Exorbitant Privilege and Exorbitant Duty*

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Abstract

We provide a quarterly time series of the historical evolution of US external assets and liabilities at market value on the 1952-2016 period. The center country of the International Monetary System enjoys an "exorbitant privilege", a sizeable excess return of gross external assets over liabilities that significantly weakens its external constraint. In exchange for this "exorbitant privilege" we document that the US provides insurance to the rest of the world, especially in times of global stress. We call this the "exorbitant duty" of the hegemon. During the 2007-2009 global financial crisis, wealth transfers from the US to the rest of the world amounted to about 19% of US GDP. We present a stylized model that accounts for these facts and links the shrinking size of the hegemon in the world economy to the decline in the world real rate of interest.

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1 Introduction

Understanding the structure of the International Monetary System is an important task. In a world becoming more multipolar and where the centre of gravity of economic activity is gradually shifting, it is paramount to analyse the role of reserve currency issuers and understand better how international financial flows and positions reflect the international monetary order. The existence of a lasting "exorbitant privilege" -a higher return on US external assets than on its external liabilities- is an important and intriguing stylized fact in international economics (see Gourinchas and Rey (2007a)). One direct consequence of the exorbitant privilege is to relax the external constraint of the U.S., allowing it to run larger trade and current account deficits without worsening its external position commensurately. Understanding the source of this exorbitant privilege is an important step in pinning down the nature of the adjustment process for the U.S., as well as analysing how the International Monetary System works. From that perspective, the recent financial crises provide new and important empirical observations: the dramatic worsening of the net foreign asset position of the United States between the fourth quarter of 2007 and the first quarter of 2009 as the financial system was melting down. The precipitous fall of a magnitude amounting to 19% of GDP is both due to flows (with the foreigners buying US assets on a net basis) and to a dramatic adjustment in valuations (the price of US holdings abroad contracting more than the rest of the world holdings in the US) in the amount of about 13%. Another large fall in the US net foreign asset position of about 17% of GDP occured when the Eurozone crisis unfolded between 2010 fourth quarter and 2012 second quarter. These last developments contrast with the usual "exorbitant privilege" whereby the US gets a transfer from the rest of the world in the form of an excess return on its net foreign asset position. In contrast, during a crisis, wealth flows from the US to the rest of the world. We call this phenomenon the "exorbitant duty" of the US: in times of global stress, the US effectively provides insurance to the rest of the world. We argue that the "exorbitant duty" and the "exorbitant privilege" are the two sides of the same coin. They reflect the structure of payments associated with an implicit insurance contract between the U.S., who is at the center of the International Monetary System and the rest of the world.

The paper has three main contributions: a) we document the "exorbitant privilege" Gourinchas and Rey (2007a) for the 1952-2016 period using in particular recently unearthed historical surveys of cross-border holdings; (b) we document the "exorbitant duty" i.e. the economic magnitude of the payments from the US to the rest of the world in the recent crises. We show that this insurance mechanism was also there during earlier episodes of global stress; (c) importantly, we provide a simple calibrated model that allows us to make sense of the patterns of external returns and of the structure of the International Monetary System since the Second World War.[1] In the model, the hegemon (the US) provides insurance to the rest of the world (ROW) since it is assumed to have a greater risk tolerance. This asymmetry in risk tolerance captures a host of potential mechanisms by which the US economy may be able to better handle economic and financial risks[2]. The model is able to reproduce the following features: (i) the US has an exorbitant privilege in normal times and an exorbitant duty in times of global stress; (ii) the US takes long positions in risky assets and short positions in safe assets; (iii) when the size of the US decreases relative to the ROW, the relative demand for insurance goes up and the US real rate goes down; (iv) the real exchange rate of the US appreciates when the probability of a crisis goes up and risk aversion in the ROW goes up: there is flight to safety.

Our interpretation of the International Monetary System is therefore one as a structure in which the hegemon provides insurance to the rest of the world in exchange for an insurance premium. A natural question arising out of this interpretation is to ask what happens to the real rate, the price of insurance and to net external exposures when the size of the hegemon shrinks in the world economy. Our calibrated model shows that in the case of a "vanishing hegemon" there is a decline in the safe real rate of interest as demand for safety increases and net exposure of the hegemon vis-a-vis the rest of the world increases. This opens up the possibility that a loss confidence in the capacity of a vanishing hegemon to deliver on its exorbitant duty could

[1] Emmanuel and Matteo (2016) interpret the exorbitant privilege as a monopolistic rent rather than as an insurance premium and analyse the optimal issuance of safe assets by the hegemon. He et al. (2015) study the preeminence of the Dollar in a coordination game.

[2] For a possible microfoundation, albeit only for the banking sector, see Maggiori (2011) who assumes that frictions in the ROW banking sector are larger than in the US.

lead to a Triffin (1960) type problem. Gourinchas and Rey (2007a)) emphasizes the possibility of this New Triffin Dilemma when there is an alternative reserve asset and writes: "In a world where the US can supply the international currency at will, and invests it in illiquid assets, it still faces a confidence risk. There could be a run on the dollar not because investors would fear an abandonment of the gold parity, as in the seventies, but because they would fear a plunge in the dollar exchange rate." [3]

In the second section of the paper we present our empirical results on exorbitant privilege and exorbitant duty. We develop our model of the international monetary system in section 3. Section 4 presents the results of the model. Section 5 concludes.

2 External balance sheet structure and returns

Financial globalization started in the 1980s and substantially accelerated in the 1990s, as evidenced by the massive surge in gross external assets and liabilities as a fraction of GDP. A recent burgeoning literature has extracted interesting stylized facts from cross country data on international investment positions (see Lane and Milesi-Ferretti (2001) for a classic contribution). Studying the composition of the balance sheet of countries is increasingly important to understand the dynamics of countries' external adjustment. The traditional trade channel of adjustment, whereby current account deficits have to be compensated by future export surpluses has to be supplemented by a valuation channel, which takes into account capital gains and losses on the foreign asset position due to fluctuations in asset prices (Gourinchas and Rey (2007b)). An asymmetric structure of assets and liabilities, for example when assets and liabilities are in different currencies, leads to a very different adjustment process than a symmetric balance sheet. US external assets are mostly denominated in foreign currencies while US external liabilities are in dollars (Tille (2004), Lane and Shambaugh (2007)). It follows that a dollar depreciation gives rise to wealth transfers from the rest of the world to the United States. Similarly, earning excess returns on average on its external asset position allows a country to run larger current account

[3] see Farhi et al. (2011), Obstfeld (2011) and Emmanuel and Matteo (2016) for recent discussions of the New Triffin dilemma.

deficits than it would otherwise, as the deterioration of the net international asset positions is muted by the capital gains.

Gourinchas and Rey (2007a) showed that the US earns an important average excess return on its net foreign asset position on the period 1952-2004. This finding fits well with the observation that in recent years, recent crisis excluded, the net international investment position of the United States has deteriorated at a speed significantly smaller than the current account deficit data would have suggested as shown clearly in Figure (5) where the net foreign asset positions is considerably less negative than the cumulation of the current account deficits would suggest. Gourinchas and Rey (2013) provides an extensive discussion of the subsequent literature estimating the excess returns of the US on its net foreign asset position. The 2007-2009 global financial crisis period saw a large negative return on US net foreign asset position (a large valuation loss) amounting to about 13% of GDP. This negative return came about because of several factors. First, it is the result of a collapse in the price of world wide risky assets held by the US at a time where the safe liabilities issued by the US either remained stable in value or increased. Second, it is a consequence of the appreciation of the US dollar. Since the currency composition of the US external balance sheet is very asymmetric (about two-thirds of external assets are in foreign currencies and almost all external liabilities are in dolllars), an appreciation of the dollar leads to a decrease in the dollar value of the assets, which translates into a valuation loss. Gourinchas et al. (2012) shows that the exchange rate appreciation accounts for about 31% of the total valuation effect on the net foreign asset position between 2007Q4 and 2009Q1. The dollar appreciation itself was contemporaneous to an increase in the demand for insurance from the rest of world: there were inflows in what the market considered as the safest assets at the time: US Treasuries and government bonds.

2.1 Data and methodology

This paper takes a fresh look at the historical evolution of the United States external position over the postwar period, including the recent crisis, by carefully constructing the US gross asset and liability positions since 1952 from underlying data and applying appropriate valuations to each components. The data construction methodology is described in Appendix A. Relative to

our former work (Gourinchas and Rey (2007a)) we improve our existing dataset along several dimensions. We have disaggregated our data into government and corporate bonds on the bonds liability side and improved our measure of income flows for each type of assets[4]. Importantly, we set initial net foreign asset positions using detailed Treasury surveys realized during the second world war. The 1943 Treasury Census of American-owned assets in foreign countries and the 1941 Treasury Census of foreign-owned assets in the US. Those surveys are detailed and reliable as they were of strategic importance for the United States while fighting against the Axis and for reparation payments after the war.[5] The post-war estimates of the US net foreign asset position are based on these surveys on positions and measures of international capital flows. Since capital controls were in place during the Bretton Woods period, the resulting estimates are quite precise as well. For the latter part of the sample we reconstruct the time series of the international investment position of the United States at market value and quarterly frequency from 1952:1 until 2012, benchmarking our series on the Bureau of Economic Analysis (BEA) official annual IIP positions. Finally, for the purpose of this paper we do not include on the asset side gold reserves as changes in their valuations do not provide insurance to the rest of the world: they are not rest of the world liabilities (see also our recent Handbook chapter (Gourinchas and Rey (2013)).The data construction is described in details in Appendix A. One issue is the reconciliation of flow and position data often coming from different sources. The discrepancy between the two, labeled 'other changes' by the BEA, has been a residual item of significant size in recent years. A correct measure of the true returns on the net foreign asset position requires that this residual item be allocated between unrecorded capital gains, unrecorded financial flows, or mismeasured initial net asset position. Appendix A discusses formally how different measures of returns can

[4]For discussions see Lane and Milesi-Ferretti (2009) and Curcuru et al. (2008)

[5]As explained in the foreword of the 1941 Survey: "On April 1940, when Germany invaded Denmark and Norway, the President of the United States issued an Executive order freezing the dollar assets of those two countries and their nationals. [...]. Tens of thousands of banks, corporations and individuals in this country were required to file, on form TFR-300, reports giving detailed information with respect to foreign owned assets and the owners [...] Never before was as complete information available for analyses of the holdings of foreigners in this country." The information contained in these surveys was of great strategic value to the United States. The 1941 Survey reports (p5) that "investigations to uncover enemy agents and enemy assets, especially after our entry into the war, were greatly facilitated by the TFR-300 information." The 1943 Survey on American owned assets abroad "had its principal use in the war settlements and the postwar period generally, although it provided much greatly needed information during the latter part of the military phases of the war."

be constructed under these different assumptions. Importantly, while the different assumptions have some impact on our calculated returns, they have no effect on our overall results: over long periods of times, the U.S. has experienced a high return on its net foreign assets, the 'exorbitant privilege'.

2.2 The "exorbitant privilege"

During the Bretton Woods era, the very special role of the United States at the centre of the international monetary system was often lamented in French quarters. Besides finance minister Giscard d'Estaing, who coined the term "exorbitant privilege" in 1965[6], economic advisor Jacques Rueff around the same time described the Dollar as a "boomerang currency": the sizable external deficits of the US were not matched by commensurate gold losses, as creditor countries reinvested the dollar gained in their exports payments into the US economy.[7] We adopt a somewhat narrower definition of the 'exorbitant privilege', consistent with our earlier contribution. In this paper as in our ealier work (Gourinchas and Rey (2007a)), it refers to the excess return of US external assets on US external liabilities.

We find that the excess total return of US gross external assets over its gross external liabilities is worth about to 2 to 3% per year between 1952 and 2016. Our results on external returns are reported in Table 1 with our benchmark estimates (at about 2.5%) in panel (c). Since exchange rate movements are an important component of capital gains and losses, we isolated the Bretton Woods and the Post Bretton Woods period. Interestingly, the magnitude of the "exorbi-

[6]The term is reported by Raymond Aron in Le Figaro, February 16, 1965, from Les Articles du Figaro, vol. II, 1994, Paris: Editions de Fallois, pp.1475.

[7]"The process works this way. When the U.S. has an unfavorable balance with another country (let us take as an example France), it settles up in dollars. The Frenchmen who receive these dollars sell them to the central bank, the Banque de France, taking their own national money, francs, in exchange. The Banque de France, in effect, creates these francs against the dollars. But then it turns around and invests the dollars back into the U.S. Thus the very same dollars expand the credit system of France, while still underpinning the credit system in the U.S. The country with a key currency is thus in the deceptively euphoric position of never having to pay off its international debts. The money it pays to foreign creditors comes right back home, like a boomerang The functioning of the international monetary system is thus reduced to a childish game in which, after each round, the winners return their marbles to the losers The discovery of that secret [namely, that no adjustment takes place] has a profound impact on the psychology of nations This is the marvelous secret of the deficit without tears, which somehow gives some people the (false) impression that they can give without taking, lend without borrowing, and purchase without paying. This situation is the result of a collective error of historic proportions."in Rueff (1971).

tant privilege" has remained realtively constant over time for our favoured estimates from about 2.6% between 1952:1-1972:4 to 2.4% during 1973:1-2016:1. At the same time the volatility of the leveraged US portfolio has increased during the fluctuating exchange rate period as evidenced in Table 2. Indeed the volatility of external liabilities -almost exclusively in Dollars- is almost unchanged over the whole sample while the volatility of external assets, very low during the Bretton Woods era increased substantially after the collapse of the fixed exchange rate system[8]. In these benchmark estimates, we have allocated the 'other changes' term of the BEA -resulting from the reconciliation of flows and position data in official statistics- in the following way. The residual error term is attributed to flow data for portfolio investment (whether debt or equity) and for bank credit. It is attributed to valuations for FDI. Our rationale for doing so is the following. Going back to the BEA's Survey of Current Business narrative account for the change in net foreign asset position, there is convincing evidence that debt inflows may have been overstated, as redemptions may not always have been accounted for properly.[9] It is also possible that portfolio equity flows and bank credit flows are mismeasured, which also leads us to conservatively allocate the error terms to the flows for these asset classes. This guarantees that we obtain conservative estimates of the excess returns as the residual items reported by the BEA tend to be negative on the liability side (reducing external liabilities) and positive on the asset side (increasing external assets). Hence excluding them from valuations tends to lower excess returns (as can be checked below). For foreign direct investment, on the other hand, it is likely that flows are better measured and that valuations are imprecise, hence our choice to allocate the residual error term to mismeasured capital gains (or losses).The estimated positive excess returns are however robust to the assumptions one could make on the the allocation of errors in the data.

For example, as a robustness check, one possible assumption is to allocate *all* mismeasured items in the evolution of the international investment position to mismeasured capital gains as in Gourinchas and Rey (2007a). As discussed in the Appendix, this is the only assumption that leaves both measured positions and the recorded net exports unchanged, which is an important

[8]We also note the large volatility of stock markets druing this period where between 11 January 1973 and 6 December 1974, the New York Stock Exchange's Dow Jones Industrial Average benchmark lost over 45% of its value and the London Stock Exchange FT30 lost 73% of its value.

[9]See Lane and Milesi-Ferretti (2009) for a thorough discussion.

benchmark. These results are reported in Table 1 panel (a). We note that for the whole period 1952:1-2016:1 the excess returns of external assets r^a over liabilities r^l are sizable at 3.1%, corroborating the statement that including the residual 'other changes' in flows rather than in the valuations tends to decrease the estimates of the excess returns.

An alternative assumption (diametricallly opposed to the previous one) would be to allocate the residuals for all asset classes to mismeasured flows, as in Curcuru et al. (2008). Since the current account balance is the counterpart of the financial account in the balance of payments, this would imply a mismeasurement of net exports on the order of 15% of exports on average in the recent period[10]. Under these assumptions, excess returns go down -as expected- but they are still positive. They amount to 1.9% on the 1952-2016 period (see Table 1 panel (c)).

Any hybrid case can of course be considered. We also present in panel (d) of Table 1 estimates in which we have completely shut down valuation effects on FDI by artificially equating returns of FDI assets to the ones of FDI liabilities. As a result we obtain what we consider possibly a lower bound for our estimates of the exorbitant privilege. It still stands at about 1% for the 1952-2016 sample.

We conclude that under a set of reasonable alternatives, the excess return of US external assets on external liabilities is large, around 2% per annum.

The country at the centre of the international monetary system acts as an international liquidity provider. As such its external balance sheet is remarkable, featuring large gross liquid liabilities and investment in mostly long term risky assets. Such a balance sheet reflects the traditional maturity transformation activity of a bank. Figures (5) and (5) report the breakdown of gross assets and liabilities into portfolio equity and debt, direct investment and bank credit. The large amount of debt, particularly government debt is striking on the liability side of the balance sheet. This is in sharp contrast to the asset side of the balance sheet where total debt constitutes

[10]This is not impossible, but may be at odds with the Bureau of Census' perception that the introduction of ARES, a new electronic system to record exports, at the end of the 1990s and its generalization after 2001 (98% coverage in 2002) has led to more accurate exports data. A 15% measurement error year-on-year would dwarf the upper bound of the Census of 10% for export mismeasurement referring to data *before* 1998 (in fact reconciliation studies produce numbers which are more in the 3 to 7% range pre-1998). Imports tend to be well-measured because of custom duties.

only a small share. US government debts is seen as a relatively safe asset by the ROW who demands large quantities of it. Portfolio equity and foreign direct investment are more prevalent on the asset side of the balance sheet. Those observations are visually conveyed by Figures (5) and (5). Figure (5) presents the importance of liquid safe liabilities (debt and bank credit) as a share of total liabilities and the share of risky assets (portfolio equity and direct investment) in total assets. Since 1952, the share of liquid liabilities amounts to between 55% and 80% of total liabilities (except during the tech bubble where it dipped to 50%) while the share of risky assets in total assets has gone up from about 40% at the beginning of the sample to 65% in the recent period (with a dip in the 1980s to about 30%). This increased exposure to risky investment is made vividly clear in Figure (5): the net portfolio equity and direct investment position of the US is shown to sizably increase over time to reach about 5 to 10% of GDP in the recent years (with a peak at about 29% during the tech bubble) while the net portfolio debt and bank credit position is shown to increase massively over time in absolute value to reach close to -53% of GDP in 2015 Q4. In a nutshell, the external leverage of the United States has increased massively since the 1990s, which coincides with a period of intense financial globalization.

2.3 The "exorbitant duty"

Since at least the summer of 2007, financial markets have been in turmoil. The subprime crisis, followed by the near default or default of several investment banks, insurance companies and nation states has driven volatility to levels not seen in the last two decades. Inspection of the data on the net foreign asset position of the United States during the period of the recent crisis is quite revealing.

We observe a dramatic collapse of most international asset positions as a fraction of GDP. Figure (5) shows that between 2007:4 and 2009:1, the net foreign asset position of the United States has dropped by *19% of GDP*. Such a precipitous fall of about 3% of GDP per quarter is unseen before in our data: *The US has provided insurance to the world when the global crisis hit.* The value of equity assets has declined by 19% of GDP between 2007:4 and 2009:1. A very similar picture emerges for FDI positions, and to a lesser extent for bank loans. US debt liabilities however

increased massively as a proportion of GDP since at least 1999. There was a sharp increase after the failure of Lehman Brothers and all markets froze. Importantly, the valuation of US Treasury Bills and bonds did not collapse during the crisis, like those of all the other assets. Figure 5 conveys clearly the contrast of safe external liabilities versus risky external assets, which is at the heart of our interpretation of the role of the United States in the centre of the international monetary system [11]. Coupled with the appreciation of the dollar, the relative stability in the value of US bonds has led to a massive wealth transfer of the US towards the rest of the world.[12]. We argue that such an insurance provision in very bad states of the world is the "exorbitant duty" of the centre country. If the US provides insurance against global shocks, it follows that the rest of the world should pay an insurance premium to the US in normal times. It also follows that the real rate of interest will reflect the relative size of the risk tolerant country compared to the more risk averse one in the world economy. As the size of the US shrinks in the world economy, the real rate of interest goes down.

We therefore sketch an unconventional view of the role of the centre country in the international monetary system and give an alternative explanation to the determinants of the global currency role, compared to the literature. Traditional views rely on liquidity effects and the medium of exchange function of money, such as Krugman (1980), Matsuyama et al. (1993), Rey (2001) or Devereux and Shi (2009), who ally medium of exchange and store of value functions in their model. In the traditional approach to international currencies, size and /or trade links are important insofar as they render a currency more liquid. Stability of the currency is also a prerequisite to foster its international use. It is also often pointed out that the synergies between medium of exchange roles, store of value and unit of account explain why the Dollar is at the same time reserve currency, vehicle currency and pegging currency. From an empirical point of view, Eichengreen and Mathieson (2000) and Chinn and Frankel (2007), for example, have provided an analysis of the composition of world reserves. For a recent model of reserve currencies,

[11]For a detailed empirical investigation of valuation effects by asset classes and countries during the global financial crisis, see Gourinchas et al. (2012)

[12]In Gourinchas et al. (2012) we estimate the mutilateral and bilateral wealth transfers that took place during the period 2007Q4-2009Q1. We find that the largest beneficiaries (positive wealth transfers) were the UK, Russia, Brazil and Emerging Asia. The largest wealth transfer by far was made by the United Sates, but the euro area and Switzerland also provided some insurance

see He et al. (2015). With a share of about 70% of observed total reserves, the US dollar has an uncontested lead. In their Mundell Fleming lectures, Shin (2012), Rey (2016) and Bernanke (2017) have all emphasized the importance of the dollar as the main currency in cross-border banking. Political scientists have focused on military might and geopolitical power of the United States as underlying determinants of the international currency. In contrast, we focus in this paper on the insurance properties of the international currency and model of the US as a *global insurer*. The US is at at the centre of the international financial system because it provides insurance to the rest of the word in times of global stress.

2.3.1 Empirical evidence on the 'exorbitant duty'

The Great Recession provided us with striking evidence of a massive wealth transfer from the US to the rest of the world during the crisis. Can we find systematic evidence of these transfers in other episodes of market turmoil? We relate empirically the net foreign asset position of the United States, and valuation gains and losses on this position to measures of market volatility. More precisely, following Bloom (2009) our measure of market volatility is the VIX index on 1986-2016 supplemented by the volatility of the MSCI US stock marlet index on 1962-1986. Figure 5 shows suggestive evidence of the negative correlation between the net foreign asset position as a share of GDP and financial market volatility, consistent with our "insurance theory" of international currencies. In bad states of the world –such as the LTCM collapse, 9/11, around the tech bubble collapse and the Lehman Brother default– the centre country transfers significant amounts of wealth to the rest of the world, while in good times, the rest of the world pays an insurance premium on US assets. We note that it does not matter whether the shock originates in the US or not as long as it is a global financial shock. As a matter of fact, large financial shocks originating in the US tend to become global shocks, against which the US then provides insurance. Following **?**, we also look at the behaviour of US consumption around the crisis and compare it to the one of the ROW. Figure **??** shows the time series of GDP for the US and for the ROW in panel a and the time series of consumption for the US and the ROW in panel b. Although the recession has been shallower in the US than in the ROW, private consumption is more volatile in the US with

a larger dip in the crisis and faster growth in normal times.

In Table 3, we regress the net foreign asset position, and the valuation on the VIX index. The recent wealth transfer is very spectacular but we do find a negative correlation is present on the whole period 1962-2016. The correlation is stronger after 1990, that is financial globalization truly took hold. As the great financial crisis started to unfold there was flight to quality and purchases of dollar assets. This translated into a real appreciation of the US dollar as shown in Gourinchas et al. (2012). This appreciation of the value of the gross (safe) dollar liabilities also contributed to the wealth transfer from the US to the rest of the world.

3 Theory

We take the following stylized facts away from the above empirical evidence:

1. There are excess returns in normal times ('*exorbitant privilege*.')
2. The US plays the role of an insurance provider to the rest of the world.
3. This insurance is particularly relevant in times of global stress ('*exorbitant duty*') where the US transfers wealth to the rest of the world.
4. There is a real appreciation of safe dollar assets at the beginning of the crisis as the Rest of the World buys more insurance.
5. The exorbitant privilege allows the US to run persistent trade deficits.
6. The real rate of interest goes down as the relative size of the insurer (the US) goes down in the world economy.

In the paper, we show that facts 1-6 are consistent with a model of *insurance provision* where the US exhibits smaller risk aversion than the rest of the world. Here, we take this lower risk aversion as given and show that the related equilibrium exhibits many of the characteristics that we observe in the data. One possible interpretation, although by no means the only one, is that

the US has access to better technology to deal with risk, a technology that it is able to'export' to the rest of the world [13]. The equilibrium allocation exploits the fact that the U.S. has access to this better technology and optimally allocates more risk to the U.S.

The full model also features rare events as in Barro (2006) with a probability of transiting from a normal state to a fragile state and to a crisis state. It also features countries with asymmetric sizes and traded and non-traded goods as in Hassan (2009). The first feature allows us to look at the impact of left skewness in the distribution of global output on the distribution of equilibrium returns. To the extent that the home country offers insurance to the rest of the world, that insurance will be more valuable when large negative shocks can happen. The second feature allows us to analyze fluctuations in the real exchange rate as well as recent declines in the real rate of interest.

3.1 Motivation with a simple example

This section presents a stylized stripped down model to illustrate how differences in risk aversion affect equilibrium portfolio allocations. Consider a world economy consisting of two countries, Home and Foreign, with equal population size equal to $1/2$. Following the usual convention, foreign variables are denoted with an asterisk '*'. Time is discrete. In each period t, Home is endowed with a stochastic amount of a single tradable good y_t per capita. Home consumption decisions are made by a representative household with additively separable preferences over consumption sequences of the form $\sum_{t=0}^{\infty} \beta^t u(c_t)$ where $\beta < 1$ is the discount factor, and $u(c)$ exhibits constant relative risk aversion (CRRA): $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$ when $\sigma \neq 1$ and $u(c) = \log(c)$ when $\sigma = 1$. Foreign receives an endowment y_t^* per capita. Foreign consumption decisions are also made by a representative household with CRRA preferences, but we assume that foreign households are more risk averse, that is: $\sigma^* \geq \sigma$. Markets are complete so that households in each country can trade state-contingent claims over all the relevant states of nature. Lastly, we assume that each country's output is i.i.d.. This implies that global output $\bar{y} = 0.5(y + y^*)$ is also

[13] Maggiori (2011) builds on our paper and microfounds the asymmetry between the US risk aversion and the rest of the world by assuming asymmetric development in the financial sector.

i.i.d with mean $E\bar{y} = Ey = Ey^*$.[14]

The equilibrium allocation can easily be derived. Setting the ratio of the marginal utility of the home and foreign households to a constant and substituting into the resource constraint, one obtains, in an ex-ante symmetric equilibrium:[15]

$$\frac{1}{2}\frac{c}{E\bar{y}} + \frac{1}{2}\left(\frac{c}{E\bar{y}}\right)^{\sigma/\sigma^*} = \frac{\bar{y}}{E\bar{y}}. \tag{1}$$

Figure 5 plots the equilibrium consumption function $c(\bar{y})$ that solve equation (1), together with foreign consumption $c^*(\bar{y})$.[16] The properties of these consumption rules are well-known: $c(\bar{y})$ is strictly convex, $c^*(\bar{y})$ strictly concave when $\sigma \neq \sigma^*$. When global output is low ($\bar{y} < E\bar{y}$, normalized to 1 in the figure), home consumption falls more than foreign consumption: $c(\bar{y}) < \bar{y} < c^*(\bar{y})$ as Home provides insurance to Foreign. The reverse obtains in good times. As a result, Home consumption is more volatile than Foreign.

It is also easy to show that this consumption rule can be locally decentralized with Home holding a leveraged portfolio $\sigma^*/(\sigma + \sigma^*) > 1/2$ of the world equity and borrowing in the risk free asset.[17] Thus, the international investment position of Home in the model resembles that of the United States in the world: long in equities and short in riskless assets.

Second, the net foreign asset position of Home worsens in bad times, since it earns a lower return on gross assets (equities) than it pays on gross liabilities (riskless debt). The deterioration in net foreign assets is necessary to reduce domestic wealth and induce Home consumption to fall more than Home output, improving Home's trade balance. This is consistent with the improvement in the trade balance and worsening in the net foreign asset position of the U.S. in times of global stress.

Third, consider the domestic autarky risk-free interest rate R_t^{aut}. Under autarky consumption

[14] It is important that global output exhibit no trend growth. Otherwise, the less risk averse agent dominates the market asymptotically. See Cvitanic et al. (2009) for details.

[15] To obtain this equilibrium condition, observe that in the symmetric equilibrium without risk, i.e. $\bar{y} = E\bar{y}$, the equilibrium would be $c = c^* = E\bar{y}$. This pins down the weights of the equivalent planner's problem.

[16] The figure assumes the following values: $\sigma = 2$, $\sigma^* = 5$, and $E\bar{y} = 1$.

[17] To see this, observe that a log-linearization of domestic consumption around its mean yields $\hat{c}/2 = \sigma^*/(\sigma^* + \sigma)\widehat{\bar{y}}$ where Jonesian 'hats' denotes log-deviations from steady state. This can be achieved locally with domestic holdings of a claim to global output equal to $\sigma^*/(\sigma^* + \sigma) > 1/2$.

equals output. It follows that the domestic autarky interest rate satisfies:

$\beta R_t^{aut} E_t \left[(y_{t+1}/y_t)^{-\sigma} \right] = 1$. Assume that domestic output is log-linearly distributed: $\ln y_{t+1} = \ln E\bar{y} + \epsilon_{t+1}$ where ϵ_{t+1} is i.i.d normal $N\left(-\sigma_\epsilon^2/2, \sigma_\epsilon^2\right)$. Then, the unconditional autarky risk-free rate satisfies:

$$E \ln R_t^{aut} = -\ln \beta - \frac{\sigma^2}{2}\sigma_\epsilon^2.$$

The second term in the above expression reflects the effect of the precautionary saving motive on equilibrium rates: as the variance of shocks or risk aversion increases, so does the demand for the safe asset, pushing down equilibrium risk free returns. Similar calculations for the foreign autarky rate imply:

$$E \ln R_t^{*aut} - E \ln R_t^{aut} = \frac{\sigma^2 - \sigma^{*2}}{2}\sigma_\epsilon^2 < 0,$$

since $\sigma < \sigma^*$. The lower autarky risk-free rates abroad reflects the stronger precautionary saving motive in the foreign country. With a lower autarky rate in Foreign than Home, financial integration implies that Home will run a trade deficit on average: $E\left[y - c\left(\bar{y}\right)\right] < 0$.[18] Again, this feature of the data accords well with the broad empirical evidence for the U.S. Differences in risk aversion play a similar role here as differences in the supply of assets in Caballero et al. (2008) or differences in the degree of domestic risks sharing in Mendoza et al. (2009) and generate 'global imbalances.'

How should we interpret differences in risk aversion between home and foreign households? Beyond a direct interpretation as differences in risk appetite stemming from deep cultural differences other interpretations are possible. For instance, suppose that Home has identical risk preferences as Foreign. However, Home has access to a technology that 'transforms' a given level of expenditures e into a consumption stream c that is then consumed by domestic households: $c = T\left(e\right)$. It is easy to check that the equilibrium allocation of expenditures is identical to the previous case, with e in place of c in equation (1), if $T\left(e\right) = e^{(1-\sigma)/(1-\sigma^*)}$. More generally, any concave transformation $T\left(e\right)$ will have the effect of increasing the *apparent* risk appetite of domestic households relative to their foreign counterparts. While Home households appear

[18] This can also be directly verified by noting that $E\left[y - c\left(\bar{y}\right)\right] = E\bar{y} - E\left[c\left(\bar{y}\right)\right]$, and $E\bar{y} = c\left(E\bar{y}\right) < E\left[c\left(\bar{y}\right)\right]$ since $c\left(.\right)$ is a strictly convex function.

less risk averse, they enjoy in fact a consumption allocation that is even less volatile than foreign households (compare $T(e)$ and c^* on figure 5).[19] The equilibrium allocation recognizes that Home households have access to a risk-reducing technology and optimally leverages Home equilibrium expenditures.[20] One possible interpretation of this risk reducing technology is that it reflects the interplay between domestic financial development and financial frictions. For instance, in a more elaborate model, financial development at home may reduce the importance of liquidity or financing constraints, increasing the perceived risk appetite of home households (see Gertler and Kiyotaki (2009) or Maggiori (2011)). It is beyond this paper to provide a full justification for observed differences in risk appetite. We simply take them as given when characterizing equilibrium returns and allocations and leave the question of their origin open for future research.

3.2 A Model of Global Disasters and Insurance

We now introduce a model of risk sharing with heterogeneity in risk aversion and size, traded and non traded goods. The model describes an endowment economy with two countries. Households differ between the two countries in their degree of risk tolerance. Home households are risk-tolerant, while foreign households are more risk averse. The economy has a full set of contingent financial instruments, so markets are complete and full risk sharing can be obtained. It follows that households will engage in trades that transfer risk from the more risk-averse to the more risk-tolerant. The purpose of the model is to characterize the equilibrium pattern of these risk-sharing trades, in terms of prices (rates of return, real exchange rate) and quantities (portfolios, valuations).

3.2.1 Setup

We consider a stationary world endowment economy with two countries, Home and Foreign. The world is populated by a continuum of households of constant mass equal to 1. A share $\alpha \in (0,1)$

[19] Since $\sigma < \sigma^*$, it is immediate that $T(e)$ is more concave than $c^*(e)$.

[20] The technology $T(.)$ alters the resource constraint of the economy, which is why the solution is not the symmetric allocation of the planner under identical preferences. The implicit assumption is that the risk altering technology is only applied to the expenditure allocation of the home country, and not to global output, otherwise the equilibrium would be $c = c^* = T(\bar{y})$.

of the world population is located in Home and the remaining share $1-\alpha$ is located in Foreign.

Time is discrete, $t = 1, 2....$ Each period t, a state of the world s_t is realized where s_t belongs to the finite set $\mathcal{S}$ with cardinal $\#\mathcal{S}$. As usual, we define $s^t = (s_0, s_1, ..., s_t) \in \mathcal{S}^t$ the aggregate history of the world and endow $\mathcal{S}^t$ with a probability distribution $\pi(s^t)$ over each history that satisfies $\pi(s^0) = 1$. The aggregate process is assumed Markovian, i.e. $\pi(s_{t+1}|s^t) = \pi(s_{t+1}|s_t)$ where $\pi(s_{t+1}|s^t)$ denotes the probability of state s_{t+1} in period $t+1$, conditional on history s^t.

In state s, each household ω receives a stochastic endowment of a traded good $y^T(\omega, s)$ and of a non-traded good $y^N(\omega, s)$. Denote $y^T(s)$ and $y^N(s)$ (resp. $y^{*T}(s)$ and $y^{*N}(s)$) the average endowment of the Home (resp. Foreign) traded and non-traded goods in state s. That is, $y^T(s) = \int_{\omega\in\Omega_H} y^T(\omega, s)\, d\nu(\omega)/\alpha$ where ν denotes the measure of households over ω and Ω_H is the set of domestic households such that $\int_{\omega\in\Omega_H} d\nu(\omega) = \alpha$. Ω_F, $y^N(s)$, $y^{*T}(s)$ and $y^{*N}(s)$ are defined similarly. Note that the assumption that the set $\mathcal{S}$ is finite and that the endowments depend only on the current realization of the state $s \in \mathcal{S}$, imposes that the aggregate endowment processes are stationary.

Each household ω has additively separable preferences defined over sequences of a consumption aggregate $\{c_\tau(\omega)\}_{\tau=t}^{\infty}$:

$$U_t(\omega) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u(c_\tau(\omega); \omega, s_\tau), \tag{2}$$

where $\beta \in (0,1)$ denotes the discount factor and $\mathbb{E}_t(.)$ the expectation conditional on time t information, defined by the appropriate filtration over histories $\{s^t\}$. The per-period utility function $u(c; \omega, s)$ exhibits a coefficient of relative risk aversion $\sigma(\omega, s)$ that is household and time-specific:

$$u(c; \omega, s) \equiv \frac{c^{1-\sigma(\omega,s)} - 1}{1 - \sigma(\omega, s)}$$

when $\sigma(\omega, s) \neq 1$ and

$$u(c; \omega, s) \equiv \ln(c)$$

when $\sigma(\omega, s) = 1$.

For simplicity, we also assume that households' risk preferences are identical within countries.[21]

Assumption 1 (Within-country preference homogeneity) *Households have identical preferences within countries:*

$$\forall \omega \in \Omega_H, s \in \mathcal{S}, \sigma(\omega, s) = \sigma(s); \quad \forall \omega \in \Omega_F, s \in \mathcal{S}, \sigma(\omega, s) = \sigma^*(s);$$

There are two important features of these preferences that we want to highlight at the outset. First, cross country differences in risk tolerance will generate 'risk-transfer' trades in equilibrium. Second, the fact that these risk preferences are state-dependent captures in a simple way the possibility of 'risk-on' and 'risk-off' behavior, i.e. time-variation in the desirability of risk-transfer trades.

We assume that markets are complete internationally, so that a full menu of state-contingent claims denominated in the traded good can be exchanged between Home and Foreign. Under this assumption each country admits a representative household, so we only need to keep track of country-level average endowments y^T, y^N and the corresponding average consumptions of traded and non-traded goods, c^T, c^N. We can therefore omit in what follows the dependency of preferences, consumption and endowments on ω.

The consumption aggregate c is defined identically in both countries as a constant elasticity index of traded and non-traded consumption:

$$c = \left[\gamma^{1/\theta} \left(c^T\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{1/\theta} \left(c^N\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

where $\theta > 0$ denotes the constant elasticity of substitution between traded and non-traded goods, and $\gamma \in (0,1)$ controls the steady state share of traded consumption expenditures.

Taking the traded good as the numeraire and denoting q the price of the domestic non-tradable

[21] Hall (2016) explores further the role of asymmetric risk aversion and heterogenous beliefs. Barro and Mollerus (2014) also analyses asymmetry in risk aversion to model the outstanding amount of risk free debt and trade in safe assets. Barro et al. (2017) look at trade and volume of safe assets in a production economy with agents having asymmetric risk aversion using the solution method developed in Villaverde and Levintal (2017).

good, the domestic price index P is defined as the Fisher-ideal deflator of domestic aggregate consumption:

$$P = \left[\gamma + (1-\gamma)\, q^{1-\theta}\right]^{1/(1-\theta)}, \tag{4}$$

with a similar definition for the foreign price index in terms of the price of foreign non-traded goods q^*.

The resource constraints are given by

$$\alpha c^T + (1-\alpha)\, c^{*T} = \bar{y}^T; \quad c^N = y^N; \quad c^{*N} = y^{*N}, \tag{5}$$

where $\bar{y}^T$ is the global supply of the traded good: $\bar{y}^T \equiv \alpha y^T + (1-\alpha)\, y^{*T}$.

Given that financial markets are complete, the conditions for the first and second welfare theorem hold and the complete market allocation solves the standard planning problem ($\mathcal{P}$) of maximizing a weighted sum of discounted utilities:

$$\max_{\left\{c_t^T, c_t^{*T}, c_t^N, c_t^{*N}\right\}} \mu\alpha\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, s_t\right) + (1-\mu)\,(1-\alpha)\,\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u^*\left(c_t^*, s_t\right) \tag{$\mathcal{P}$}$$

given the consumption aggregators in (3) and subject to the resource constraints (5), where $\mu \in [0,1]$ represents the weight given by the planner to Home households.

Because there is no capital in the model, ($\mathcal{P}$) can be solved state-by-state: for each state $s \in \mathcal{S}$, the first-order condition of the problem imposes that the marginal utility of tradable consumption be proportional across states and countries:

$$c^{(1/\theta - \sigma(s))} \left(c^T\right)^{-1/\theta} \kappa^{-1/\theta} = c^{*(1/\theta - \sigma^*(s))} \left(c^{*T}\right)^{-1/\theta}, \tag{6}$$

where $\kappa = \left(\mu/\left(1-\mu\right)\right)^{-\theta}$ is a constant that reflects the relative importance of Home vs. Foreign for the planner. According to the risk sharing condition (6), shocks to the endowment of non-traded goods will shift the marginal utility of traded good consumption when preferences are non-separable, i.e. when $\sigma \neq 1/\theta$. When $\sigma > 1/\theta$, traded and non-traded goods are *gross substitutes*: a decline in the endowment of non-traded good increases the marginal utility of traded

good consumption. Conversely, when $\sigma < 1/\theta$, the traded and non-traded goods are *gross complements*: a decline in the endowment of the non-traded good reduces the marginal utility of traded good consumption.

Definition 1 (Planner's allocation) *An equilibrium allocation of the planner's problem ($\mathcal{P}$) is a p-uple* $(c(s), c^T(s), c^N(s), c^*(s), c^{*T}(s), c^{*N}(s))$ *that solves state-by-state (a) the risk sharing conditions (6), (b) the resource constraints (5) and where (c) aggregate consumption is defined by the consumption-aggregator (3).*

Given an equilibrium allocation, the price of the non-traded good can be obtained as the ratio of marginal utilities for traded and non traded goods, where we substituted $c^N = y^N$:

$$q = \left(\frac{\gamma y^N}{(1-\gamma)\, c^T} \right)^{-1/\theta}. \tag{7}$$

Furthermore, from (6), the –common– stochastic discount factor between periods t and $t+1$ is given by

$$M_{t,t+1} = \beta \left(\frac{c_{t+1}}{c_t} \right)^{\frac{1}{\theta} - \sigma(s_t)} \left(\frac{c^T_{t+1}}{c^T_t} \right)^{-1/\theta}, \tag{8}$$

and satisfies

$$\mathbb{E}_t \left[M_{t,t+1} R_{t+1} \right] = 1, \tag{9}$$

for *any* traded asset with gross return R_{t+1} in terms of the traded good.

3.2.2 Characterization

The analysis of the equilibrium allocation can be simplified if we define:

$$x(s) = \kappa c(s)^{\sigma(s)\theta - 1} / c^*(s)^{\sigma^*(s)\theta - 1}. \tag{10}$$

Using this definition, the risk sharing condition (6) becomes $c^{*T}(s) = x(s) c^T(s)$ and the resource constraint yields $c^T(s) = \bar{y}^T(s) / \left[\alpha + (1-\alpha)\, x(s) \right]$. $x(s)$ controls the equilibrium allocation of the global endowment of traded goods between Home and Foreign in state s. When $x(s) = 1$,

tradable consumption per capita is equated: $c^T(s) = c^{*T}(s) = \bar{y}^T(s)$. When $x(s) > 1$, Foreign obtains a larger share of the traded good: $c^{*T}(s) > \bar{y}^T(s) > c^T(s)$ while the converse is true when $x(s) < 1$.

Substituting the previous expression into the definition of the domestic and foreign consumption index, $x(s)$ is the state-by-state implicit solution to the following equation:

$$\left(\frac{x(s)}{\kappa}\right)^{\frac{\theta-1}{\theta}} = \frac{\left(\gamma^{1/\theta}\left(\frac{\bar{y}^T(s)}{\alpha+(1-\alpha)x(s)}\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{1/\theta}\left(y^N(s)\right)^{\frac{\theta-1}{\theta}}\right)^{\sigma(s)\theta-1}}{\left(\gamma^{1/\theta}\left(\frac{x(s)\bar{y}^T(s)}{\alpha+(1-\alpha)x(s)}\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{1/\theta}\left(y^{*N}(s)\right)^{\frac{\theta-1}{\theta}}\right)^{\sigma^*(s)\theta-1}}. \tag{11}$$

This expression highlights how $x(s)$ varies with the realizations of both traded and non-traded goods endowments. Consider the case where $\sigma(s) > 1/\theta$ and $\sigma^*(s) > 1/\theta$ so that traded and non-traded are gross substitute in both countries. A decline in $y^N(s)$ raises Home's marginal utility of traded good consumption. Risk sharing requires that Home consumes relatively more of the traded good, a decrease in $x(s)$. A similar effect occurs when $y^{*N}(s)$ increases. A fall in the global endowment of tradable good $\bar{y}^T(s)$ impacts relatively more the more risk averse country. If $\sigma^*(s) > \sigma(s)$, risk sharing requires that $x(s)$ increases, allocating more traded consumption to Foreign.

It is immediate from (11) that as long as endowments follow a stationary process, so does $x(s)$. In that case, the equilibrium distributions of home and foreign consumption is also stationary case.[22] We summarize these results in the following proposition.

Proposition 1 (Characterization of the Planner's allocation) *With a stationary endowment process* $\mathbf{y}(s) = (\bar{y}^T(s), y^N(s), y^{*N}(s))$*, the equilibrium allocation that solves the Planner's problem* $(\mathcal{P})$ *is fully characterized by the process* $x(s)$ *that solves equation (11) state-by-state. Consumption allocations are recovered from* $c^T(s) = \bar{y}^T(s)/\left[\alpha + (1-\alpha)\, x(s)\right]$ *and* $c^{*T}(s) = x(s)\bar{y}^T(s)/\left[\alpha + (1-\alpha)\, x(s)\right]$, $c^N(s) = y^N(s)$ *and* $c^{*N}(s) = y^{*N}(s)$.

Equation (11) admits an analytical solution in two special cases. First, when $\sigma(s) = \sigma^*(s) =$

[22] If endowments are non-stationary, it is easy to check that $x(s)$ converges to 0 or 1: the less risk averse households dominate aggregate consumption asymptotically. See Cvitanic et al. (2009).

$1/\theta$, one can check that the solution is $x(s) = \kappa$. The consumption of traded goods in each country is a constant fraction of the global endowment of traded goods, and the stochastic discount factor simplifies to the usual formula $M_{t,t+1} = \beta \left(\bar{y}^T_{t+1}/\bar{y}^T_t\right)^{-\sigma}$.

Second, when $\theta = 1$ and $\alpha = 1$ (the large country limit),

$$x(s)^{1+\gamma(\sigma^*(s)-1)} = \kappa \bar{y}^T(s)^{\gamma(\sigma(s)-\sigma^*(s))} \left(\frac{y^N(s)^{\sigma(s)-1}}{y^{*N}(s)^{\sigma^*(s)-1}}\right)^{1-\gamma}.$$

This expression illustrates that in the large country limit, the allocation of traded goods between Home and Foreign depends upon the global endowment of traded good $\bar{y}^T$ only to the extent that risk tolerance differs across countries ($\sigma(s) \neq \sigma^*(s)$).

In the general case, defining $\mathbf{y}_t = \mathbf{y}(s_t)$ and $x_t = x(s_t)$ with a slight abuse of notation, we can write the stochastic discount factor as:

$$\begin{aligned} M_{t,t+1} &\equiv M\left(\mathbf{y}_t, \mathbf{y}_{t+1}\right) \\ &= \beta \left(\frac{\gamma^{1/\theta}\left(\bar{y}^T_{t+1}/\left[\alpha + (1-\alpha)\, x_{t+1}\right]\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{1/\theta}\left(y^N_{t+1}\right)^{\frac{\theta-1}{\theta}}}{\gamma^{1/\theta}\left(\bar{y}^T_t/\left[\alpha + (1-\alpha)\, x_t\right]\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{1/\theta}\left(y^N_t\right)^{\frac{\theta-1}{\theta}}}\right)^{\frac{1-\sigma\theta}{\theta-1}} \\ &\quad \cdot \left(\frac{\bar{y}^T_{t+1}}{\bar{y}^T_t}\frac{\alpha + (1-\alpha)\, x_t}{\alpha + (1-\alpha)\, x_{t+1}}\right)^{-1/\theta}. \end{aligned} \tag{12}$$

This expression illustrates one of Hassan (2009)'s central points: as α increases, the stochastic discount factor increasingly reflects the endowments shocks of the larger economy. In the large country limit ($\alpha = 1$),

$$\lim_{\alpha \to 1} M_{t,t+1} = \beta \left(\frac{\gamma^{1/\theta}\left(\bar{y}^T_{t+1}\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{1/\theta}\left(y^N_{t+1}\right)^{\frac{\theta-1}{\theta}}}{\gamma^{1/\theta}\left(\bar{y}^T_t\right)^{\frac{\theta-1}{\theta}} + (1-\gamma)^{1/\theta}\left(y^N_t\right)^{\frac{\theta-1}{\theta}}}\right)^{\frac{1-\sigma\theta}{\theta-1}} \left(\frac{\bar{y}^T_{t+1}}{\bar{y}^T_t}\right)^{-1/\theta},$$

and the stochastic discount factor responds exclusively to Home's endowment shocks.[23]

The stochastic discount factor constructed in (12), can be used to calculate the value of any financial asset. Consider a generic asset with beginning-of-period price $P(s)$ (cum-dividend) in

[23] In the limit of $\alpha = 1$, $\bar{y}^T = y^T$.

state s that pays a dividend $D(s)$. $P(s)$ must satisfy:

$$\begin{aligned} P(s) &= D(s) + \mathbb{E}\left[M(\mathbf{y}(s), \mathbf{y}(s'))P(s')|s\right] \\ &= D(s) + \sum_{s' \in \mathcal{S}} \pi(s'|s) M(\mathbf{y}(s), \mathbf{y}(s'))P(s') \end{aligned} \tag{13}$$

where the second line spells out the expectation over future states. Define the $\#\mathcal{S} \times 1$ vector $\overline{\mathbf{P}} = (P(s_1), ..., P(s_{\#\mathcal{S}}))'$. Define similarly the vector $\overline{\mathbf{D}}$ and the $\#\mathcal{S} \times \#\mathcal{S}$ matrix $\overline{\mathbf{M}}$ whose (s, s') element is $\pi(s'|s)M(\mathbf{y}(s), \mathbf{y}(s'))$. Equation (13) can then be rewritten in matrix form and solved for $\overline{\mathbf{P}}$:

$$\overline{\mathbf{P}} = \left(\overline{\mathbf{I}} - \overline{\mathbf{M}}\right)^{-1} \overline{\mathbf{D}}, \tag{14}$$

where $\overline{\mathbf{I}}$ is the $\#\mathcal{S} \times \#\mathcal{S}$ identity matrix.

We can use equation (14) to characterize the domestic financial wealth of each country, the value of a claim to their current and future endowment and, by subtraction, the value of their net international investment position. Denote $W(s)$ the beginning-of-period domestic financial wealth of Home (in units of the traded good). From the country's budget constraint under complete markets, it satisfies:

$$W(s) = P(s)c(s) + \mathbb{E}\left[M(\mathbf{y}(s), \mathbf{y}(s'))W(s')|s\right] \tag{15}$$

That is, domestic financial wealth is the value of a tail-claim to current and future consumption expenditures and can be characterized using equation (15). Similarly, denote $V(s)$ the value of a claim to current and future domestic endowment, i.e. the value of the domestic traded and non-traded Lucas trees (in units of the traded good). It satisfies:

$$V(s) = y^T(s) + q(s)y^N(s) + \mathbb{E}\left[M(\mathbf{y}(s), \mathbf{y}(s'))V(s')|s\right].$$

Finally, Home's net foreign investment position $NA(s)$ (in units of the traded good) is simply the

difference between the country's wealth and the value of its assets. It follows that:

$$NA(s) = W(s) - V(s) = c^T(s) - y^T(s) + \mathbb{E}\left[M(\mathbf{y}(s), \mathbf{y}(s'))NA(s')|s\right]$$

The first term on the right hand side is simply the opposite of the trade balance. This expression states that –under complete markets– the net foreign investment position is the value of a claim to current and future trade deficits and can be evaluated using equation (16). In matrix notation:

$$\overline{\mathbf{NA}} = -\left(\bar{\mathbf{I}} - \overline{\mathbf{M}}\right)^{-1}\left(\bar{\mathbf{y}}^T - \bar{\mathbf{c}}^T\right) \tag{16}$$

3.2.3 Business cycles and disasters

To illustrate the impact of heterogeneity in risk aversion and size in times of global stress, we assume the following process for traded and non-traded domestic output

$$\ln y_t^T = \ln \gamma + \epsilon_t^T + \upsilon_t, \tag{17a}$$

$$\ln y_t^N = \ln (1-\gamma) + \epsilon_t^N + \upsilon_t, \tag{17b}$$

and

$$\ln y_t^{*T} = \ln \gamma + \epsilon_t^{*T} + \upsilon_t, \tag{18a}$$

$$\ln y_t^{*N} = \ln (1-\gamma) + \epsilon_t^{*N} + \upsilon_t, \tag{18b}$$

for traded and non-traded foreign output.

The random terms ϵ^T, ϵ^N and $\epsilon^{*T}, \epsilon^{*N}$ are uncorrelated, i.i.d normally distributed shocks with mean $-\sigma_\epsilon^2/2$ and variance σ_ϵ^2. These terms capture regular business cycle fluctuations in output. These fluctuations in output trigger a precautionary saving motive whose strength varies across countries when $\sigma \neq \sigma^*$.

The random term υ_t captures low-probability disasters, as in Barro (2006). As in that paper, disasters are independent from ϵ shocks. Unlike Barro (2006), we assume that the output process is

stationary in levels: disasters are –eventually– followed by recoveries. This assumption is made mostly for tractability since it ensures that the consumption process remains stationary, even when home and foreign households have different risk appetite. However, this assumption has also substantive merits. Nakamura et al. (2010) found that roughly half of the fall in consumption during disasters is subsequently reversed, indicating partial recovery. Given the curvature of the utility function it remains true that disasters matter much more than recoveries for equilibrium asset returns.

We allow for variation in the probability of disaster risk. To do so, we model v_t as a three-state Markov process. We label the three states 'safe', 'fragile' and 'disaster', with values v_s, v_f and v_d and transition probabilities $P(v_j|v_i) = p_{ij}$ that satisfy $\sum_j p_{ij} = 1$, and denote $\bar{p}_i$ the unconditional probability of state i given the transition matrix P. We assume further that $\{v_i\}$ satisfy:

$$v_s = v_f = -\ln\left(\bar{p}_d\left(1-b\right)+1-\bar{p}_d\right) \tag{19a}$$

$$v_d = \ln(1-b)+v_f \tag{19b}$$

where $b \in [0,1)$. This representation of the disaster process ensures that output drops by a factor $(1-b)$ when a disaster occurs, a number that has been estimated in the literature, and that $Ey^T = Ey^{*T} = \gamma$, and $Ey^N = Ey^{*N} = 1-\gamma$ regardless of b. In other words, by varying b, we are changing the left skewness of the output process, keeping expected output constant.

For given realization of ϵ, output is the same in the safe and fragile states (since $v_s = v_f$). However, we assume that the probability of transition towards the disaster state increases in the fragile state ($p_{sd} < p_{fd}$). Thus, a transition towards the fragile state triggers precautionary demands, even though the disaster has not occurred yet. In addition, our model allows risk appetite to change as a function of the state of the world. For instance, the occurrence of a fragile state may be associated with a re-assessment of risk tolerance (i.e. $\sigma_f \geq \sigma_s$ and $\sigma_f^* \geq \sigma_s^*$). In our simulations, we will explore the role of these 'risk-on' and 'risk-off' transitions.

Our specification implies that rare events are *global*: when a disaster occurs, output collapses

in the same proportion in all sectors and countries. It would be straightforward to extend the analysis to the case of country-specific disasters.[24] The empirical evidence discussed in Nakamura et al. (2010) supports the notion that some disasters are local and others global. In the context of the model, it is immediate that patterns of risk sharing resembling what we observe in the data would emerge if disasters are either more severe or more frequent in Foreign. But this hardly seems a reasonable assumption considering that the recent crisis originated in the U.S., not in the rest of the world. Instead, our approach explores the extent to which Home is able to provide insurance in times of global stress. Under equations (17)-(19), the stochastic process for endowments follows a stationary Markov process and we can solve for $x(\mathbf{y};\kappa)$ given κ, and select κ to calibrate Home's net foreign assets in steady state.

Appendix E.1 provides analytical results for the case without disaster shocks. We show in particular that our framework nests the model of Hassan (2009) when there is no asymmetry in risk aversion and there is separability between non traded and traded goods consumption. More generally, we find in our generalized model that when the size of the US is large, the asset with the best hedging properties is the US bond as the world is effectively dominated by shocks to the US non traded sector. US bonds have high returns during bad times for the non traded goods production because of real appreciation when the production of non traded goods is low.

4 Quantitative results

As is well known, disaster risk has the potential to generate realistic risk premia. To the extent that domestic and foreign government bonds have different risk exposure from the point of view of the marginal investor, this may magnify expected excess returns, potentially accounting for the 'exorbitant privilege'. Furthermore, when risk appetite differs across countries, the occurrence of a symmetric disaster triggers a reallocation of resources and associated valuation adjustments that resembles what happened in 2007-2008 between the U.S. and the rest of the world. In other words, the model can potentially also account for the 'exorbitant duty'. We also allow the coeffi-

[24] See Guo (2007).

cient of risk aversions of home and foreign to increase in fragile times (when the probability of a catastrophic shock increases) compared to safe times. This corresponds to the observed risk-on risk-off attitude of financial markets, whose psychological determinants may be hard to embed in standard models. We now explore the quantitative predictions of the model in terms of optimal consumptions, real exchange rate, wealth dynamics and stochastic discount factor.

4.1 Calibration

Our approach is to adopt fairly standard values for γ, θ, σ and σ_ϵ , β and to vary α and σ^*. γ measures the share of traded goods in consumption expenditures around the steady state. We assume a low value $\gamma = 0.25$, consistent with the indirect evidence on high trade costs (see Obstfeld and Rogoff (2005) for a discussion). θ measures the elasticity of substitution between traded and non-traded goods. Estimates in the literature are fairly low, between 0.5 and 1.3.[25] We adopt a value $\theta = 1$, towards the higher end of that range. Reasonable values for the coefficient of relative risk aversion σ vary between 2 and 5. We choose $\sigma = 3$ as a benchmark and will vary $\sigma^* \geq \sigma$. We pick σ^* = 3.5 in normal times for our benchmark and allow for the possibility that it may go up in fragile times when "risk is on". Finally, the model requires an estimate of the volatility of output around its steady state. As argued above, the assumption of stationarity is mostly maintained to ensure stationarity of consumption allocations. Consequently, we need to input the standard deviation of log output deviations from a Hodrick-Prescott filter. Using annual data, a common value is $\sigma = 0.02$. We set the discount factor β so as to generate the same price-earning ratio as in a model with output growth, or $\beta = 0.923$ in our benchmark calibration.[26] Next, we set $\alpha = 0.25$ reflecting the relative importance of the U.S. economy for global outcomes and capturing traditional arguments in favor of the U.S. as the issuer of the reserve currency.

We then need to calibrate the process for disaster events. We set the Markov transition matrix

[25] See the discussion in Obstfeld and Rogoff (2005).

[26] In a model without shocks (disasters or otherwise) and no growth, the PE ratio is $1/\left(\beta^{-1}-1\right)$. With a growth rate of g, the PE ratio becomes $1/\left(\beta^{-1}-1+(\sigma-1)\,g\right)$. Consequently, we set $\beta^{-1} = 1.03 + (\sigma-1)\,0.025$.

for disaster shocks as follows:

$$P = \begin{bmatrix} 0.955 & 0.04 & 0.005 \\ 0.200 & 0.768 & 0.032 \\ 0.200 & 0.00 & 0.800 \end{bmatrix} \tag{20}$$

In the safe state, the conditional probability of a moving to the fragile state is 4% while that of a disaster occurring is only 0.5% per year. In the fragile state, the conditional probability of a disaster increases markedly, to 3.2%. We also set the probability of recoveries from disasters to 20% percent from the disaster state.[27] Next, we calibrate the size of the disaster by setting $b = 0.42$. Barro (2006) estimates a similar parameter between 0.15 and 0.65. Our estimate is around the middle of that range.

Lastly, as in Barro (2006), we allow for the possibility that government T-bills experiences a partial default when a disaster occurs. That is, we assume that real government bonds pay the local consumer price index in safe or fragile times, but only a fraction of the promised payment in periods of global stress. An important parameter is the expected recovery rate r^i on government bonds from country i. The face value in terms of traded goods in state j (safe or fragile) of a government bond that pays P^i_{t+1} the following period is then:

$$P^{bi}_t = (1 - p_{jd})\, \mathbb{E}_t \left[M_{t,t+1} P^i_{t+1} | s \text{ or } f \right] + p_{jd} r^i \mathbb{E}_t \left[M_{t,t+1} P^i_{t+1} | d \right],$$

and the expected return on the government bond is

$$\ln \mathbb{E}_t R^{bi}_{t+1} = \ln \left[(1 - p_{jd})\, \mathbb{E}_t \left[P^i_{t+1} | s \text{ or } f \right] + p_{jd} r^i \mathbb{E}_t \left[P^i_{t+1} | d \right] \right] - \ln P^{bi}_t.$$

We interpret r^i as capturing in a simple way the '*fiscal capacity*' of country i, i.e. the capacity for the government of that country to honor it's debt obligations through taxation of the domestic

[27] The unconditional probability of a disaster is $\bar{p}_d = 4.29\%$, higher than Barro (2006) estimate of 1.7%. This is because disasters are temporary in our set-up. However, this number has no impact on allocations (consumption and trade balances) in safe and fragile states, and a minimal impact on excess returns in safe times. It influences expected returns during a disaster since a higher p_{ds} implies a larger chance of a recovery and incipient high return.

economy. Barro (2006) documents that the real return on T-bills in many countries was low during rare events, either because of outright default or expropriation, or –a more common scenarion– because of the real depreciation of nominal claims through high inflation. r, therefore, represents another important parameter, conceptually separate but not entirely unrelated to size or risk appetite. For instance, r captures implicitly a host of political economy factors that determine a country's 'willingness to pay' as opposed to its 'ability to pay' as measured by its size. Most of the literature on sovereign debt emphasizes the important of a country's 'willingness to pay' in understanding episodes of sovereign default. We allow for differences in recovery rates across countries. Specifically, we assume that Home can enforce repayment ($r = 1$) while Foreign may suffer from partial implicit or explicit default ($r^* < 1$).

4.2 Model Solution and Portfolios

To solve the model, we discretize the state space and solve for the optimal consumption allocation in each state such that there are initially no net external positions: $NA(x_0; \kappa) = 0$. We then construct the stochastic discount factor $M_{t,t+1}$ and use this SDF to price government real bonds and global equities. We can solve for the law of motion for wealth at home and abroad (see Equation (15)) and compute the returns on domestic and foreign wealth R_t^w and R_t^{w*}. Assuming a set of traded assets (global equity, domestic and foreign bonds) we can then obtain the portfolio weights implementing the planner's allocation by replicating the wealth dynamics. For example for the home country we have to find portfolio weights on global equity, domestic and foreign bonds such that

$$R_{t+1}^w = b_t R_{t+1} + b_t^g R_{t+1}^g + b_t^{hb} R_{t+1}^{hb} + b_t^{fb} R_{t+1}^{fb}$$

We impose the following realistic constraints on the portfolio allocation. First, we assume that only a fraction δ of equities are globally traded and can be held by foreign investors in their portfolios. This reflects the fact that there are many wealth components which are non traded in the real world, such as for example non listed companies. Varying δ allows us to parameterize the size of the gross equity position as a percentage of GDP and hence to generate portfolios which

are not too leveraged. We set δ to 0.05 in our benchmark simulations. We also impose the following short sale constraints: Home and Foreign cannot short sale equities; Foreign cannot issue domestic real bonds (i.e. bonds that pay Home's price index in terms of tradables); Symmetrically, Home cannot issue foreign real bonds (i.e. bonds that pay Foreign's price index in terms of tradable). Details are given in Appendix F.

Table 5 reports the results on asset pricing, net foreign asset positions, trade balance and real exchange rate under different scenarios. The equity premium is defined as $\ln \mathbb{E}_t \left[\bar{R}^e_{t+1}\right] - \ln \mathbb{E}_t \left[R^f_{t+1}\right]$ where $\bar{R}^e_{t+1}$ is the gross return on a claim to current and future total endowment (global equity) and R^f_{t+1} is the gross return on Home's real bonds. The bond excess return is defined as $\ln \mathbb{E}_t \left[R^{*f}_{t+1}\right] - \ln \mathbb{E}_t \left[R^f_{t+1}\right]$ where R^{*f}_{t+1} is the gross return on real foreign bonds. All returns are measured in terms of tradable goods. The domestic trade balance-output ratio is defined as $\left(y^T - c^T\right) / \left(y^T + qy^N\right)$. The net foreign asset position is defined as $NA_t = W_t - V_t$ where W_t is the value of a claim to current and future domestic consumption: $W_t = P_t c_t + E_t \left[M_{t,t+1} W_{t+1}\right]$ and V_t is the value of a claim to current and future domestic endowment, $V_t = y^T_t + q_t y^N_t + E_t \left[M_{t,t+1} V_{t+1}\right]$.

In each column, we report properties of the equilibrium allocation under a set of parameters. Column (1) is our benchmark, which features the following asymmetries between Home and Foreign. Home and Foreign are different with respect to size ($\alpha = 0.25$) and risk aversion (σ is 3 for home remaining constant during fragile times while the coefficient of risk aversion σ^* is 3.5 for the foreign country and increases to 4.5 during fragile and disaster times). Furthermore there is a 50% haircut on the foreign bond in catastrophic times while the domestic country bond is safe ($r^* = 0.5$ and $r = 1$).

The model is broadly able to account for the stylized facts emphasized earlier. Thanks to the catastrophic shock, the equity premium and the price earning ratio are in line with the data. Because of the asymmetry in risk aversion between Home and Foreign, Home provides insurance against global risks to Foreign. This is reflected in the pattern of trade deficits/surpluses of the Home country. In normal times, home is running a trade deficit, of about 0.33 percent of output which increases to 0.52 $\%$ in fragile times. When a disaster occurs, however, this trade deficit

becomes a large trade surplus (10.57%). This pattern of trade deficits has a counterpart in the domestic net foreign asset position.[28] In normal times, Home has a negative net foreign asset position, of about 16.81 percent of output. The net foreign asset position and its deterioration in fragile times are of the same order of magnitude as in the data; the model generate sizable wealth transfers of the order of 13 $\%$ of GDP during fragile times. Foreigners buy more insurance during fragile times as reflected in a massive increase in the net debt liabilities. With reasonable parameters, the model can therefore reproduce the size of the net wealth transfer from Home to Foreign during times of global stress, the '*exorbitant duty*.'

How can Home stabilize its net foreign asset position in normal times despite repeated trade deficits? The answer is that Home's net foreign position benefits from valuation gains in normal times that offset trade deficits. In terms of portfolios, Home now holds a very leveraged portfolio, with large net debt liabilities, that are reinvested in global equities. This portfolio delivers small positive excess returns in good times –enough to offset the trade deficits –. Hence, the model also delivers the '*exorbitant privilege*' that we documented earlier. The excess return on the net foreign asset position (the exorbitant privilege) is about 0.4% in normal times and 3.3% in fragile times. While this number seems smaller than the average 2 $\%$ excess return found in the data one should remember that this excess return is substantially leveraged since the return on the net foreign asset position is $R^{NA} = R^L + \mu_A \left(R^A - R^L\right)$ where $\mu_A = A/\left(A - L\right)$ can be a large number.

Interestingly the real exchange rate of the home country appreciates between the normal and fragile times, reflecting flight to safety and purchases of home safe assets as the probability of disaster increases. There is a safety premium of the home bond compared to the foreign bonds because of the haircut on foreign bond in catastrophic times. The higher return is required to induce agents to hold foreign real government debt. The allocation is unchanged by this assumption, since we are only modifying the payoff structure of one assets (foreign real bonds) but keeping the overall market structure unchanged. Therefore, the pattern of trade deficits in good times, trade surpluses in bad times, remains the same, as does the structure of excess returns on

[28] Recall that $NA_t = -NX_t + \mathbb{E}_t\left[M_{t,t+1} NA_{t+1}\right]$.

gross assets and liabilities.

Finally the safe real rate decreases significantly in fragile times as it is driven down by the precautionary demand for safe assets.

Moving across the rows, we shut down our "risk-on" parameter in fragile times in column (2). All the results remain qualitatively the same (though the magnitudes are smaller) except for the exchange rate which does not appreciate between normal and fragile times anymore since the equilibrium consumptions do not change; the trade balance remains now constant for the same reason. In Column (3), we go back to our benchmark but now allow the coefficient of risk aversion of the home country to increase between safe and fragile states (it increases from 3 to 3.5). This decreases the asymmetry between home and foreign in fragile times and therefore decreases the insurance provision (net debt liabilies are smaller, the real appreciation is smaller). This also increases the real rate of interest. In column (4) we increase the size of the home country from 0.25 to 0.5. This means that teh world is on average less risk averse and translates into a lower equity premium and price earning ratio, a lower T-Bill excess return and exorbitant privilege (hence a lower trade deficit). We also observe a lower transfer (exorbitant duty) and a smaller exchange rate appreciation. Finally this leads to a higher safe real rate. We explore the role of size on all these variables in more details below.

Columns (5) and (6) reports results when there is no haircut to the foreign bond in disaster times. This affects only the excess return on bonds which flips sign.To understand this result, observe that when a disaster strikes global endowment fall proportionately in all sectors and countries. Since Home insures Foreign, however, Home's consumption of traded goods falls relatively more than Foreign's. Hence Home runs a smaller trade deficit, or trade surplus. This implies that the domestic non-traded goods are relatively more abundant and their price falls more, relative to Foreign. The net result is that the domestic real bond is not a particularly good hedge against global shocks since the domestic price index will fall more than foreign. In column (6) we also take out the disaster shock. As expected the risk premium collapses and by construction fragile times and disaster times become identical. The real rate is much larger. Column (7) explores the effect of an increase in the elasticity of substitution across goods keeping everything

else constant. This tends to increase the equity premium and increase the transfers in disaster time as the traded and non traded goods become better substitute. This also mutes somewhat the response of the real exchange rate. Finally column (8) relaxes the assumption that only a fraction of equities are traded. As a result, portfolios are very leveraged (net debt liabilities are very large).

4.2.1 The real safe rate, the real exchange rate and the Vanishing Hegemon

In Figure (8) we plot the real safe rate as a function of α the relative size of the home economy in the world. As the size of the world insurer shrinks (alpha close to zero) the real safe rate goes down as the world becomes on average more risk averse and the demand for safe assets becomes large relative to the size of the hegemon. One interpretation of the trend decline in the real rate since the 1980 could therefore be linked to the decrease of the relative size of the US in the world economy[29]. In a world where the global insurer is small, a large world demand for safe asset drives their price up. The mirror image of this increased demand for safe assets is a larger exposure of the hegemon to world risk materialized by a large leveraged porftolio. In Figure (9) we show the net debt exposure and the valuation losses as percentage of output for the hegemon. They both increase sizably in magnitude to be as large as around 25 % of GDP when the hegeomn becomes small in the world economy. When the hegemon is only about 6 percent of the world economy the valuation losses incurred during fragile times become as high as about 24 percent of output (they were about 13 percent of output in the benchmark case where the size of the US is a quarter of the world economy). The real appreciation incurred by the hegemon as the rest of the world piles up on US assets for safety is logically also larger as the size of the hegemon shrinks (from about 28 percent in the benchmark case to a 32 percent appreciation in the fragile state when the hegemon is only 6 $\%$ of the world GDP). Hence in a world where the hegemon gradually vanishes the real rate becomes lower, the insurance transfers become larger as a share of the hegemon GDP and the volatility of the real exchange rate is on the rise. This brings front and centre the possibillity that the safe asset provider may face a 'New Triffin dilemma' as argued in Gourinchas and Rey (2007a) (see Triffin (1960)). The gold value of the dollar is no longer fixed,

[29] Gourinchas and Rey (2016) argues that the decline in real rate may be due to the conjugation of - possibly related- boom-bust cycle (see also Rey (2017)) and the trend decline described above

but we may still live in a Triffin world (see Farhi et al. (2011); Obstfeld (2011); Emmanuel and Matteo (2016)). In the 1960s, the source of the problem was the mismatch between the amount of gold held by the US Federal Reserve (the 'backing' of the dollar) and the outstanding dollars held abroad. Similarly, in a world of a shrinking hegemon there is a growing asymmetry between the fiscal capacity of the United States (the 'backing' of US Treasury bills) and the stock of debt assets held abroad. In our model the US government bond is safe and never faces a haircut (unlike the rest of the world bond) but the large external balance sheet exposure of the US can generate potentially large valuation losses in the event of a global crisis. As the exposure keeps growing, it could even threaten the fiscal capacity of the hegemon, or the loss absorbing capacity of its central bank, leading to a run equilibrium.

5 Conclusion

In this paper we present new estimates of the exorbitant privilege on the 1952-2016 period. We also argue that this exorbitant privilege is an insurance premium and is associated to an exorbitant duty which is an insurance transfer during global crises as the US, hegemon of the international financial system, provides insurance to the rest of the world. We construct a model to explain those facts and is also able to account for the external balance sheet composition of the hegemon as well as deliver interesting insights on the behaviour of the real exchange rate, net exposure of the hegemon to world risk and behaviour of the real safe rate. Our central assumption is that the centre country (the US) is in aggregate less risk averse than the rest of the world. The interpretation of the structure of the international financial system in terms of a global insurer making transfers to the rest of the world in crisis times and the interpretation of the exorbitant privilege as an insurance premium are novel. Our general equilibrium model opens the way to a research agenda that could analyse more deeply the underlying structural characteristics of the hegemon and discuss the dynamics following large shocks to the existing monetary order.

average returns	1952:1-2015:4	1952:1-1972:4	1973:1-2015:4
		(a) : Valuations	
$r^a - r^l$	3.13%	2.38%	3.49%
r^a	6.81%	6.59%	6.92%
r^l	3.68%	4.20%	3.43%
		(b) : Financial Flows	
$r^a - r^l$	1.90%	2.56%	1.58%
r^a	5.26%	6.16%	4.82%
r^l	3.36%	3.60%	3.24%
		(c) : Mixed	
$r^a - r^l$	2.46%	2.56%	2.42%
r^a	5.77%	6.16%	5.57%
r^l	3.30%	3.60%	3.16%
		(d) : Equal FDI returns	
$r^a - r^l$	1.02%	1.81%	0.64%
r^a	4.33%	5.41%	3.80%
r^l	3.30%	3.60%	3.16%

Table 1: Annualized total real returns on external assets and liabilities. In Panel (a) all "Other changes" are allocated to valuations; in Panel (b) "Other changes" are allocated to financial flows; in Panel (c) "Other changes" are allocated to financial flows except for FDI assets and liabilities for which "Other changes" are allocated to valuations; Panel (d) is the same as Panel (c) but we equated the rates of returns of FDI assets to the ones on liabilities. r^a refers to the total return on gross assets, r^l to the total return on gross liabilities. Returns are quarterly (annualized).

Std. Dev.	1952:1-2015:4	1952:1-1972:4	1973:1-2015:4
$r^a - r^l$	10.19%	6.71%	11.53%
r^a	15.24%	9.54%	17.38%
r^l	11.78%	10.75%	12.28%

Table 2: Standard Deviation of Quarterly Returns. The table reports the quarterly standard deviation of total returns on gross external assets and liabilities.r^a refers to the total return on gross assets, r^l to the total return on gross liabilities. "Other changes" are allocated to financial flows except for FDI assets and liabilities for which "Other changes" are allocated to valuations.

	r^o	r^d	r^{di}	r^e
		(a) : valuations		
1952:1-2015:4	1.88%	4.15%	4.04%	3.70%
1952:1-1972:4	1.35%	3.78%	1.48%	4.31%
1973:1-2015:4	2.15%	4.34%	5.3%	3.41%
		(b) : financial flows		
1952:1-2015:4	1.43%	1.96%	2.32%	2.00%
1952:1-1972:4	1.69%	3.04%	1.48%	2.80%
1973:1-2015:4	1.30%	1.44%	2.73%	1.61%
		(c) : mixed		
1952:1-2015:4	1.43%	1.96%	4.04%	2.00%
1952:1-1972:4	1.69%	3.04%	1.48%	2.80%
1973:1-2015:4	1.30%	1.44%	5.30%	1.61%
		(d) : Equal FDI returns		
1952:1-2015:4	1.43%	1.96%	0%	2.00%
1952:1-1972:4	1.69%	3.04%	0%	2.80%
1973:1-2015:4	1.30%	1.44%	0%	1.61%

Table 3: Excess Returns by Asset Class. In Panel (a) "Other changes" are allocated to valuations; in Panel (b) "Other changes" are allocated to financial flows; in Panel (c) "Other changes" allocated to financial flows except for FDI assets and liabilities for which "Other changes" are allocated to valuations. Panel (d) is the same as Panel (c) but we equated the rates of returns of FDI assets to the ones on liabilities. r^o refers to the annualized quarterly excess return on 'other assets'; r^d to 'debt'; r^{di} to direct investment and r^e to equities.

Table 4	$nagdp$	$vagdp$		$nagdp$	$vagdp$
1962:2-2015:4			1990:1-2015:4		
vix	-0.60**	-0.05**		-0.50**	-0.09**
	(.11)	(.02)		(.09)	(.03)
c	-1.75	1.28**		-1.75	2.52
	(2.1)	(.36)		(2.1)	(.70)
N	190	190		80	80
Adj. R^2	0.14	0.04		0.26	0.11

Table 4: Exorbitant Duty over Time. The table reports the results from an OLS regression of the U.S. net foreign asset position relative to GDP ($nagdp$) on the VIX index extended before 1986 with the volatility of the MSCI-ex US index. $vagdp$ refers to the valuation component (relative to GDP) defined as $VA_t = NA_t - NAt - 1 - FX_t$ where FA_t represents the net financial flows in period t.

Parameters	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
α	0.25	0.25	0.25	0.5	0.25	0.25	0.25	0.25
θ	1	1	1	1	1	1	2	1
σ_s^*	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5
$\sigma_{f \text{ or } d}^* - \sigma_s^*$	1	0	1	1	1	1	1	1
$\sigma_{f \text{ or } d} - \sigma_s$	0	0	0.5	0	0	0	0	0
b	0.42	0.42	0.42	0.42	0.42	0	0.42	0.42
δ	0.05	0.05	0.05	0.05	0.05	0.05	0.05	1
r^*	0.5	0.5	0.5	0.5	1	1	0.5	0.5
Equity Premium (%)	2.41	1.82	2.96	2.1	2.1	0.093	2.65	2.65
Fragile	13.88	10.43	16.94	12.09	12.09	0.123	15.09	15.09
Disaster	48.84	36.41	51.03	42.44	42.44	0.123	48.61	48.61
Price Earning Ratio (%)	17.78	17.39	18.16	17.57	17.57	16.18	17.95	17.95
Fragile	18.86	18.12	19.61	18.47	18.47	16.18	19.17	19.17
Disaster	6.58	7.32	6.09	6.98	6.98	16.18	6.35	6.35
T-bill excess return (%)	0.73	0.93	1.24	0.73	-1.45	-0.07	1.07	1.07
Fragile	4.31	5.69	7.1	4.13	-7.46	-0.12	6.14	6.14
Disaster	7.44	11.87	10.65	7.25	-7.37	-0.12	10.05	10.05
External Excess Return (%)	0.388	0.21	0.322	0.467	0.57	0.012	0.33	0.33
Fragile	3.32	1.26	2.75	3.28	3.68	0.014	2.78	2.78
Disaster	21.86	10.08	19.13	19.71	21.86	0.014	25.47	25.47
Trade Balance (% of output)	-0.33	-0.33	-0.33	-0.22	-0.22	-0.22	-0.3	-0.3
Fragile	-0.52	-0.33	-0.39	-0.33	-0.22	-0.21	-0.55	-0.55
Disaster	10.57	3.16	6.21	6.86	6.86	-0.21	12.81	12.81
Net Foreign Assets (% of output)	-16.81	0	-10.18	-9.28	-9.28	3.68	-24.38	-24.38
Fragile	-30.26	-3.42	-19.93	-17.05	-17.05	3.62	-42	-42
Disaster	-46.14	-12.82	-26.87	-29.79	-29.79	3.62	-56.3	-56.3
Net Debt Liabilities (% of output)	-18.38	-10.29	-6.06	-16.03	-16.03	-10.24	-0.8	-63
Fragile	-33.62	-11.49	-18.95	-23.18	-23.16	-10.09	25.11	-116
Disaster	-47.31	-15.99	-26.59	-31.47	-31.47	-10.09	-51.58	-164
Real Exchange Rate (log)	-0.59	-0.59	-0.59	-0.58	-0.58	-0.4	-0.25	-0.25
Fragile	-0.93	-0.59	-0.71	-0.88	-0.88	-0.38	-0.47	-0.47
Disaster	15.92	5.21	9.84	15.08	15.08	-0.38	11.3	11.3
Safe Real Rate (%)	3.02	3.61	2.4	3.33	3.33	5.92	2.77	2.77
Fragile	-11.64	-8.02	-15.52	-9.76	-9.76	5.92	-13.21	-13.21
Disaster	24.68	23.93	25.23	24.31	24.31	5.92	24.94	24.94
All time	1.89	2.85	0.86	2.39	2.39	5.92	1.47	1.47

Table 5: Model solutions under different sets of parameters.

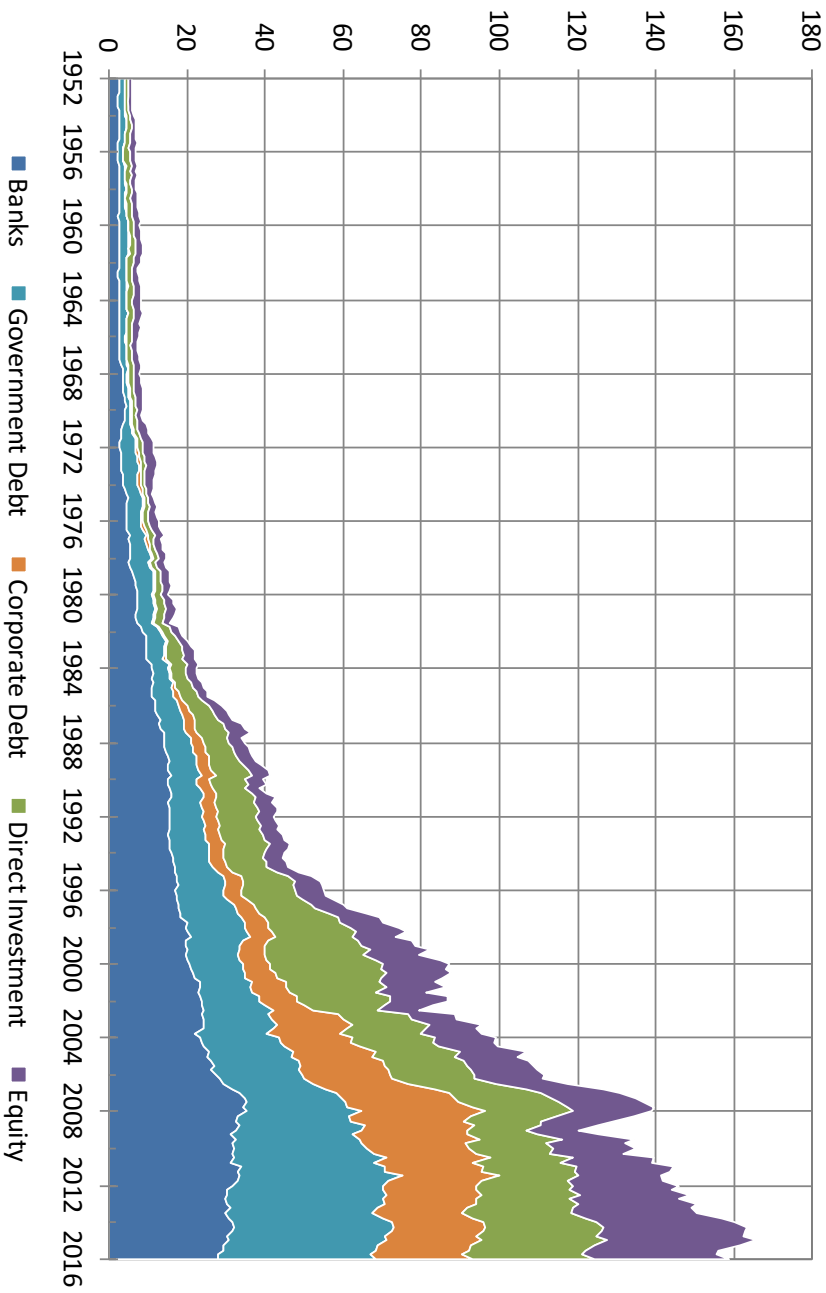

Figure 1: Gross Liabilities of the United States, percent of GDP

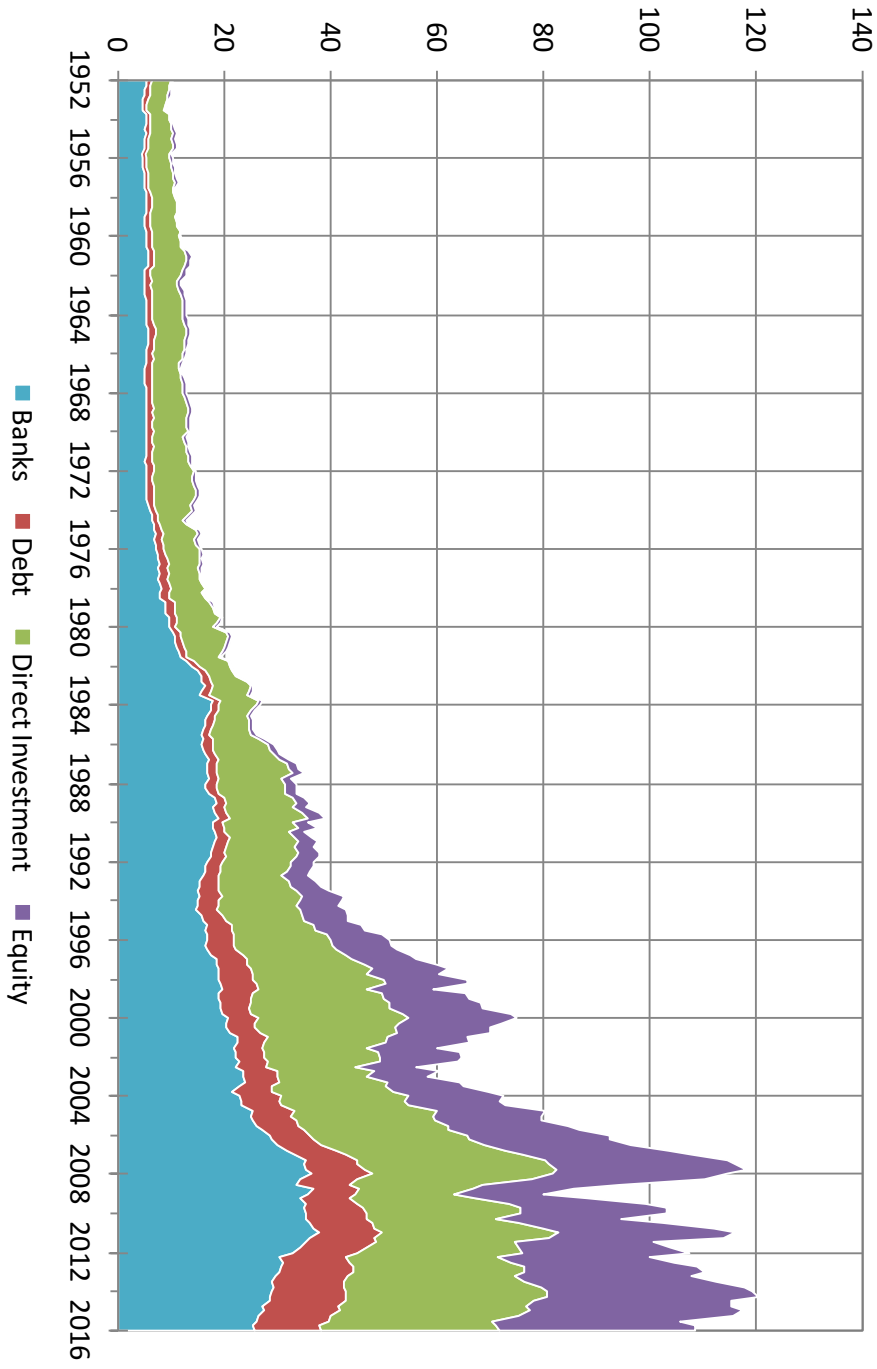

Figure 2: Gross Assets of the United States, percent of GDP

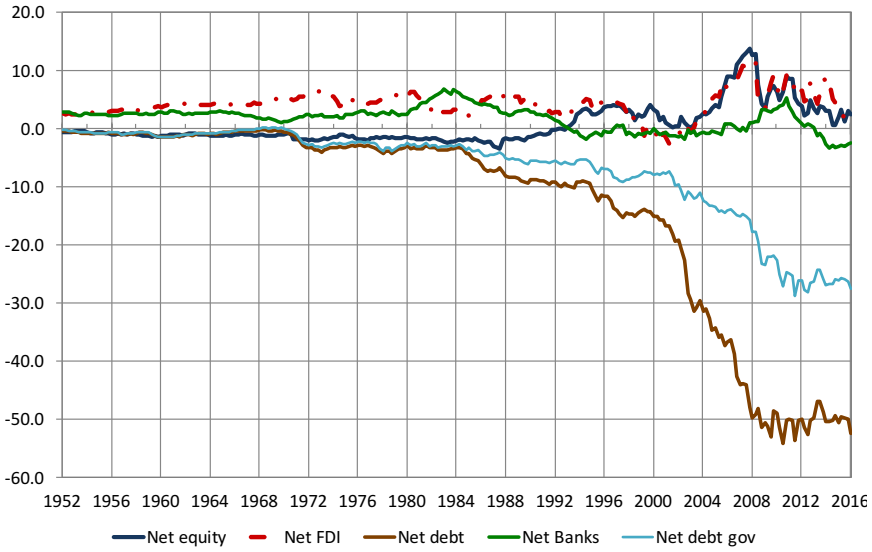

Figure 3: U.S. Net Positions by asset classes, percent of GDP

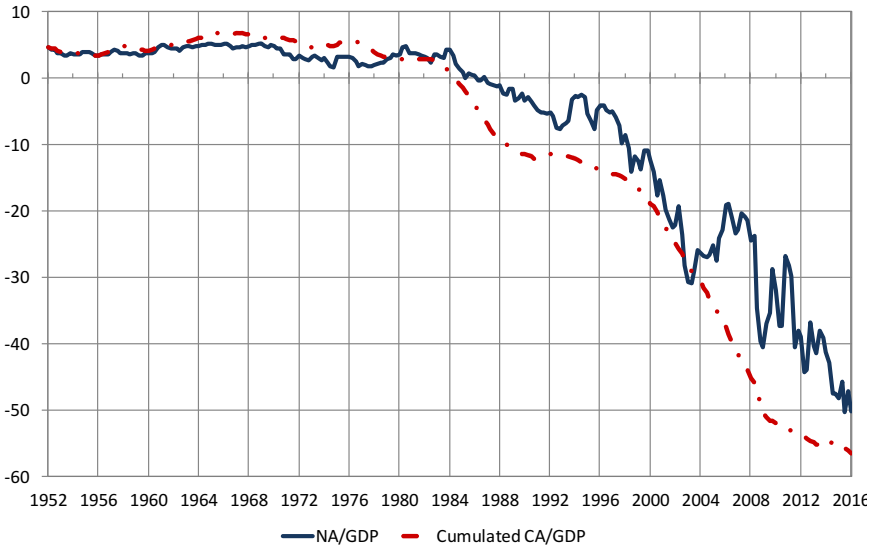

Figure 4: U.S. Net Foreign Asset Position, percent of GDP

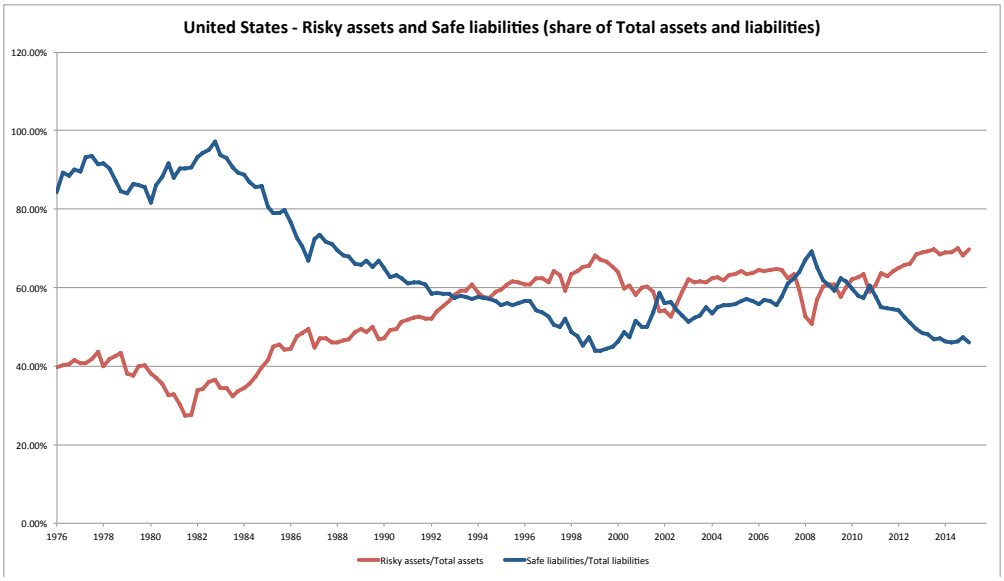

Figure 5: Liquid liabilities (percent of total liabilities) and risky assets (percent of total assets)

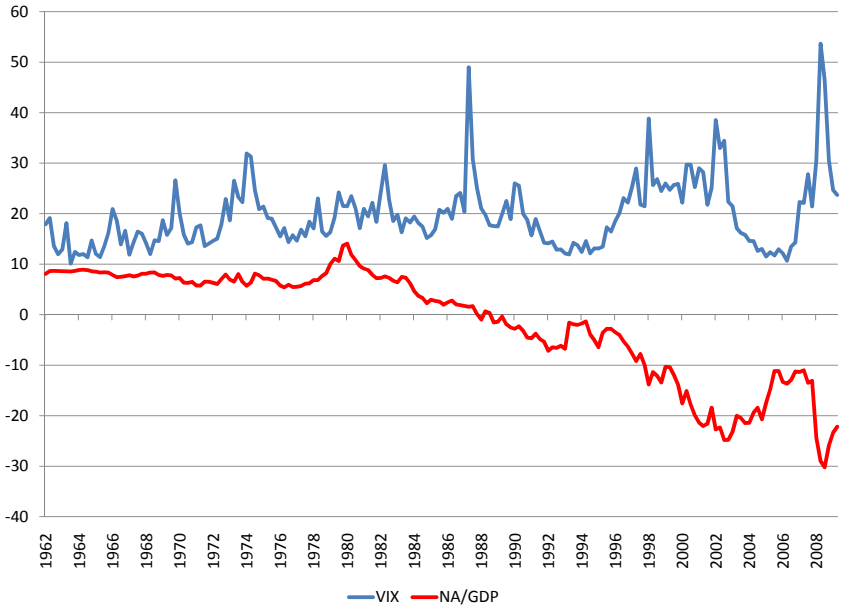

Figure 6: VIX and Net Foreign Asset Position (percent of US GDP)

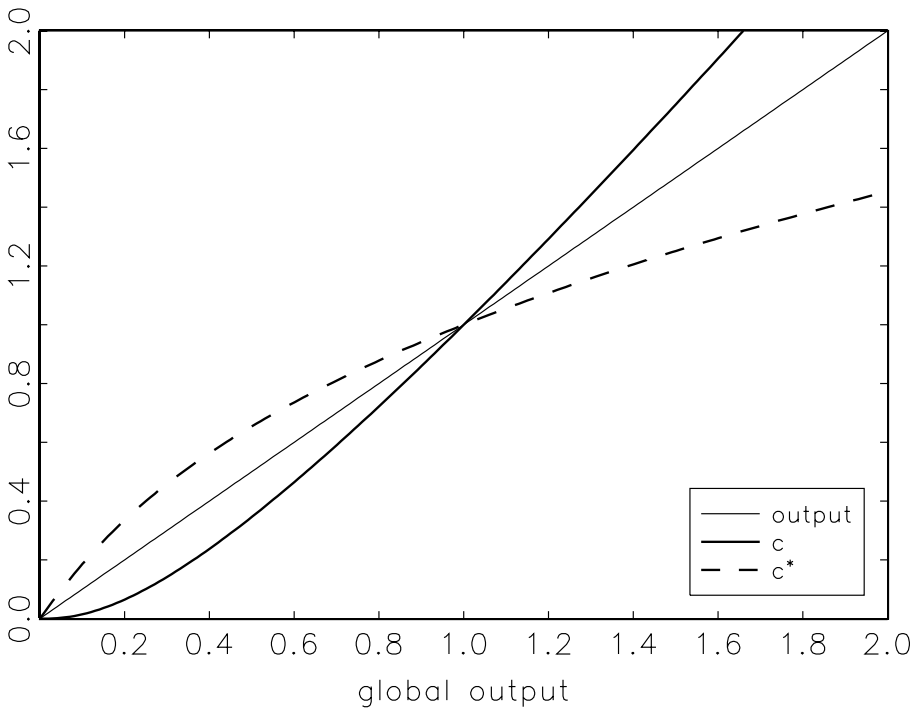

Figure 7: Risk sharing with heterogenous risk aversion. Home provides insurance to Foreign. The figure is drawn under the following assumptions: $E\bar{y} = 1$, $\sigma = 2$, $\sigma^* = 5$.

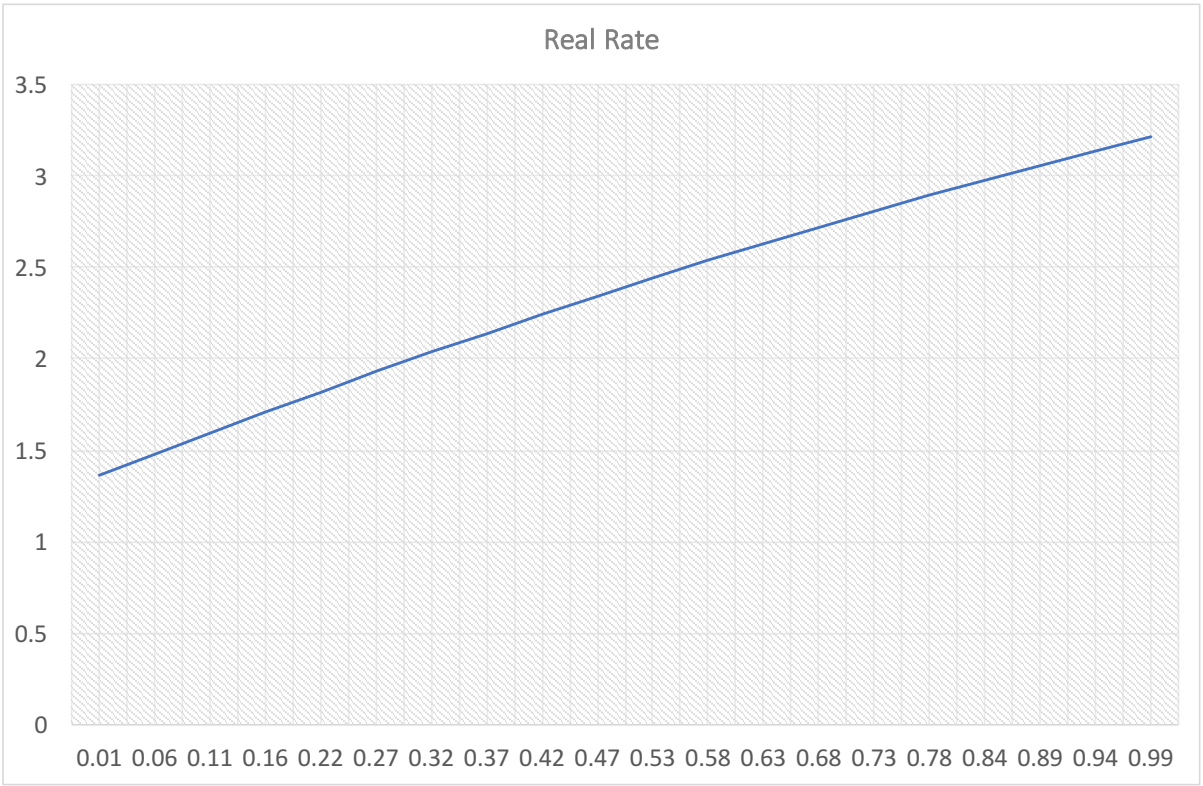

Figure 8: Real safe rate and relative size of the Hegemon. Small number means Hegemon is small in the world economy.

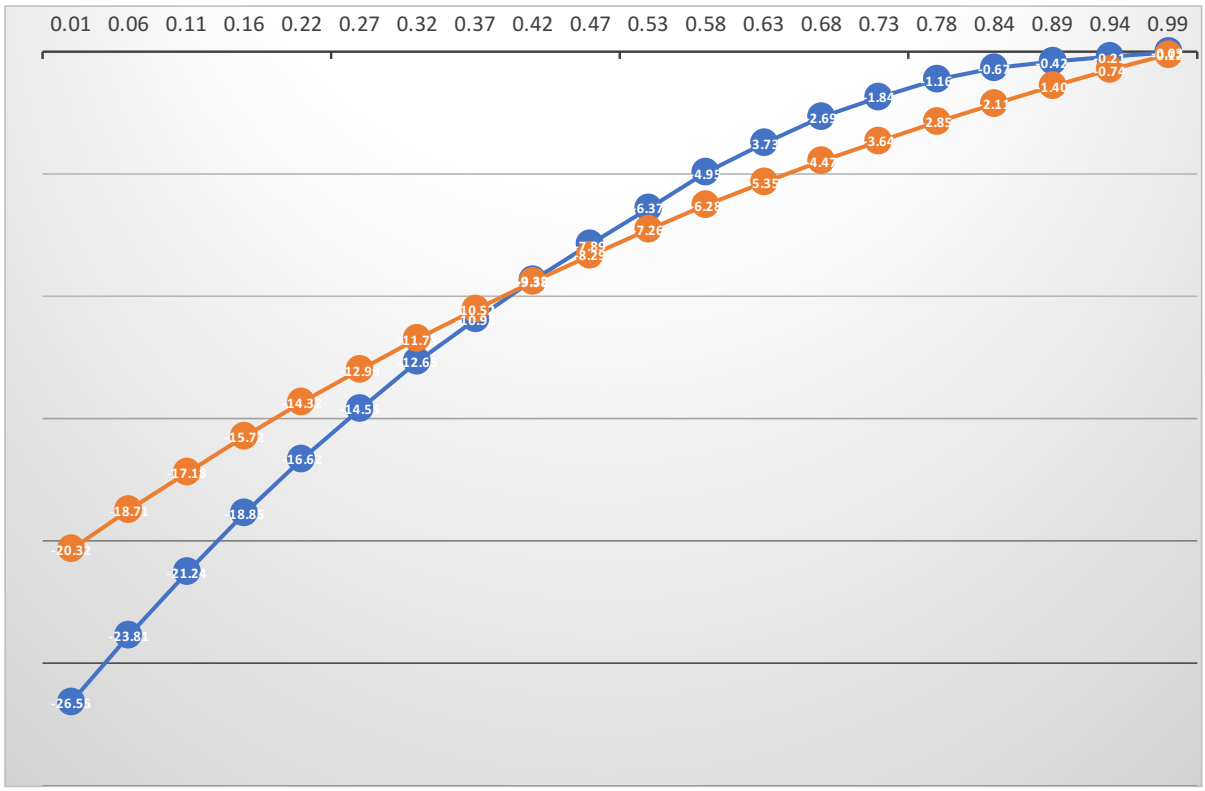

Figure 9: Net debt exposure (% of output, in blue) and valuation losses (% of output, in orange) as a function of the relative size of the Hegemon. Small number means Hegemon is small in the world economy.

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Appendix

A Data

To illustrate our methodology, consider the following stock-flow equation, describing the law of motion for a given class of assets i. Assets i include all the broad categories of assets classified as in the balance of payments: portfolio debt (with a distinction between corporate and government bonds whenever the data allow us to do so), direct investment, portfolio equity investments, and other assets (bank loans and trade credit)

$$PX^i_{t+1} = PX^i_t + FX^i_{t+1} + VX^i_{t+1} + OC^i_{t+1}. \tag{21}$$

In writing equation (21), we adopt the representation of the BEA: PX^i_t represents the position given by the BEA at the *end of period* t for assets i, FX^i_t the financial flow during period t, VX^i_t the explicit valuation gain that can be attributed to currency and asset-price movements, while OC^i_t is a residual item for 'other changes'.[30] For a given class of asset i, we can compute an *explicit* total return $R^{ex,i}_{t+1}$ as

$$\left(R^{ex,i}_{t+1} - 1\right) PX^i_t = I^i_{t+1} + VX^i_{t+1},$$

where I^i_{t+1} is the distributed yield, as measured by net income receipts for asset i. Summing over all asset classes, measured (or *explicit*) total returns on the net foreign asset positions are given by

$$\left(R^{ex}_{t+1} - 1\right) NFA_t = I_{t+1} + VAL_{t+1}$$

where I_{t+1} is the net income balance (including interest income, distributed dividends and direct investment earnings) and VAL_{t+1} is the sum across all assets of the net valuation changes reported by the BEA (currency and asset prices).[31]

The final step is to go back to balance of payment accounting to insure consistency of the data. Substituting financial flows using the fundamental Balance of Payment equation gives us the international investment position at the end of period $t+1$, NA_{t+1} as:[32]

$$NFA_{t+1} = R^{ex}_{t+1} NFA_t + NX_{t+1} + SD_{t+1} + OC_{t+1}$$

where SD_t is the statistical discrepancy between trade and financial flows reported in the balance of payments, NX_t is the trade balance. Other changes OC_t can represent either mismeasured valuations (as is assumed in Gourinchas and Rey (2007a)), mismeasured financial flows (as in Curcuru et al. (2008) and Forbes (2010)), mismeasured initial positions, or any combination

[30] In the BEA's IIP reconciliation table 3, other changes represent "changes in coverage due to year-to-year changes in the composition of reporting panels, primarily for bank and nonbank estimates, and to the incorporation of survey results. Also includes capital gains and losses of direct investment affiliates and changes in positions that cannot be allocated to financial flows, price changes, or exchange-rate changes."

[31] According to the Balance of Payments manual, direct investment income in the current account includes distributed earnings as well as the share of reinvested earnings. So there is an entry in the current account for reinvested earnings and an offsetting entry in the financial account.

[32] For more details, see Appendix A. In these derivations, we ignore the capital account as well as unilateral transfers. Both components are small components of the US balance of payments.

thereof.[33]

If we allocate OC_{t+1} to mismeasured valuations, we get a new estimate of total (implicit) returns R_{t+1} such that:

$$\begin{aligned} NFA_{t+1} &= R_{t+1}NFA_t + NX_{t+1} + SD_{t+1} \\ (R_{t+1} - 1)\, NFA_t &= I_{t+1} + VAL_{t+1} + OC_{t+1} \end{aligned}$$

If 'other changes' reflect mismeasured financial flows, the return on the net foreign position is unchanged, but the Balance of Payments identity requires that net exports NX_t are mismeasured by a commensurate amount:

$$\begin{aligned} NFA_{t+1} &= R^{ex}_{t+1}NFA_t + NX'_{t+1} + SD_{t+1} \\ NX'_{t+1} &= NX_{t+1} + OC_{t+1} \end{aligned}$$

Finally, if other changes represent mismeasured initial positions, both initial position and returns are mismeasured, with:

$$\begin{aligned} NFA_{t+1} &= R'^{ex}_{t+1}NFA'_t + NX_{t+1} + SD_{t+1} \\ NFA'_t &= NFA_t + \frac{OC_{t+1}}{(I_{t+1} + VAL_{t+1})/NA_t + 1} \\ \left(R'^{ex}_{t+1} - 1\right) NFA'_t &= I_{t+1} + VAL_{t+1} \end{aligned}$$

B Data Appendix

The data methodology follows that of Appendix A in Gourinchas & Rey (2007). We focus here on:

- a description of the series taken to compute each categories
- a description of the formulas taken to reallocate the residuals.

We provide a Line by Line Description of Flows, Positions, and Return Data.

Since the original paper, new breakdowns have been introduced:

- on debt liability, between Government Debt and Corporate Debt;
- on other asset, between Gold and non Gold.

B.1 Computation of Quarterly Positions on Assets

B.1.1 Equity

Flows Before 1982, from FFA, table F106 Rest of the World line 50 (FU263164103.Q, ROW foreign corporate equities liability , including ADRs, NSA). After 1982Q1 from BEA (USIT Table 8b line A2 before 1998Q1 then USIT Table 8a line A4).

[33] Lane and Milesi-Ferretti (2009) propose an allocation of these 'other changes' based on best judgement.

Positions End of years levels from BEA. Before 1976, various lines from Survey of Current Business (thereafter named SCB). After 1976, from International Investment Position (thereafter named IIP) Table 2, line 21.

Capital gains We first choose a list of countries to compute an aggregate capital gain. The choice is based upon the major countries represented in the portfolio, subject to availability of series. Hence the stock indices (in USD) taken in GFD are from UK (FTSE All-Share Index), Japan (Japan Nikkei 225 Stock Average), Netherlands (Netherlands All-Share Price Index), France (France SBF-250 Index), Canada (Canada S & P/TSX 300 Composite), Germany (Germany CDAX Composite Index), Mexico (Mexico SE Indice de Precios y Cotizaciones), Switzerland (Switzerland Price Index), Hong Kong (Hong Kong Hang Seng Composite Index), Australia (Australia ASX All-Ordinaries), Italy (Banca Commerciale Italiana Index), and Brazil (Brazil Bolsa de Valores de Sao Paulo). Those 12 countries represent at least 64% of equity holdings over the period.

Then we weigh the country returns by the adjusted shares of each country in the equity asset portfolio of the USA. The country positions are recorded in the US Treasury Reports on US Holdings of Foreign Securities after 1994. Before 1994, we estimate them according to Direct Investment positions (as recorded by the BEA), following the same ratios of Direct Investment vs. Equity holdings.

B.1.2 Debt

Flows Before 1982, from FFA, Table F106 line 44 (FU263163003.Q, Bonds, NSA). After 1982Q1, from BEA (USIT Table 8b line A13 before 1999Q1, then USIT Table 8a line A18).From [CEM] (p99): "Foreign bonds include securities issued by foreign governments and their political subdivisions, by foreign corporations, and by international and regional financial institutions. Types of bonds and notes, both in registered and in bearer form, include convertible debt, zero-coupon debt, index-linked debt securities, medium-term notes, note issuance facilities, floating rate notes, variable rate notes, structured rate notes, asset-backed securities, and all other long-term debt securities. Asset-backed securities include collateralized mortgage obligations (CMOs), collateralized bond obligations (CBOs), collateralized loan obligations (CLOs), collateralized debt obligations (CDOs), other securities backed by pools of mortgages, credit card receivables, automobile loans, consumer and personal loans, and commercial and industrial loans. Asset-backed securities are reportable if the issuer se- curitizing the assets is a foreign resident." [CEM] also documents some underreporting of US foreign bond purchases prior to 2007, especially asset-backed bonds issued by Caribbean financial centers. Gross purchases were adjusted to account for this phenomenon. It is not clear if the flowswere subsequently revised.

Positions End of years levels from BEA. Before 1976, various lines from Survey of Current Business as follows: 1951Q4 from 1963 SCB supplement, assuming that 50% of 'other securities' represent fixed income instruments. 1955, 1960, 1962 to 1969 end of year positions from October SCB, various years. 1970 to 1975 end of year positions from August 1984 SCB. After 1976, from International Investment Position Table 2, line 20. The debt asset series only includes **long term** fixed income instruments.

Capital gains We first choose a list of countries to compute an aggregate capital gain. The choice is based upon the major countries represented in the portfolio, subject to availability of

series. Hence bond series are taken from Canada, UK, Germany, Japan, Mexico, France, Netherlands, Spain, Ireland, Italy, Australia, Sweden and Denmark. For each country, we compute a capital gain on dollar-denominated bonds and on local currency ones. The country capital gain is weighted according to the share of dollar-denominated bonds, computed from US Treasury Surveys on US Portfolio Holdings of Foreign Securities. Dollar bonds capital gains are those of Barclays Eurodollar series when available, US 10-year otherwise. Local currency bonds capital gains are derived from changes in GFD 10-year government bond yields (see formula in Campbell et al. (1997), equation 10.1.19 p408). Then we weigh the country returns by the adjusted shares of each country in the long-term debt asset portfolio of the USA. The country positions are recorded in the US Treasury Reports on US Holdings of Foreign Securities after 1997. Before 1997, we set them to their value of 1997.

The source data for the local currency government bond yields are as follows:

- Canada 10-year Government Bond Yield: IGCAN10D. Source: bank of canada. duration: assumed 10 years.
- United Kingdom 10-year Government Bond Yield: IGGBR10D. Starts in 1958. Sources: Central Statistical Office, Annual Abstract of Statistics, London: CSO (1853-), the Financial Times, and the Bank of England. The benchmark bond is used for this series. The benchmark bond is the bond that is closest to the stated maturity without exceeding it. When the government issues a new bond of the stated maturity, it replaces the bond used for the index to keep the maturity as close to the stated time period as possible. Depending on how much of a difference there is in maturities, there may be some adjustment in the yield when the new bond is introduced.
- Germany 10-year Benchmark Bond: IGDEU10D. Source: Bundesbank The benchmark bond is used for this series. The benchmark bond is the bond that is closest to the stated maturity without exceeding it. When the government issues a new bond of the stated maturity, it replaces the bond used for the index to keep the maturity as close to the stated time period as possible. Depending on how much of a difference there is in maturities, there may be some adjustment in the yield when the new bond is introduced.
- Japan 10-year Government Bond Yield: IGJPN10D. Sources: Industrial and Commercial Semi-Annual Report (1948-57), Bank of Japan, Economic Statistics Monthly (1969-).
- Mexico 10-year Government Bond Yield: IGMEX10D. Starts in 2001. Source: Banco de Mexico
- France 10-year Government Bond Yield: IGFRA10D. Sources: INSEE, Bulletin Mensuel Statistique, Paris: INSEE (1800-). The Tec-10 Index Yields for Constant Maturity French Bonds is used beginning April 9, 1996.
- Netherlands 10-year Government Bond Yield: IGNLD10D. Sources: Central Bureau of Statistics, Maandschrift (1946-). The 2 1/2% consol is used from 1946 until 1954, the 3 1/4% issue of 1948 is used from 1955 until October 1964, and an index of the three or five longest running issues of the Dutch government begins in November 1964.
- Spain 10-year Government Bond Yield: IGESP10D. Sources: Banco d'Espana, Bulletin Mensual, Madrid: Banco d'Espana (1948-). An index of private bonds is used from 1958 through

March 1978 with monthly data starting in July 1959, and an index of government bonds with a maturity of over 2 years is used beginning from April 1978 until March 1992. The 10-year bond is used beginning in 1986.

- Ireland 10-year Government Bond Yield: IGIRL10D. Sources: Central Bank of Ireland, Quarterly Bulletin (1947-) Notes: The Ireland 5s from the 2nd National Loan due in 1950 is used from 1928 through 1953. Data are annual from June 1950 through 1952, quarterly from I/53 until II/54 and monthly in all other periods. The 3.5% exchequer bonds of 1950-51 maturing in 1970 are used from 1951 until 1970, and an index of 15-year government bonds is used from 1971 to 1980 when 10-year bonds are used.

- Italy 10-year Government Bond Yield: IGITA10D. Sources: Banca d'Italia, Bolletino (1946-). The 3.5% consol is used from 1900 until 1953, and the 5% reconstruction loan of 1978 is used beginning in 1954. Data are annual for 1900 through 1902. An index of government treasury bonds begins in 1955. The average maturity of these bonds is six years. Data for 10-year bonds begins in 1980.

- Australia 10-year Government Bond Yield:IGAUS10D. Sources: Reserve Bank of Australia, Monthly Statistical Bulletin (1956-). 12 years from 1941 to May 1959, 20 years from June 1959 through 1980, 15 years from 1981 through 1990, and 10 years since 1991 to produce the theoretical yield on a perpetual ten-year bond, and 20 years again beginning in June 1997.

- Sweden 10-year Government Bond Yield: IGSWE10D. Sources: Sveriges Riksbank, Quarterly Review (1946-). Notes: Beginning in 1957, the average yield on bonds maturing in 15 years or more is used, and beginning in 1980, 10-year government bonds are used.

- Denmark 10-year Government Bond Yield: IGDNK10D. Sources: Danmarks Bank, Monetary Review (1946-) Notes: The consolidated bond is used through 1976. In 1977, the 10-year bond is used.

B.1.3 Direct Investment

Flows Before 1960Q1, from FFA Table F106 line 56 (FU263192005.Q, U.S. direct investment abroad). Note that through 1992Q4, FFA US direct investment abroad excludes net inflows from corporate bonds issued by Netherlands Antillean financial subsidiaries. There is no discrepancy here since these bonds issues start after 1978. Between 1960Q1 and 1981Q4 from BEA (USIT Table 1 line 51). After 1982Q1 from BEA (USIT Table 1 line 51) from which we deduct reinvested earnings (USIT Table 7b line 4 before 1999Q1, then USIT Table 7a line 4). Indeed after 1982, since reinvested earnings are explicit, we deduct them from flows and incomes so as to count them only once in valuations.

Positions End of year levels from BEA. Year 1951 comes from Survey of Current Business (recorded on historical cost) and we do not benchmark until 1982 when data on market value starts being available from IIP Table 2 line 43 (A break in series in 1994 reflects the reclassification from the direct investment accounts to the nonbank investment accounts of intercompany debt positions between parent companies and affiliates that are not depository institutions and that

are primarily engaged in financial intermediation. Estimates for 1976 forward are linked to the 1977, 1982, 1989, 1994, 1999, and 2004 benchmark surveys of U.S. direct investment abroad).

Capital gains We first choose a list of countries to compute an aggregate capital gain. The choice is based upon the major countries represented in the portfolio, subject to availability of series. Hence the stock indices (in USD) taken in GFD are from Australia (Australia ASX All-Ordinaries), Belgium (Brussels All-Share Price Index), Bermuda (Bermuda SE Index), Brazil (Brazil Bolsa de Valores de Sao Paulo), Canada (Canada S & P/TSX 300 Composite), France (France SBF-250 Index), Germany (Germany CDAX Composite Index), Hong Kong (Hong Kong Hang Seng Composite Index), Ireland (Ireland ISEQ Overall Price Index), Italy (Banca Commerciale Italiana Index), Japan (Japan Nikkei 225 Stock Average), Luxembourg (Luxembourg SE LUXX Index), Mexico (Mexico SE Indice de Precios y Cotizaciones), Netherlands (Netherlands All-Share Price Index), Panama (Panama Stock Exchange Index), Singapore (Singapore FTSE All-Share Index), Spain (Madrid SE General Index), Switzerland (Switzerland Price Index), UK (FTSE All-Share Index), and Venezuela (Caracas SE General Index). Those 20 countries represent at least 74% of direct investments over the period 1966-2011 (and at least 80% after 1974).

Then we weigh the country returns by the adjusted shares of each country in the direct investment portfolio of the USA. The country positions are recorded in the BEA. Before 1965, we set the shares to their values of 1966.

Double counting of Investment (FDI) earnings. One has to be careful not to double count reinvested earnings as they are counted both in the current account income, as well as in the financial account flows. We have to count them either in the capital gain part of the return, or in the yield part (in they enter the yield part theay are also in the financial flow part). To understand this, let us write the change in positions between two periods:

$$POS_t = POS_{t-1} + V_t + F_t$$

where V_t is the valuation which does not include reinvested earnings, and $F_t = FLOW_t + RE_t$ the total flows reported by the U.S. international transactions, including real flows and reinvested earnings RE_t. Let us define a 'small' capital gain cg_t by $cg_t = \frac{V_t}{POSt-1}$ and a 'big' capital gain CG_t by $CG_t = \frac{V_t+RE_t}{POSt-1}$.Before 1982 we do not have the breakdown of reinvested earnings in the flows nor the income so we have to use F_t. The implicit rate of return in the data (before 1982) is given by $\frac{POS_t - POS_{t-1} - F_t}{POS_{t-1}} = cg_t$.

We can write

$$POS_t = POS_{t-1}(1 + cg_t) + FLOW_t + RE_t = POS_{t-1}(1 + CG_t) + FLOW_t$$

so

$$CG_t = cg_t + \frac{RE_t}{POS_{t-1}}$$

To recap, we have:

1. the 'small capital gain' definition, where flows contain the reinvested earnings; this needs keeping the reinvested earnings part also in income flows.

2. the 'big capital gain' definition, where reinvested earnings are part of the capital gain, and therefore have to be removed from the income part.

After 1982, we are able to disaggregate distributed and reinvested earnings in flows and income. We use the CG_t definition of capital gain, because estimating the 'big' capital gain is likely more accurate than the 'small' one. Indeed, the 'big' capital gain is closely linked to capital gain on generic equity. It can be approximated by general market return indices. The 'small' capital gain definition is needed, though, to estimate an average proportion of reinvested earnings within the aggregate capital gain and to use this estimate pre 1982 where we have to adjust the capital gain as we cannot take reinvested earnings out of income and flows. We therefore do the following. We regress the evolution of direct investment implicit return (ie the discrepancy between BEA recorded positions and flows, divided by position) against the estimated capital gain based on market estimates on 1982-2007 (Note: we exclude the crisis). We find that $1+cg = 0.939(1+CG)$ where cg denotes the implicit BEA capital gain and CG the market estimated capital gain. We use this coefficient to scale down the estimated capital gain before 1982, corresponding to the fact that before 1982, reinvested earnings are part of financial and income flows. Post 1982, we subtract reinvested earnings from the income and the financial flow data and we compute CG_t using stock index data.

B.1.4 Gold

Flows Gold flow is defined as the difference on stocks (measured in million of fine troy ounces) from IFS series 111.1AD.ZF..., valued at market price using GFD price of gold (series __XAU_D. description: From January 30, 1934 until March 1968, the gold transactions of the U.S. Government for both monetary and industrial purposes were made at $35 per fine ounce, plus or minus a handling charge of one quarter of one percent and less mint charges. In 1968 a two-tiered gold system was instituted under which the private commodity price of gold was permitted to fluctuate without official intervention, while the official price and role of monetary gold remained unchanged among monetary authorities. This system was terminated on November 10, 1973 because of the wide variations between "official" prices and market prices. From 1973 on, separate series are provided for the daily PM Fixing price for gold in London and the daily price of gold in New York.). The IFS data is annual until 1957, quarterly after. Note that the IMF values the stock of gold using a price of $35/oz before 1972 and $42.2 after.

Positions Gold stock is taken from IMF International Financial Statistics, 'Gold in million of fine troy ounces', and valued at market price.using GFD series for the price of gold (series __XAU_D)

Capital gains Gold valuation arises from GFD market price for Gold.

B.1.5 Other (excluding gold)

Flows

- Before 1960Q1, other asset flows are constructed to match the BEA definition. We start with other asset flows defined from FFA: FFA total assets (Table F106 line 39, FU264190005.Q, Net increase in U.S. liabilities of the rest of the world) minus the sum of flows on equity, direct investment, debt and gold described above. Then we adjust back to the BEA definition of total flows with an estimate of change in net interbank claims and adding Gold and SDR

flows. However, the estimated change in net interbank claims is zero before 1960Q1. Gold and SDR are from FFA Table F106 line FU263011005.Q (sum of monetary gold and SDR).

- Between 1960Q1 and 1981Q4, (opposite of) BEA Table 1 line 40 (US-owned assets abroad) minus estimated flows on equity, direct investment, debt and gold as described above.
- After1982Q1, (opposite of) BEA Table 1 line 40 (US-owned assets abroad) minus estimated flows on equity, direct investment, debt and gold as described above, minus BEA Table 7b line 4/ Table 7a line 4 (direct investment abroad reinvested earnings)

For future reference: change in net interbank claims is calculated by two methods, then averaged. The first method, from the liability side is constructed as BEA Table 1 line 55 (foreign owned assets in the US) + FFA Table F106 line FU263011005.Q (sum of monetary gold and SDR) + estimated bond issuance from Neth. Ant. subsidiaries of US firms minus FFA Table F106 line 13 FU264090005.Q (net acquisition of financial assets). The estimated bond issuance from Neth. Ant. bond issued by US subsidiaries is the difference between minus FFA-based direct investment assets (Table F106 line 56, FU263192005.Q, U.S. direct investment abroad) and BEA-based direct investment assets (USIT Table 1 line 51), and is set to 0 before 1979Q1 (see bond asset flows for the offsetting adjustment). The second method, from the asset side, is constructed as BEA Table 1 line 40 (U.S. owned assets abroad) + FFA Table F106 line FU263011005.Q (sum of monetary gold and SDR) + estimated bond issuance from Neth. Ant. subsidiaries of US firms minus FFA Table 106 line 39 FU264190005.Q (net increase in liabilities) minus BEA Table 1 line 39 (capital account transactions, net). The two methods coincide exactly

Positions

- Before 1961: from various SCB and SCB supplements. 1951-54 and 1956-59 from 1963 SCB Supplement (Sum of US Government assets, short term securities, and other long term securities); These positions do not include Gold reserves of the US. 1955 from Oct. 1969 SCB which offers more disaggregated estimates. To these SCB estimates, we add the IMF gold tranche from IFS (series 111.1C.SZF...)
- 1962 to 1969: from various SCB and SCB supplements. The SCB data now includes reserve position in the Fund.
- 1970-1975: from SCB August 1984, US official reserves (line 3) minus Gold (line 4), other US government assets (line 8) + US claims on unaffiliated foreigners (line 18) and claims reported by US banks (line 19).
- After 1976: from BEA IIP Table 2 US official reserves (line 7) minus Gold (line 8) plus US government assets (line 12) + US claims on unaffiliated foreigners (line 22) and US claims reported by banks (line 23).

Capital gains None.

B.2 Computation of Quarterly Positions on Liabilities

B.2.1 Equity

Flows Before 1973, FFA equity (FU263064105.Q, Net purchases of U.S. corporate equity by the rest of the world, plus FU263064203.Q, Net purchases of U.S. mutual fund shares by the rest of the world). The FFA data includes equity purchased by foreign official agencies (reported separately by the BEA). After 1982Q1 from BEA (USIT Table 8b lines B2 and memo 4 until 1997Q4, USIT Table 8a lines B4 and memo 4 after 1998Q1).

Positions Until end 1969, various lines from BEA SCB. After 1970, BEA reports separately foreign official assets and other foreign assets. But in line 'Other foreign official assets' are considered both holdings of stock and corporate bonds. IMF reports, starting end 1982, the complete equity liability position, which enables to estimate the amount of stock included in line 'Other foreign official assets' of BEA. The ratio of this line is 0.82 of equity and 0.18 on debt on average after 1982. Therefore we consider for the position on equity liability, 0.82 times line 27 together with line 33 between 1970 and 1975 from the BEA SCB.We use 0.82 times line 33 together with line 39 between 1976 and 1981 from IIP Table 2. After 1982, position comes from IMF IFS end of year position on Equity Securities Liability.

Capital gains From S & P 500 Composite Price Index (from GFD).

B.2.2 Government Debt

We include in this category holdings and flows on US Treasury securities of all maturities (short and long), as well as holdings of agencies by both private foreign residents and foreign officials. The main issues here are that (a) the BEA data does not report holdings of ST agencies. We use the FFA data after 2001, which includes estimates based on the Treasury Surveys; (b) the BEA data does not isolate the agency flows and holdings of private foreign residents (only official holdings of agencies). We recover the flows using the FFA data after 2001. We do similar calculations for positions using BEA and FFA estimates (based on the Treasury Surveys).

Flows. detailed descriptions: From CEM [p105], BEA line 65 consists of "net purchases of U.S. Treasury bonds, notes, bills, and certificates by private foreign residents and international organizations. *U.S. Treasury securities of all maturities are included.* Excluded from these accounts are net purchases by foreign official agencies, which are included in line 58." From CEM [p121], line 58 consists of "net transactions by foreign official agencies in U.S. Treasury bills and certificates and in bonds and notes (both marketable and non-marketable)." and line 59: "This account measures net transactions by foreign official agencies in bonds, notes, and other *long- term* obligations of U.S. government corporations and federally sponsored agencies (U.S. agency bonds). Transactions in U.S. agency bonds, other than those with foreign official agencies, are covered in part of line 66 as U.S. federally sponsored agency bonds." From [CEM] p103, Table 8 line B30: "U.S. federally sponsored agency bonds include transactions in obligations of U.S. government- sponsored agencies and corporations. Transactions include new issues, redemptions, and transactions in outstanding issues. Both guaranteed and nonguaranteed obligations are included. Obligations

Type	Instrument	Investor	Maturity	period	source
Flow	Treasuries	Official	All	.–1960Q1	FFA Table F106.24, FU263061130.Q, treasury secur
Flow	Treasuries	Official	All	1960Q1–.	BEA USIT Table 1 line 58. US Treasury Securities
Flow	Agencies	Official	LT	.–1960Q1	FFA Table F106.27, FU263061713, Agency and GSE
Flow	Agencies	Official	LT	1960Q1–.	BEA USIT Table 1 line 59. Other gov. securities.
Flow	Agencies	Official	ST	2001Q2–.	difference between FFA F106.l27 and BEA T1.59
Flow	Treasuries	Private	All	.–1960Q1	FFA Table F106.25, FU263061140.Q, treasury secur
Flow	Treasuries	Private	All	1960Q1–.	BEA USIT Table 1 line 65. US Treasury Securities
Flow	Agencies	Private	LT	.–1982Q1	FFA Table F106.28, FU263061723, Agency and GSE
Flow	Agencies	Private	LT	1981Q1–98Q1	BEA USIT Table 8b line B12. US agency bonds
Flow	Agencies	Private	LT	1998Q1–.	BEA USIT Table 8a line B30. US agency bonds
Flow	Agencies	Private	ST	.–2001Q1	difference between FFA F106.28 and BEA USIT T8

Table 6: Government debt data sources: flows

may take the form of straight debt, participation certificates, pass-through debt, and collateralized mortgage obligations. U.S. agency bonds exclude privately issued mortgage-backed securities even if the underlying collateral is government guaranteed." Table 8 only refers to long term securities.

Type	Instrument	Investor	Maturity	period	source	c
Pos.	All	All	All	.–1969	Old issues of Survey of Current Business	s
Pos.	Treasuries	Official	All	1970–76	SCB August 1984. US government securities (line 23)	a
Pos.	Agencies	Official	LT	1970-76	SCB August 1984. US government securities (line 24)	a
Pos.	Treasuries	Official	All	.–1976	BEA IIP Table 2 line 29	
Pos.	Agencies	Official	LT	.–1976	BEA IIP Table 2 line 30	
Pos.	Agencies	Official	ST	.–2001	difference between FFA L106.14 and BEA IIP T2.30	a
Pos.	Treasuries	Private	All	1970-76	SCB August 1984. US government securities (line 30)	a
Pos.	Treasuries	Private	All	1976–.	BEA IIP Table 2 line 36	
Pos.	Agencies	Private	All	1952–.	FFA Table L106.15 FL263061723.Q. Agency and GSE	c

Table 7: Government debt data sources: positions

Positions Note that the Flow of Fund series on agency holdings by foreigners does not provide a breakdown between short term and long term. We assume that before 2001, this series only include long term agencies, as is likely since there was no separate information on short term agency holdings at the time.

Capital gains Long term capital gain computed the same way as for debt assets on government bonds. Short term is zero, and share of short and long term bonds is taken in US Treasury Surveys on Foreign Holdings of US Securities. Source data (GFD): USA 10-year Bond Constant Maturity Yield. Sources: 10 year bonds are used beginning in 1941. Yields on Treasury nominal securities at 'constant maturity' are interpolated by the U.S. Treasury from the daily yield curve for non-inflation-indexed Treasury securities. This curve, which relates the yield on a security

to its time to maturity, is based on the closing market bid yields on actively traded Treasury securities in the over-the-counter market. These market yields are calculated from composites of quotations obtained by the Federal Reserve Bank of New York. The constant maturity yield values are read from the yield curve at fixed maturities, currently 1, 3 and 6 months and 1, 2, 3, 5, 7, 10 and 20 years. This method provides a yield for a 10-year maturity, for example, even if no outstanding security has exactly 10 years remaining to maturity. Similarly, yields on inflation-indexed securities at 'constant maturity' are interpolated from the daily yield curve for Treasury inflation protected securities in the over-the-counter market. The inflation-indexed constant maturity yields are read from this yield curve at fixed maturities, currently 5, 7, 10, and 20 years. Yields on treasury securities at constant, fixed maturity are constructed by the treasury department, based on the most actively traded marketable Treasury securities. Yields on these issues are based on composite quotes reported by U.S. government securities dealers to the Federal Reserve Bank of New York. To obtain the constant maturity yields, personnel at treasury construct a yield curve each business day and yield values are then read from the curve at fixed maturities.

B.2.3 Corporate Debt

For corporate debt flows and positions, we need to exclude holdings of LT agencies by private foreign residents.

Flows

- Before 1982, FFA U.S. corporate bonds (Table F106.30, FU263063005.Q, includes net issues by Netherland Antillean financial subsidiaries of U.S. corporations) minus our own estimate of net issues of corporate bonds from Netherland Antillean financial subsidiaries of U.S. corporations. This estimate is the difference between minus FFA-based direct investment assets (Table F106 line 56, FU263192005.Q, U.S. direct investment abroad) and BEA-based direct investment assets (USIT Table 1 line 51), and is set to 0 before 1979Q1.
- Between 1982Q1 and 1997Q4, from BEA USIT Table 8b line B10 (corporate and other bonds) minus USIT Table 8b line B12 (agency debt) plus memo 3 (foreign official holdings of US corporate and other bonds)
- after 1998Q1; USIT Table 8a lines B16 minus USIT Table 8a plus memo 3 foreign official holdings of US and other corporate bonds.

Positions

- before 1969 from various SCB supplements. The positions need to be corrected for holdings of agency debt by private foreign residents, i.e. subtracting FFA Table L106.15 FL263061723.Q
- Between 1970 and 1975 IIP from SCB August 1984, line 32, corporate and other bonds + 0.32 times line 27 (other foreign official assets) minus FFA Table L106.15 FL263061723.Q (agency debt held by private agents). See paragraph on equity liability position for explanation of the coefficient 0.32.
- Between 1976 and 1981, BEA IIP table 2 line 38 + 0.32 times line 33 (other foreign official assets) minus FFA Table L106.15 FL263061723.Q (agency debt held by private agents).

- After 1982, BEA IIP Table 2 lines 33 and 37 minus position on equity liability, minus FFA Table L106.15 FL263061723.Q (agency debt held by private agents).

Capital gains From GFD Dow Jones Corporate Bond Price Index. Description: "There have been three Dow Jones Bond Averages. The original index was subdivided into four indices that were also kept on a daily basis back to 1915. It consisted of ten bonds from four groups: high-grade rails, second-grade rails, utility bonds and industrial bonds. The index was discontinued in its existing form on June 30, 1976 because of the reduction in the number of rail bonds and the new DJBA, which excludes railroad bonds, was introduced. A new Dow Jones Bond Average replaced the old bond average of 40 bonds in July 1976. The old bond average used a composite of 40 bonds and four sub-indices of 10 high-grade rails, 10 second-grade rails, 10 public utilities, and 10 industrials. The shrinkage in the number of railway bonds required the computation of a new average. These bond indices were discontinued on April 5, 2002 when the new Dow Jones Corporate Bond Averages were introduced. The Dow Jones Coporate Bond Return Index was introduced on April 5, 2002. Data were calculated back to 1997 with the base set at December 31, 1996 = 100. The index includes only bullet bonds that cannot be called, and is equally weighted rather than capitalization weighted. Separate indices are calculated for Industrials, Utilities and Finance, with sub-indices calculated for average maturities of 2, 5, 10, and 30 (17.5 years+)-year bonds. The index in theory includes 96 bonds, 32 from each sector. Historical prices and yields for the Dow Jones Bond Indices have been used to calculate the total return index on a weekly basis back to April 1915."

B.2.4 Direct Investment

Flows Before 1960Q1, FFA series (Table F106 line 37, FU263092001.Q, foreign direct investment in the U.S.). Between 1960Q1 and 1981Q4 from BEA (USIT Table 1 line 64). After 1982Q1 from BEA (USIT Table 1 line 64) from which we deduct reinvested earnings (USIT Table 7b line 45 before 1999Q1, then USIT Table 7a line 69). Indeed after 1982, since reinvested earnings are explicit, we deduct them from flows and incomes so as to count them only once in valuations.

Positions End of year levels from BEA. Year 1951 comes from Survey of Current Business (recorded on historical cost) and we do not benchmark until 1982 when data on market value starts being available from IIP Table 2 line 44 (A break in series in 1994 reflects the reclassification from the direct investment accounts to the nonbank investment accounts of intercompany debt positions between parent companies and affiliates that are not depository institutions and that are primarily engaged in financial intermediation. Estimates for 1976 forward are linked to the 1980, 1987, 1992, 1997, and 2002 benchmark surveys of foreign direct investment in the United States.)

Capital gains Same series as for Equity Liability after 1982. Before 1982 we proceed as on the asset side to avoid double counting reinvested earnings (see FDI assets).

In order to get values before 1982, we regress the evolution of direct investment implicit return (ie the discrepancy between BEA recorded positions and flows, divided by position) against the estimated capital gain above, after 1982 (regression between 1982 and 2007 to avoid crisis period). We find that $1 + cg = 0.951(1 + CG)$ where cg denotes the implicit BEA capital gain and CG the

S & P capital gain. We use this coefficient to scale down the estimated capital gain before 1982, corresponding to the fact that before 1982, reinvested earnings are part of financial and income flows.

B.2.5 Other

Flows

- Before 1960Q1: constructed from FFA to match the BEA definition. FFA F106 line 13 (FU264090005.Q net acquisition of financial assets) minus calculated equity, direct investment and debt (corporate and government) flows. Then subtract FFA F106 line 14 and 5 (FU263011005.Q).
- After 1960Q1: from BEA Table 1 line 55 (foreign owned assets in the US) minus calculated flows on equity, direct investment and debt.

Positions

- Before 1970: from various SCB and SCB supplements
- Between 1970 and 1975: from August 1984 SCB.
- After 1976: from BEA IIP Table 2 line 31 (Other US government liabilities), line 32 (US liabilities to foreign official assets reported by US banks), line 40 (US currency), line 41 (US liabilities to unaffiliated foreigners) and line 42 (US liabilities reported by US banks not included elsewhere)

Capital gains None.

B.3 Computation of Quarterly Returns: Disaggregation of Income Flows

Income flows are available for each category after 1986 but we need to disaggregate them before 1986. We therefore compute for each category an estimated yield, and then before 1982 (and between 1982 and 1986 for all categories except Direct Investment) we split the aggregate income according to shares of each category's estimated income (position mutiplied by yield).

B.3.1 Yield estimation before 1986 through disaggregation of income flow

We review in the following paragraphs the series taken to compute estimated yields before 1986.

Equity Asset Countries and positions are the same as for capital gain on equity asset. Yields are taken from GFD: UK FT-Actuaries Dividend Yield, Tokyo SE Dividend Yield, Netherlands CBS All x/Royal Dutch Dividend Yield, France Dividend Yield, Canada S & P/TSX-300 Dividend Yield, Germany Dividend Yield, Mexico SE Dividend Yield, Switzerland Dividend Yield, Hong Kong Hang Seng Index Dividend Yield, Australia ASX Dividend Yield, Italy Dividend Yield, Brazil Dividend Yield.

Debt Asset Income:

- Before 1982Q1: share of total income receipts from Table 1 line 13 corrected for reinvested earnings. The share is the weight of debt assets in position* yields (over equity, debt and other assets).
- Between 1982Q1 and 1986Q1: share of total income receipts from Table 1 line 13 corrected for reinvested earnings, minus estimated FDI asset income. The share is the weight of debt assets in position* yields (over equity, debt and other assets)
- Between 1986Q1 and 1999Q1: from IFS series 1112350..9... (PI BONDS AND NOTES INCOME: CRE).
- After 1999Q1: from BEA Table 4 line 8 (Interest). According to [CEM] p43: "For income receipts, estimates are prepared for interest and dividends received from foreigners on U.S. holdings of foreign long-term debt and equity securities, interest received by banks and securities brokers from foreigners, and interest received by U.S. non- banks from foreigners."

The methodology is the same as described for capital gain on debt asset. Countries' yield series are derived from Barclays Eurodollar Redemption Yield series and 10-year government bonds yield series.

Direct Investment Asset Income:

- Before 1982Q1: Share of total income receipts from Table 1 line 13 corrected for an estimate of reinvested earnings (more details needed here cg vs. CG) to which the estimated reinvested earnings is added back. The share is the weight of direct investment in position * yields.
- After 1982Q1: BEA Table 1 line 14 (Income receipts on direct investment abroad) minus reinvested earnings from BEA Table 7a/b line 4.

The methodology is the same as described for capital gain on direct investment abroad. Yields are taken from GFD: same yields as for equity asset, and Belgium SE Dividend Yield, Ireland Dividend Yield, Singapore Se Dividend Yield, Madrid SE Dividend Yield, Venezuela Dividend Yield.

Gold Asset None.

Other (excluding Gold) Asset Income:

- Before 1982Q1: share of total income receipts from Table 1 line 13 corrected for reinvested earnings. The share is the weight of other assets in position* yields (over equity, debt and other assets).
- Between 1982Q1 and 1986Q1: share of total income receipts from Table 1 line 13 corrected for reinvested earnings, minus estimated FDI asset income. The share is the weight of other assets in position* yields (over equity, debt and other assets)

- Between 1986Q1 and 1999Q1: From IFS BOPS series 1112370..9... (OTHER INVESTMENT INCOME: CRE) [Note that this series aggregates other, equity and debt before 1986Q1]
- After 1999Q1: Defined as a residual to total income receipts from Table 1 line 13,minus reinvested earnings from BEA Table 7a/b line 4 and estimated flows on debt, equity and direct investment. Equivalently, this is equal to investment income on US government receipts (BEA Table 4 line 13), Interest on claims reported by nonbanking concerns (Table 4 line 12), and interest on claims reported by banks (Table 4 line 10).

Yield from GFD on USA 3-month commercial paper.

Equity Liability Yield from GFD, SYUSAYM.

Government Debt Liability Income:

- Before 1960Q1: estimated as a share of total income payments (Table 1 line 30) minus estimated reinvested earnings. The share is the weight of government debt assets in position* yields.
- Between 1960Q1 and 1999Q1: BEA Table 1 line 33 (US Government payments). The ST agencies are less than 1% of government payments as of 1999, so we neglect them before.
- After 1999Q1: BEA Table 1 line 33 (US Government payments) + share of BEA Table 4 line 24 (Interest on liabilities for customers liabilities). The share is estimated as the ratio ST Agency securities/(Oth ST Neg secs + Other Custody Liab in dollars) from Treasury Survey (TIC) data, and is constant before 2001. This additional term proxies the income on ST agencies (not included in line 33)

From GFD, yield on 10-year government bond and on 3-month Treasury bill; weighted as for capital gain, between short-term and long-term.

Corporate Debt Liability Income:

- Before 1960Q1: estimated as a share of total income payments (Table 1 line 30) minus estimated reinvested earnings. The share is the weight of corporate debt assets in position* yields
- Between 1960Q1 and 1973Q1: estimated as a share of total income payments (Table 1 line 30) minus estimated reinvested earnings and government debt income. The share is the weight of corporate debt assets in position* yields (for equity, DI, other and corporate debt).
- Between 1973Q1 and 1982Q1: estimated as a share of total income payments (Table 1 line 30) minus estimated reinvested earnings, government debt income and other income. The share is the weight of corporate debt assets in position* yields (for equity, DI, and corporate debt).

- Between 1982Q1 and 1986Q1: estimated as a share of total income payments (Table 1 line 30) minus estimated reinvested earnings, minus FDI income and government debt income. The share is the weight of corporate debt assets in position* yields (for equity, other and corporate debt).
- Between 1986Q1 and 1999Q1: from IFS BOPS series 1113354..9... (PI OTH SECT INCOME BONDS & NOTE: DEB)
- After 1999Q1: from BEA Table 4 line 21 (Interest)

From GFD, Dow Jones Corporate Bond Yield.

Direct Investment Liability Same as Yield on Equity Liability.

Other Liability income:

- Before 1960Q1: estimated as a share of total income payments (Table 1 line 30) minus estimated reinvested earnings. The share is the weight of corporate debt assets in position* yields.
- Between 1960Q1 and 1981Q4: estimated as a share of total income payments (Table 1 line 30) minus estimated reinvested earnings and government debt income. The share is the weight of corporate debt assets in position* yields (for other, corporate debt, equity and di)
- Between 1982Q1 and 1985Q4: estimated as a share of total income payments (Table 1 line 30) minus government debt income and FDI income. The share is the weight of corporate debt assets in position* yields (for other, corporate debt and equity)
- Between 1986Q1 and 1998Q4: defined as residual. Calculated as Total income payments (BEA Table 1 line 30) minus reinvested earnings (Table 7a/b line 69) minus estimated equity, direct and debt income.
- After 1999Q1: defined as residual. Calculated as Total income payments (BEA Table 1 line 30) minus reinvested earnings (Table 7a/b line 69) minus estimated equity, direct and debt income.

Yield from GFD on USA 3-month commercial paper.

B.3.2 Yield series after 1986

After 1986 (1982 for Direct Investment), yield series are the following, extracted from IMF Balance Of Payments Statistics (BOPS):

Asset - Equity: B2340@C111.Q

- Debt: B2350@C111.Q. This series contains yield on short term and long term debt, as can be seen after 1999 on BEA USIT Table 4 'Foreign Securities'. Since BEA data on the whole sample only concern long term debt (both for positions and flows), we scale down the BOPS series according to the estimated yields on long term and on short term debt; from the US Treasury reports on US holdings of foreign securities, we assume a share of 21% of short term debt contained in the debt position considered by IMF.
- Direct Investment: B2330@C111.Q (minus reinvested earnings, as identified in flow series)
- Other: B2370@C111.Q (plus B2360@C111.Q after 2003) and the short term part of B2350@C111.Q as described above.

Liability - Equity: B3340@C111.Q
- Government Debt: B3352@C111.Q
- Corporate Debt: B3354@C111.Q
- Direct Investment: B3330@C111.Q (minus reinvested earnings, as identified in flow series)
- Other: B3370@C111.Q (plus B3360@C111.Q after 2003)

C Reallocating the Other Change Term

We reallocate the residual between flows, valuations and initial positions.

C.1 Benchmark of Other Change with BEA

In order to do proper reallocations, we first benchmark our valuations to those of the BEA whenever they are available. This is possible starting end 1989 thanks to Gohrband & Howell (2011) for the following categories:

Asset Debt, Equity, Direct Investment.

Liability Equity, Direct Investment.

For the categories mentioned above, each year after 1989 we set our other change to that of the BEA. For more precision, the difference on valuation between our quarterly estimates and the annual BEA estimate, is split according to the share of the absolute value of estimated valuations on each quarter (same idea as for the reallocation methodology below).

For all other categories, no breakdown is provided, therefore we have the choice between keeping our valuations (and hence our other change), or re-aggregate some categories (Government Debt, Corporate Debt and Other Liability on one side; Long term Debt, Gold and Other Asset on the other side).

C.2 Reallocation Methodology

We start by writing the stock-flow equation:

$$PX_t^i = PX_{t-1}^i + FX_t^i + VX_t^i + OC_t^i$$

Since data are quarterly and residuals only appear on end of years, we have to write this formula for the quarter before (no residual if t is end of year) :

$$PX_t^i = \left(PX_{t-2}^i + FX_{t-1}^i + VX_{t-1}^i\right) + FX_t^i + VX_t^i + OC_t^i$$

And hence, on four quarters:

$$PX_t^i = PX_{t-4}^i + \Sigma_{q=0}^3 FX_{t-q}^i + \Sigma_{q=0}^3 VX_{t-q}^i + OC_t^i$$

Here is how the reallocation takes place: the user chooses, for each year t, the shares a_t, b_t and c_t of the residual, with $a_t + b_t + c_t = 1$, to reallocate respectively on flows, valuations and initial position. This is an annual residual to reallocate among quarterly series; for more precision, for each series we choose to split the reallocated residual according to the share of the absolute value on the quarter over the sum of absolute values of the four quarters.

Then the final series after reallocation are for $q \in \{0, 1, 2, 3\}$

$$FX_{t-q}^{i'} = FX_{t-q}^i + a_t OC_t^i \frac{\mid FX_{t-q}^i \mid}{\Sigma_{p=0}^3 \mid FX_{t-p}^i \mid}$$

$$VX_{t-q}^{i'} = VX_{t-q}^i + b_t OC_t^i \frac{\mid VX_{t-q}^i \mid}{\Sigma_{p=0}^3 \mid VX_{t-p}^i \mid}$$

There remains

$$PX_t^i = PX_{t-4}^i + \Sigma_{q=0}^3 FX_{t-q}^{i'} + \Sigma_{q=0}^3 VX_{t-q}^{i'} + c_t OC_t^i$$

Therefore

$$PX_{t-4}^{i'} = PX_{t-4}^i + c_t OC_t^i$$

which can be backwards written for all quarters: $\forall k \geq 0$

$$PX_{T-k-1}^{i'} = PX_{T-k}^i - FX_{T-k}^{i'} - VX_{T-k}^{i'}$$

D Treatment of offshore financial centers

The main offshore centers are in our sample. Though the reporting is spotty (see **?** for a thorough study) there are some important cross-border positions between some offshore centers and advanced economies. It is very unlikely that the ultimate owners of financial assets bought by

offshore centers are actual residents of off-shore centers. Rather, offshore centers act as intermediaries to channel funds across the globe, reflecting, among other things, tax "optimization" and tax evasion. **?** shows that a significant amount of rich countries wealth seem to evaporate via those channels. Because, by design, the traceability of the geography of financial flows emanating from and going into offshore centers is limited, it sis probably the case that some financial transactions are done to go around domestic fiscal authorities, legally or illegally. Hence most of those transactions are really domestic transactions intermediated offshore. For US debt assets where the Cayman islands and Bermudas are important destinations and we lack good bond price data, we apply the same rate of return as US debt. For FDI capital gains we include Bahamas, Cayman, Panama shares valued with local stock market indices. For equity assets, we apply the cut-off rule described above.

E Characterization of the model

E.1 Approximate analytical results of the model without disasters

As a starting point, consider the case without disaster shocks ($b = p_d = 1 - p_n = 0$). For small ϵ shocks around the steady state, we can obtain approximate the solution to the planner's problem and characterize analytically the properties of the allocation.

Allocation, and Asset Returns. For small ϵ shocks, the return on any traded asset R_{t+1}, in terms of tradable, satisfies approximately

$$\ln E_t R_{t+1} \approx -\ln E_t M_{t,t+1} - cov_t\left(\hat{m}_{t,t+1}, \hat{r}_{t+1}\right),$$

where $\hat{z}$ denotes the log-deviation of variable Z_t from its steady state $\bar{Z}$. It follows that the difference in log-expected return between two assets R^i_{t+1} and R^j_{t+1} satisfies

$$\ln E_t R^j_{t+1} - \ln E_t R^i_{t+1} \approx cov_t\left(\hat{m}_{t,t+1}, \hat{r}^i_{t+1}\right) - cov_t\left(\hat{m}_{t,t+1}, \hat{r}^j_{t+1}\right). \tag{22}$$

As Hassan (2009) noted, while the returns R^i and R^j are measured in terms of tradable, the excess return is invariant to the choice of numeraire. Assuming that a country's risk-free government bond is simply an asset that pays that country's price index, and applying (22), the expected excess return between domestic and foreign real risk-free interest rates is[34]

$$\begin{aligned} \ln E_t R^{*f}_{t+1} - \ln E_t R^f_{t+1} &= cov_t\left(\hat{m}_{t,t+1}, \hat{p}_{t+1}\right) - cov_t\left(\hat{m}_{t,t+1}, \hat{p}^*_{t+1}\right) \\ &= (1-\gamma)\, cov_t\left(\hat{m}_{t,t+1}, \hat{q}_{t+1} - \hat{q}^*_{t+1}\right), \end{aligned} \tag{23}$$

where the second line makes use of the log-approximation $\hat{p} = (1-\gamma)\,\hat{q}$. This expression makes clear that domestic real interest rates will –on average– be lower when the ratio of domestic to foreign non traded price is positively correlated with the stochastic discount factor, that

[34] To see this, observe that, in terms of tradables, $R^f_{t+1} = P_{t+1}/E_t\left[M_{t,t+1}P_{t+1}\right]$ with a similar definition for R^{*f}_{t+1}. Although each real interest rate is risk free in terms of its own consumption price index, they are not riskless in terms of the tradable good, or any common numeraire.

is when times of relative scarcity ($\hat{m} > 0$) are also times when domestic non-traded prices are high ($\hat{q} > \hat{q}^*$).

As discussed in the empirical section, one definition of the 'exorbitant privivege' is that the U.S. experiences a lower risk free return: $\ln E_t R^{*f}_{t+1} - \ln E_t R^f_{t+1} > 0$. Can our model of risk-sharing reproduce this feature? Log-linearizing equations (7), (11) and (12), appendix B derives the following equilibrium allocations in deviation from steady state:

$$\begin{aligned}
\hat{x}_t &= \frac{\gamma\theta\left(\sigma-\sigma^*\right)\widehat{\widehat{y}}^T_t + (1-\gamma)\left(\left(\sigma\theta-1\right)\hat{y}^N_t - \left(\sigma^*\theta-1\right)\hat{y}^{*N}_t\right)}{1+\gamma\left(\sigma\theta-1\right)(1-\alpha)+\gamma\left(\sigma^*\theta-1\right)\alpha} \\
\hat{c}^T_t &= \widehat{\widehat{y}}^T_t - (1-\alpha)\,\hat{x}_t \;;\; \hat{c}_t = \gamma\hat{c}^T_t + (1-\gamma)\,\hat{y}^N_t \\
\hat{q}_t &= \frac{1}{\theta}\left(\hat{c}^T_t - \hat{y}^N_t\right) \;;\; \hat{p}_t = (1-\gamma)\,\hat{q}_t \\
\hat{m}_{t,t+1} &= (1/\theta-\sigma)\left(\hat{c}_{t+1}-\hat{c}_t\right) - 1/\theta\left(\hat{c}^T_{t+1}-\hat{c}^T_t\right).
\end{aligned}$$

After simple but tedious algebra, one can show that expected excess return on risk free bonds can be expressed as

$$\begin{aligned}
\ln E_t R^{*f}_{t+1} - \ln E_t R^f_{t+1} = \quad & \left(\left(1-\gamma\right)/\left[1+\left(\sigma\theta-1\right)\gamma\left(1-\alpha\right)+\left(\sigma^*\theta-1\right)\gamma\alpha\right]\right)^2 \frac{\sigma^2_\epsilon}{\theta} \\
& \left[-\gamma\left(\sigma^*-\sigma\right)\left(1+\left(\sigma\theta-1\right)\gamma\right)\left(1+\left(\sigma^*\theta-1\right)\gamma\right)/\left(1-\gamma\right)\right. \\
& +\alpha\left(\sigma\theta-1\right)\left(1+\left(\sigma^*\theta-1\right)\gamma\right)\left(\sigma+\left(\sigma^*-\sigma\right)\gamma\alpha\right) \\
& \left.-\left(1-\alpha\right)\left(\sigma^*\theta-1\right)\left(1+\left(\sigma\theta-1\right)\gamma\right)\left(\sigma^*-\left(\sigma^*-\sigma\right)\gamma\left(1-\alpha\right)\right)\right].
\end{aligned} \tag{24}$$

The first term inside the brackets reflects the covariance between the marginal utility of wealth and the relative price of non-traded goods, conditional on shocks to traded output. It is proportional to the difference in risk aversion $(\sigma^* - \sigma)$ and is always negative: when the global output of traded goods declines ($\widehat{\widehat{y}}^T_t < 0$), domestic consumption of traded goods fall more than foreign consumption ($\hat{x} > 0$) because foreign is more risk averse. Consequently, the price of domestic non-traded goods falls more than the price of foreign non-traded goods: $\hat{q} < \hat{q}^*$ (non-traded goods are relatively more abundant at home than abroad). The domestic price index declines more than the foreign one, hence foreign real bonds provide a comparatively better hedge. This effect is scaled by the size of the traded sector γ, disappears when home and foreign have the same risk appetite ($\sigma = \sigma^*$) and is independent of size α. The second term in brackets reflects the effect of shocks to domestic non-traded output y^N. A decline in the endowment of domestic non-traded goods $\left(\hat{y}^N < 0\right)$ increases the domestic marginal utility of traded goods when the goods are gross substitutes ($\sigma\theta > 1$). Home traded consumption increases ($\hat{x} < 0$), so the relative price of domestic non-traded goods increases: $\hat{q} > \hat{q}^*$ (domestic non-traded are relatively scarcer at home) and the domestic real bond provides a good hedge. The last term represents the effect of shocks to the endowment of foreign non-traded. By a similar argument, the foreign bond provides a good hedge against shocks to the foreign non-traded good. The last two terms are proportional to the size of the home and foreign country (α and $(1-\alpha)$). This extends the

results from Hassan (2009) to the case where countries have different risk appetites.[35]

E.2 General Model: Portfolio weights

Portfolio equations. Consider the following notations: W_t^h is the beginning of period (cum-dividend) claim to domestic consumption expenditures (tail asset). It satisfies:

$$W_t = P_t^h C_t^h + E_t \left[\mathcal{M}_{t,t+1} W_{t+1} \right]$$

Similarly, define V_t^h as the cum-dividend claim to domestic output, i.e.

$$V_t = y_t^T + q_t y_t^N + E_t \left[\mathcal{M}_{t,t+1} V_{t+1} \right]$$

We assume that only a fraction $0 \leq \delta \leq 1$ of V_t can be traded. By varying δ we can control the size of the net equity position (relative to output).

Define beginning of period net foreign assets as

$$\begin{aligned} NA_t &= W_t - V_t \\ &= P_t^h C_t^h - y_t^T - q_t y_t^N + E_t \left[\mathcal{M}_{t,t+1} NA_{t+1} \right] \\ &= -NX_t + E_t \left[\mathcal{M}_{t,t+1} NA_{t+1} \right] \end{aligned}$$

The return on domestic wealth is:

$$R_{t+1}^w = \frac{W_{t+1}}{W_t - P_t^h C_t^h} = \frac{\tilde{W}_{t+1} + P_{t+1}^h C_{t+1}^h}{\tilde{W}_t}$$

where $\tilde{W}_t$ is the end of period price of the tail asset (ex-dividend), defined as:

$$\tilde{W}_t = W_t - P_t^h C_t^h$$

so that, substituting into the law of motion for W_t, we obtain:

$$\tilde{W}_t R_{t+1}^w = W_{t+1}$$

A similar return on the foreign consumption tail asset can be defined as

$$R_{t+1}^{w*} = \frac{W_{t+1}^*}{W_t^* - P_t^f C_t^f} = \frac{\tilde{W}_{t+1}^* + P_{t+1}^f C_{t+1}^f}{\tilde{W}_t^*}$$

[35] When $\sigma = \sigma^*$, (24) collapses to Hassan (2009)'s formula

$$\ln E_t R_{t+1}^{*f} - \ln E_t R_{t+1}^f = \frac{\sigma \left(\sigma - 1/\theta \right) \left(1 - \gamma \right)^2}{1 + \left(\sigma\theta - 1 \right) \gamma} \left(2\alpha - 1 \right) \sigma_\epsilon^2$$

The general idea is to map the returns R^w_{t+1} and R^{w*}_{t+1} into an existing menu of assets, while satisfying resource constraints. Suppose we can write the following projection:

$$R^w_{t+1} = b_t R_{t+1} + b^g_t R^g_{t+1} + b^{hb}_t R^{hb}_{t+1} + b^{fb}_t R^{fb}_{t+1}$$

where R^g_{t+1} denotes the return to a global equity claim:

$$\begin{aligned} R^g_{t+1} &= \frac{V^g_{t+1}}{V^g_t - \bar{y}^T - \alpha q_t y^N_t - (1-\alpha) q^*_t y^{*N}_t} \\ &= \frac{\alpha \tilde{V}_t}{\tilde{V}^g_t} R_{t+1} + \frac{(1-\alpha)\tilde{V}^*_t}{\tilde{V}^g_t} R^*_{t+1} \end{aligned}$$

where $\tilde{V}_t$ is the ex-dividend value of domestic equity:

$$\tilde{V}_t = V_t - y^T_t - q_t y^N_t$$

similarly

$$\tilde{V}^*_t = V^*_t - y^{*T}_t - q^*_t y^{*N}_t = V^*_{t+1}/R^*_{t+1},$$

and

$$V^g_t = \alpha V_t + (1-\alpha) V^*_t$$

is the (cum dividend) value of a claim to global equity, similarly

$$\tilde{V}^g_t = V^g_t - \bar{y}^T_t - \alpha q_t y^N_t - (1-\alpha) q^*_t y^{*N}_t$$

while

$$R_{t+1} = \frac{V_{t+1}}{V_t - y^T_t - q_t y^N_t},$$

$$R^*_{t+1} = \frac{V^*_{t+1}}{V^*_t - y^{*T}_t - q^*_t y^{*N}_t}$$

are the returns to the domestic and foreign trees respectively.

Suppose $\tilde{W}_t$ is invested as follows (where the first term represents the non-traded equities):

$$\tilde{W}_t = (1-\delta)\tilde{V}_t + \delta s^g_t \tilde{V}^g_t + s^{hb}_t \tilde{P}^{hb}_t + s^{fb}_t \tilde{P}^{fb}_t$$

Substituting, we obtain:

$$W_{t+1} = (1-\delta) R_{t+1} \tilde{V}_t + \delta s^g_t R^g_{t+1} \tilde{V}^g_t + s^{hb}_t R^{hb}_{t+1} \tilde{P}^{hb}_t + s^{fb}_t R^{fb}_{t+1} \tilde{P}^{fb}_t = R^w_{t+1} \tilde{W}_t$$

and therefore:

$$(1-\delta) = b_t \frac{\tilde{W}_t}{\tilde{V}_t}$$
$$s_t^g = b_t^g \frac{\tilde{W}_t}{\tilde{V}_t^g}$$
$$s_t^{hb} = b_t^{hb} \frac{\tilde{W}_t}{P_t^{hb}}$$
$$s_t^{fb} = b_t^{fb} \frac{\tilde{W}_t}{P_t^{fb}}$$

It follows that

$$b_t + b_t^g + b_t^{hb} + b_t^{fb} = 1$$

So we can run the following regression, imposing this restriction (e.g. subtracting domestic bond returns):

$$\left(R_{t+1}^w - R_{t+1}^{hb}\right) - b_t \left(R_{t+1} - R_{t+1}^{hb}\right) = b_t^g (R_{t+1}^g - R_{t+1}^{hb}) + b_t^{fb} (R_{t+1}^{fb} - R_{t+1}^{hb})$$

But we must also impose the resource constraint that the home and foreign portfolios must add up to the total resources.
Let's write the foreign projection as:

$$\left(R_{t+1}^{*w} - R_{t+1}^{hb}\right) - b_t^* \left(R_{t+1}^* - R_{t+1}^{hb}\right) = b_t^{*g} (R_{t+1}^g - R_{t+1}^{hb}) + b_t^{*fb} (R_{t+1}^{fb} - R_{t+1}^{hb})$$

Define total wealth $W_t^g = \alpha W_t + (1-\alpha) W_t^*$, and $\tilde{W}_t^g = \alpha \tilde{W}_t + (1-\alpha)\tilde{W}_t^*$. Substituting from the portfolio allocation, we have:

$$\tilde{W}_t^g = \alpha \left((1-\delta)\tilde{V} + \delta s_t^g \tilde{V}_t^g + s_t^{hb}\tilde{P}_t^{hb} + s_t^{fb}\tilde{P}_t^{fb}\right) + (1-\alpha)\left((1-\delta)\tilde{V}_t^* + \delta s_t^{*g}\tilde{V}_t^g + s_t^{*hb}\tilde{P}_t^{hb} + s_t^{*fb}\tilde{P}_t^{fb}\right) = \tilde{V}_t^g$$

from which it follows that

$$\alpha s_t^g + (1-\alpha) s_t^{*g} = 1$$
$$\alpha s_t^{hb} + (1-\alpha) s_t^{*hb} = 0$$
$$\alpha s_t^{fb} + (1-\alpha) s_t^{*fb} = 0$$

Substituting, we obtain:

$$\alpha b_t^g \tilde{W}_t + (1-\alpha) b_t^{*g} \tilde{W}_t^* = \delta \tilde{V}_t^g$$
$$\alpha b_t^{hb} \tilde{W}_t + (1-\alpha) b_t^{*hb} \tilde{W}_t^* = 0$$
$$\alpha b_t^{fb} \tilde{W}_t + (1-\alpha) b_t^{*fb} \tilde{W}_t^* = 0$$

Now compute the *size-adjusted* excess return:

$$\alpha\frac{\tilde{W}_t}{\delta\tilde{V}_t^g}\left[\left(R_{t+1}^w - R_{t+1}^{hb}\right) - b_t\left(R_{t+1} - R_{t+1}^{hb}\right)\right] - (1-\alpha)\frac{\tilde{W}_t^*}{\delta\tilde{V}_t^g}\left[\left(R_{t+1}^{*w} - R_{t+1}^{hb}\right) - b_t^*\left(R_{t+1}^* - R_{t+1}^{hb}\right)\right] =$$
$$\alpha\frac{\tilde{W}_t}{\delta\tilde{V}_t^g}\left(b_t^g(R_{t+1}^g - R_{t+1}^{hb}) + b_t^{fb}(R_{t+1}^{fb} - R_{t+1}^{hb})\right) - (1-\alpha)\frac{\tilde{W}_t^*}{\delta\tilde{V}_t^g}\left(b_t^{*g}(R_{t+1}^g - R_{t+1}^{hb}) + b_t^{*fb}(R_{t+1}^{fb} - R_{t+1}^{hb})\right)$$

Substituting the expression for b_t^{*g} and b_t^{*fb}, we get:

$$\alpha\frac{\tilde{W}_t}{\delta\tilde{V}_t^g}\left[\left(R_{t+1}^w - R_{t+1}^{hb}\right) - b_t\left(R_{t+1} - R_{t+1}^{hb}\right)\right] - (1-\alpha)\frac{\tilde{W}_t^*}{\delta\tilde{V}_t^g}\left[\left(R_{t+1}^{*w} - R_{t+1}^{hb}\right) - b_t^*\left(R_{t+1}^* - R_{t+1}^{hb}\right)\right] =$$
$$\left(\alpha\frac{\tilde{W}_t}{\delta\tilde{V}_t^g}b_t^g - (1-\alpha)\frac{\tilde{W}_t^*}{\delta\tilde{V}_t^g}b_t^{*g}\right)(R_{t+1}^g - R_{t+1}^{hb}) + \left(\alpha\frac{\tilde{W}_t}{\delta\tilde{V}_t^g}b_t^{fb} - (1-\alpha)\frac{\tilde{W}_t^*}{\delta\tilde{V}_t^g}b_t^{*fb}\right)(R_{t+1}^{fb} - R_{t+1}^{hb}) =$$
$$\left(2\alpha\frac{\tilde{W}_t}{\delta\tilde{V}_t^g}b_t^g - 1\right)(R_{t+1}^g - R_{t+1}) + \left(2\alpha\frac{\tilde{W}_t}{\delta\tilde{V}_t^g}b_t^{fb}\right)(R_{t+1}^{fb} - R_{t+1}^{hb})$$
$$= \beta_1(R_{t+1}^g - R_{t+1}) + \beta_2(R_{t+1}^{fb} - R_{t+1}^{hb})$$

This weighted regression recovers b_t^g and b_t^{fb} and imposes the resource constraint.

Imposing short sales constraint

Strategy

Define $y_t = \alpha\frac{\tilde{W}_t}{\delta\tilde{V}_t^g}\left[\left(R_{t+1}^w - R_{t+1}^{hb}\right) - b_t\left(R_{t+1} - R_{t+1}^{hb}\right)\right] - (1-\alpha)\frac{\tilde{W}_t^*}{\delta\tilde{V}_t^g}\left[\left(R_{t+1}^{*w} - R_{t+1}^{hb}\right) - b_t^*\left(R_{t+1}^* - R_{t+1}^{hb}\right)\right]$, and $x_t = (R_{t+1}^g - R_{t+1}^{hb}, R_{t+1}^{fb} - R_{t+1}^{hb})'$. The previous regression can be expressed as

$$y_t = x_t'\beta$$

where

$$\beta_1 = 2\alpha\frac{\tilde{W}_t}{\delta\tilde{V}_t^g}b_t^g - 1 \quad ; \quad \beta_2 = 2\alpha\frac{\tilde{W}_t}{\delta\tilde{V}_t^g}b_t^{fb}$$

which we can solve for b_t^g and b_t^{fg} as a function of β:

$$(1+\beta_1)\frac{\tilde{V}_t^g}{\tilde{W}_t}\frac{\delta}{2\alpha} = b_t^g \quad ; \quad \beta_2\frac{\tilde{V}_t^g}{\tilde{W}_t}\frac{\delta}{2\alpha} = b_t^{fb}$$

We impose the constraints directly on the OLS estimates. The general problem is of the fol-

lowing form:

$$\begin{aligned} \min_{\beta} \quad & \frac{1}{2}(y - x\beta)'(y - x\beta) \\ \text{s.t.} \quad & C\beta \geq D \end{aligned}$$

This can be expressed as a Linear Quadratic problem:

$$\begin{aligned} \min_{\beta} \quad & \frac{1}{2}\beta'(X'X)\beta - \beta'X'Y + \frac{1}{2}Y'Y \\ \text{s.t.} \quad & C\beta \geq D \end{aligned}$$

The solution is:

$$\hat{b} = \hat{b}_{OLS} + (X'X)^{-1}C(C(X'X)^{-1}C')^{-1}(D - C\hat{b}_{OLS})$$

We implement this directly in GAUSS with QProg.
The short sale constraints are the following:

- long on foreign debt: $s_t^{fb} \geq 0$ or equivalently $b_t^{fb} \geq 0$ or $\beta_2 \geq 0$
- short on domestic debt: $s_t^{hb} < 0$ or equivalently $1 - b_t - b_t^g - b_t^{fg} \leq 0$ which can be rewritten as:

  $$1 - (1 - \delta)\frac{\tilde{v}_t}{\tilde{W}_t} - (1 + \beta_1 + \beta_2)\frac{\tilde{V}_t^g}{\tilde{W}_t}\frac{\delta}{2\alpha} \leq 0$$

  and solved for

  $$\frac{2\alpha}{\delta}\left[\frac{\tilde{W}_t}{\tilde{V}_t^g} - (1 - \delta)\right] - 1 \leq \beta_1 + \beta_2$$

- home has a long position in equity: $s_t^g > 0$ or:

  $$\beta_1 > -1$$

- foreign has a long position in equity: $s_t^{*g} > 0$ or

  $$\beta_1 < 1$$

Recovering the Gross External Portfolio Shares
After estimating the portfolio shares (b, b^g, b^{hb}, b^{fb}) or wealth shares $((1 - \delta), s^g, s^{hb}, s^{fb})$, we can

construct the net foreign wealth (end of period) as:

$$\begin{aligned}\tilde{NA}_t &= \tilde{W}_t - \tilde{V}_t \\ &= \left[(1-\delta)\tilde{V}_t + \delta s_t^g \tilde{V}_t^g + s_t^{hb}\tilde{P}_t^{hb} + s_t^{fb}\tilde{P}_t^{fb}\right] - \tilde{V}_t \\ &= (\alpha s_t^g - 1)\delta\tilde{V}_t + \delta s_t^g(1-\alpha)\tilde{V}_t^* + s_t^{hb}\tilde{P}_t^{hb} + s_t^{fb}\tilde{P}_t^{fb}\end{aligned}$$

Now, given the short-sale constraints, we have:

$$\alpha s_t^g - 1 \leq 0 \quad ; \quad s_t^{hb} \leq 0 \quad ; \quad (1-\alpha)s^g \geq 0 \quad ; \quad s^{fb} \geq 0$$

so the gross assets and liabilities positions can be defined as:

$$\tilde{A}_t = (1-\alpha)s_t^g \delta\tilde{V}_t^* + s_t^{fb}\tilde{P}_t^{fb}$$

$$\tilde{L}_t = \delta(1-\alpha s_t^g)\tilde{V}_t - s_t^{hb}\tilde{P}_t^{hb}$$

By varying δ we can control the size of the gross equity position relative to GDP.