explore a fourth measure of financial stability M^4: the *probability that a fraction κ of the capital K_t^L is held by distressed intermediaries in the next period*. M_t^4 is the solution to the equation $\int_{M_t^4}^{\overline{\alpha}} k_{it} dG(\alpha^i) = \kappa K_t^L$. This measure would be equivalent to the baseline measure M^1 if we set $\kappa = 1$, so it can be seen as a generalization of the first measure. Setting this fraction to a lower value captures some of the skewness effects mentioned. We implement this measure with a fraction arbitrarily set at $\kappa = 0.5$, so the probability that half of the capital is held by distressed intermediaries in the next period.

Finally we also calculate a fifth measure, M^5: *the expected share of capital held by defaulting intermediaries at $t + 1$*. This measure relates to the costly default described in Section 6. If the deadweight loss is proportional to the share of capital held by

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defaulting intermediaries, then this measure gives us a sense of the expected efficiency costs of decreasing financial stability. Figure 18 shows theses measures as a function of the interest rate in partial equilibrium. All of them show a significant adverse effect of an interest rate decrease on financial stability when interest rates are low. In contrast financial stability does not worsen with a decrease in the interest rate when the level of the interest rate is high.
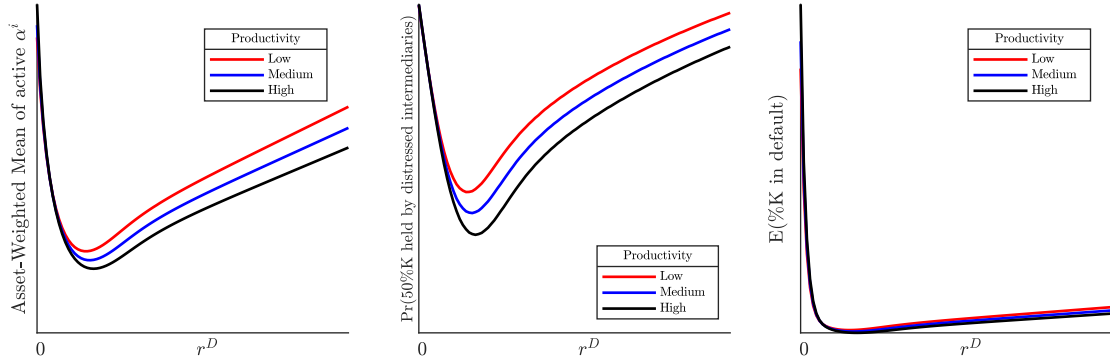

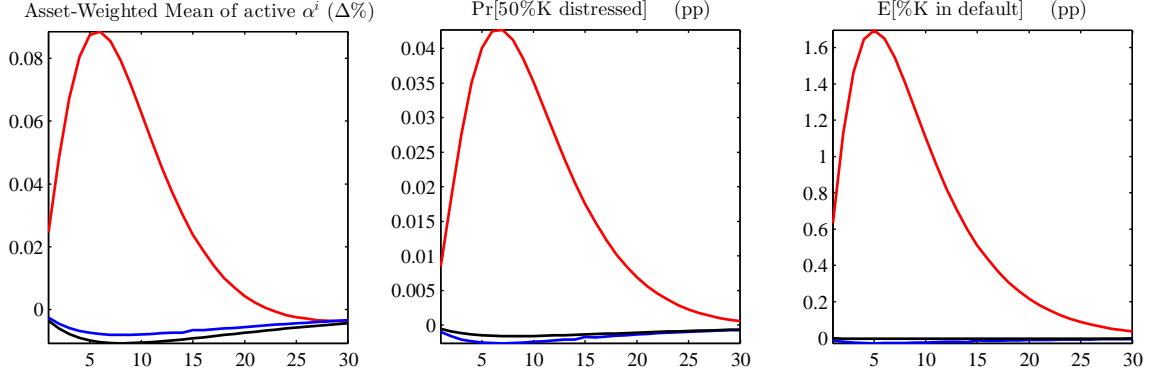Figure 18: *Alternative measures of financial stability and interest rate*

Figure 19: *Monetary policy shock of 100 basis points to γ_t: Alternative measures*

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Appendix D. Data Description

Bank balance sheet data uses annual data from the Bankscope database. Bank return data are from Datastream. Market returns were calculated using the MSCI World Index data available from Bloomberg. The Effective Federal Funds Rate and the CPI are from the Federal Reserve Economic Data.

The leverage ratio is defined as the ratio at book value of total assets to total equity, here defined as common equity. We drop negative equity from the dataset, and institutions with assets worth less than 1 million USD. We also remove institutions that have leverage larger than 1000 at least once across the sample.

For the leverage series, we compute both unweighted and weighted averages of the leverage ratio for each quarter. For the weighted average we use total assets as weights. We checked using total equity as weights and results are qualitatively unchanged.

Year	Assets		Equity		Leverage			#Obs
	Mean	St.Dev.	Mean	St.Dev.	Mean	St.Dev.	Skewness	
1993	12419	39962	737	1930	14.71	13.77	10.67	276
1994	13725	44263	825	2179	14.81	10.95	5.47	329
1995	14638	42323	898	2200	14.33	11.16	5.73	349
1996	14278	35918	912	2109	14.82	16.26	9.48	364
1997	16778	42793	1073	2469	14.57	16.96	10.44	369
1998	18661	47154	1290	3158	13.71	10.69	5.67	392
1999	21478	64999	1526	4632	14.30	13.26	7.95	437
2000	24176	77576	1768	5599	14.04	15.11	10.79	456
2001	27648	87629	2079	6843	14.00	17.40	11.10	446
2002	28693	91765	2224	7328	14.09	25.57	13.34	458
2003	32012	103752	2414	8030	14.52	28.92	11.23	454
2004	44022	149190	3160	10731	14.58	17.96	7.65	444
2005	46877	152941	3287	10794	16.28	27.52	7.77	468
2006	56112	193439	3697	12586	15.69	24.76	8.28	422
2007	71684	245968	4458	14634	15.23	19.00	5.78	396
2008	67652	258317	4359	16032	15.59	27.13	13.73	374
2009	63755	240442	4872	17701	14.54	17.88	6.21	401
2010	66402	232329	5191	17920	12.90	11.93	4.77	448
2011	64599	235938	5318	18441	12.96	20.23	13.19	461
2012	65011	238222	5570	19317	12.51	21.56	13.29	467
2013	69559	249110	6468	22352	10.69	7.47	3.18	469
2014	70429	245651	6734	23223	10.58	7.71	4.31	475
2015	67587	228692	6923	23808	10.43	7.31	3.49	439

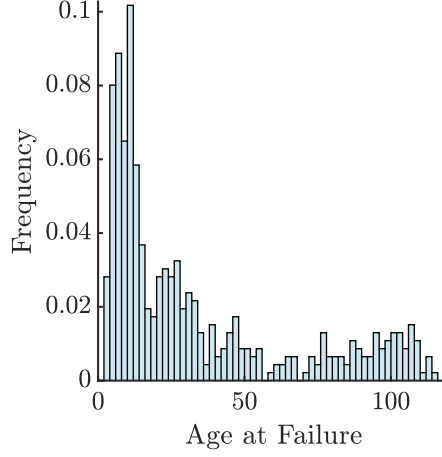Table 3: Descriptive cross-sectional statistics by period (unweighted).

Figure 20: *Histogram of age of banks at closing date (in years). Data for failures in the US since October, 2000. Source: FDIC.*

Appendix E. Interbank market

In this appendix, we present a version of the baseline model where intermediaries can supply funds to each other through deposits. The main difference in the financial intermediary problem, is that inactive intermediaries will optimally choose to deposit their net worth, thus supplying funds to leveraged banks. These deposits are also guaranteed by the government and therefore the same asset as household deposits from the point of view of the borrowing bank.

Whenever $R_t^D > 1$, storage is dominated by deposits and will never be used. Inactive intermediaries will also optimally prefer to hold deposits over shares of the capital stock. Since intermediaries are risk-neutral, the presence of an option value of default implies that in equilibrium $R_t^D > E(R_{t+1}^K)$. Since inactive intermediaries will not be able to exploit the option value of default, they strictly prefer deposits over shares of the capital stock, implying $\alpha_t^N = \alpha^L$ for all t. The balance sheet of an intermediary i that chooses to lend its net worth is then:

Assets	Liabilities
$-d_{it}$	ω_{it}

where to maintain consistency in notation, deposits held as assets are noted as negative d_{it}. The intermediary program is as before:

$$V_{it} = \max \ \mathbb{E}_t(c_{i,t+1}) \tag{49}$$

$$\text{s.t.} \ \ \Pr(\pi_{i,t+1} < \omega_{it}) \leq \alpha^i \tag{50}$$

$$k_{it} + s_{it} = \omega_{it} + d_{it} \tag{51}$$

$$c_{i,t+1} = \max\left(0, \pi_{i,t+1}\right) \tag{52}$$

$$\pi_{i,t+1} = R^K_{i,t+1} k_{it} + s_{it} - R^D_t d_{it}$$

Since borrowing to deposit is revenue neutral, it follows that Proposition 3.1 again holds in this case. Each intermediary will choose to leverage up to its VaR constraint or not raise deposits at all. Writing the value functions under this case we have

$$V^L_{it} = \mathbb{E}_t \left[\max\left(0, R^K_{i,t+1} k_{it} - R^D_t d_{it}\right) \right] \tag{53}$$

$$V^N_{it} = R^D_t \omega \tag{54}$$

$$V^O_{it} = \omega \tag{55}$$

The deposit market clearing equation is as before:

$$D_t = \int d_{it} dG(\alpha^i) = D^H_t \tag{56}$$

With the difference that now D_t is the *net* borrowing from the financial sector as a whole. The market clearing is $D_t = D^H_t$, where D^H_t are total household deposits. We also define $D^L_t = \int_{\alpha^L_t}^{\overline{\alpha}} d_{it} dG(\alpha^i)$ as the total deposit liabilities in levered intermediaries. Equation (41) then becomes:

$$F_t = \frac{D^L_t}{1 + \chi} \tag{57}$$

The rest of the equations of the model are exactly the same, but underlying them are a few key differences. All capital is now held by levered intermediaries, which implies that

no fraction of the capital stock is ever free from potential distress at $t + 1$. Moreover, the extensive margin now also affects the deposit supply. The more intermediaries drop out from levered markets, the larger is aggregate deposit supply (ceteris paribus). Partial equilibrium results are very similar to the ones without the interbank market, as can be seen in Figure 21. Note that for the aggregate capital stock supply curve, the two models are almost indistinguishable.
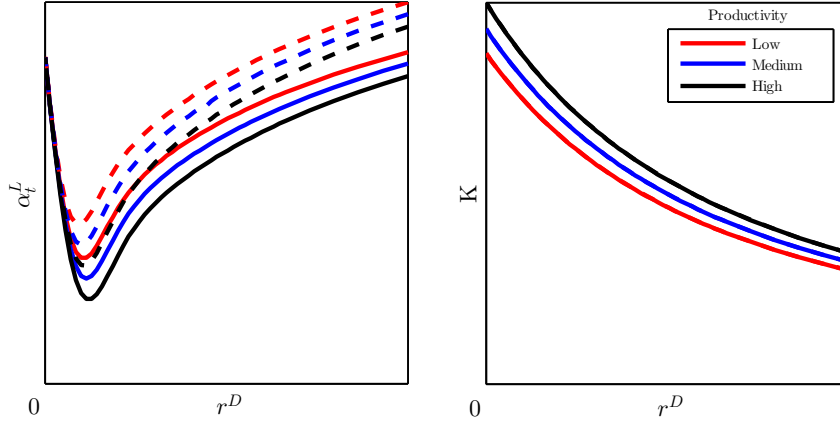

Figure 21: *Cut-off level α_t^L and aggregate capital stock as a function of deposit rates r_t^D in the model with an interbank market (full lines). For comparison, the baseline model is also plotted (dotted lines).*

The main difference in partial equilibrium is that for a given interest rate, the cut-off is now lower. Non-active intermediaries no longer invest directly in the capital stock. Had leverage and the cut-off remained the same the capital stock would be smaller and returns higher. This leads to both higher leverage from intermediaries above the cut-off (intensive margin) and a lower cut-off (extensive margin).

In general equilibrium, the main results are extremely similar to our baseline model as can be seen in figures 23 and 24. The main difference seems to be in the behaviour of the cut-off where the baseline model seems to have additional amplification, particularly away from the steady-state.
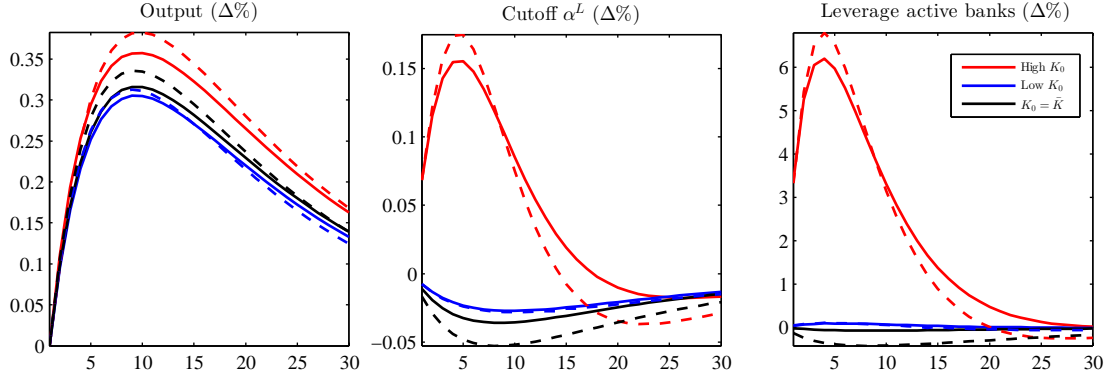
Figure 22: *Monetary policy shock of 100 basis points to γ_t in the model with an interbank market (full lines). For comparison, the baseline model is also plotted (dotted lines).*
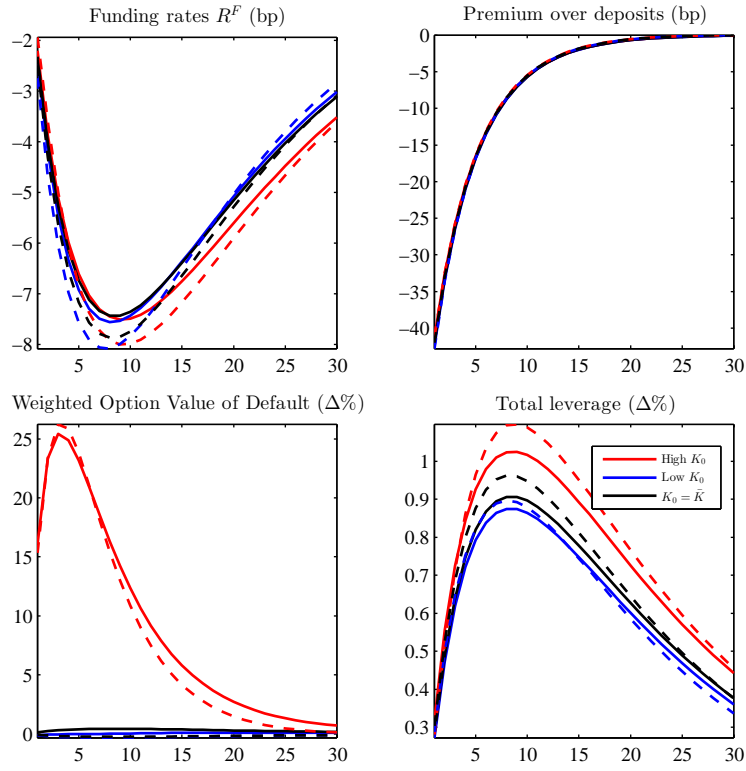

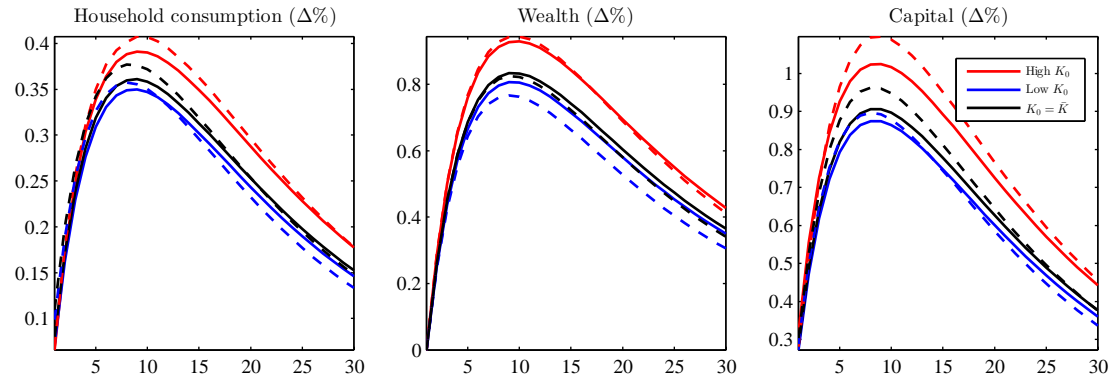Figure 23: *Monetary policy shock of 100 basis points to γ_t: Financial variables*

Figure 24: *Monetary policy shock of 100 basis points to γ_t: Real variables*

Appendix F. Comparative statics on size of equity and volatility

Here we explore the role of volatility and net worth in the financial block of the model. We perform two exercises. In the first one we change the parameter σ_z, governing the exogenous volatility of the TFP process. As we can see in figure 25, the main change is in the composition of the financial sector. When volatility is higher, the VaR is tighter and therefore the intensive margin is reduced. Leverage from active intermediaries is lower, which leads to both lower capital stock and cut-off α^L. As it turns out, the lower is volatility, the easier it is for more risk-taking intermediaries to capture more of the market due to the loosening of VaR constraints.

We also look at the effect of changing the parameter ω, the endowment of net worth received by intermediaries. As can be seen in Figure 26, the effect is almost purely compositional with almost no effect on the total amount of capital (differences too marginal to show up in the graph). Given that the right hand side of equation (21) is independent of ω, then changing net worth is just allowing the more risk-taking intermediaries to acquire more assets (given aggregate variables). As with lower volatility, the higher ω is the easier it is for more risk-taking intermediaries to capture a larger share of the market.
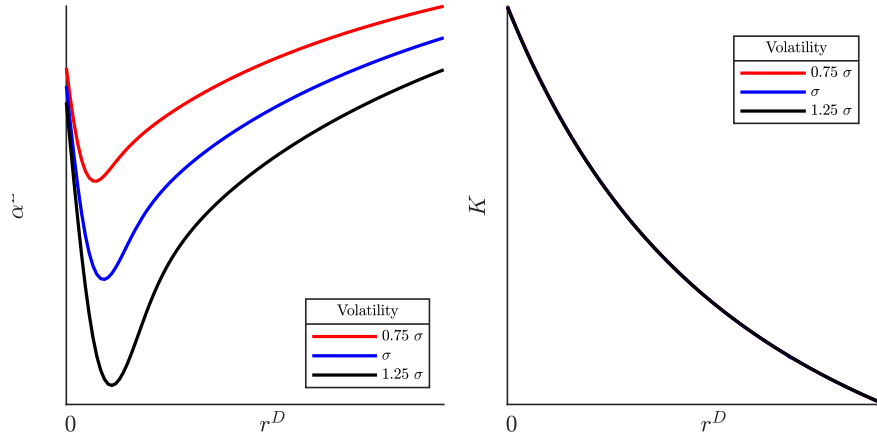

Figure 25: Comparative statics on volatility and interest rates for the financial sector block
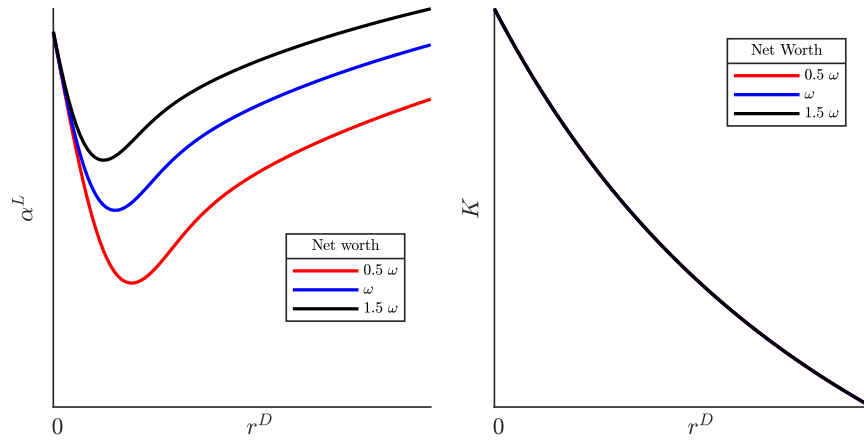
Figure 26: Comparative statics on net worth and interest rates for the financial sector block